Foundations of Artificial Intelligence

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Exercise Sheet 6
Due: Tuesday, June 16, 2009

Exercise 6.1 (Syntax and Semantics of Predicate Logic)

(a) Classify the following expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. In the expressions, a and b are constants, x and y are variables, f and g are functions, and P and Q are predicates.

    (a) $P(x, y)$ (d) $\mathcal{I}, \alpha \models P(a, f(x))$

    (b) $f(a, b)$ (e) $f(g(x), b)$

    (c) $\mathcal{I} \models P(a, f(b))$ (f) $Q(x)$ is satisfiable.

    (g) $\exists x (P(x, y) \wedge Q(x)) \vee P(y, x)$
    (h) $\forall x (\exists y (P(x, y) \wedge Q(x)) \vee P(x, y))$
    (i) $\forall x \forall y (P(x, y) \wedge Q(x) \vee P(f(y), x))$
    (j) $Q(x) \vee P(x, y) \equiv P(x, y) \vee Q(x)$

(b) Consider the following set of formulae:

$$\Theta = \left\{ \begin{array}{l} \forall x \neg P(x, x) \\ \forall x \forall y \forall z ((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z)) \\ \forall x \forall y (P(x, y) \vee x = y \vee P(y, x)) \end{array} \right\}$$

Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models \Theta$ (i.e., $\mathcal{I} \models F$ for all $F \in \Theta$). Why is it not necessary to specify a variable assignment α to state a model of Θ?

(c) Are there also models of Θ with an infinite $\mathcal{D}$?

Exercise 6.2 (Substitutions and Unification)

(a) Compute the substitutions

    (i) $P(x, y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,
    (ii) $P(x, y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,
    (iii) $P(x, y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$, and
    (iv) $P(x, y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals:
$\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$. In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 6.3 (Resolution in First Order Logic)

From "Horses are animals" it follows that "The head of a horse is the head of an animal". Demonstrate that this inference is valid by carrying out the following steps:

(a) Translate the premise and the conclusion into the language of first-order logic. Use three predicates: $HeadOf(h, x)$ (meaning "h is the head of x"), $Horse(x)$, and $Animal(x)$.

(b) Negate the conclusion, and convert the premise and the negated conclusion into clause form.

(c) Use first-order resolution to show that the conclusion follows from the premise.

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

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[1] http://www.informatik.uni-freiburg.de/~ki/teaching/ss09/gki/coverSheet-english.pdf