Foundations of Artificial Intelligence

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Exercise Sheet 5

Due: Tuesday, June 9, 2009

Exercise 5.1 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$

$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$

$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$

$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$

$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.
Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show that $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 5.2 (Davis-Putnam Procedure)

Use the *Davis-Putnam* procedure to find a model for each of the following formulae, or prove that the particular formula has no model. At each step, indicate which rule you have applied.

(a) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$

(b) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge \neg P \wedge \neg R \wedge \neg U$

Exercise 5.3 (Modelling in First-Order Logic)

Represent the following sentences in first-order logic, using a consistent vocabulary.

(a) Everyone who has a good sense of hearing is able to sing.

(b) Nobody is a real musician, if he is not able to inspire his audience.

(c) Nobody, except for a real musician can write a symphony.

Which properties does someone necessarily possess when he wrote a symphony?

Exercise 5.4 (Skolem Normal Form)

Transform the following formulae to Skolem normal form:

(a) $F_1 = \forall x(\exists y R(x,y) \land \exists y R(y,x))$

(b) $F_2 = \forall x \forall z(R(x,z) \Rightarrow \exists y(R(x,y) \land R(y,z)))$

(c) $F_3 = \forall x \exists z(R(x,z) \land \neg \exists y(R(x,y) \land R(y,z)))$

Exercise 5.5 (Herbrand Expansion)

Let $F = \forall x \forall y(P(x, f(x, g(y))) \land P(h(y), f(y,y)))$.

(a) State ten minimally large terms from the Herbrand universe of F.

(b) State five minimally large formulae from the Herbrand expansion of F.

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] `http://www.informatik.uni-freiburg.de/~ki/teaching/ss09/gki/coverSheet-english.pdf`