Foundations of AI

4. Informed Search Methods

Heuristics, Local Search Methods, Genetic Algorithms

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- Best-First Search
- A* and IDA*
- Local Search Methods
- Genetic Algorithms

Best-First Search

Search procedures differ in the way they determine the next node to expand.

Uninformed Search: Rigid procedure with no knowledge of the cost of a given node to the goal.

Informed Search: Knowledge of the cost of a given node to the goal is in the form of an *evaluation function* f or h, which assigns a real number to each node.

Best-First Search: Search procedure that expands the node with the "best" f- or h-value.

General Algorithm

```
function BEST-FIRST-SEARCH(problem, EVAL-FN) returns a solution sequence
   inputs: problem, a problem
           Eval-Fn, an evaluation function

   Queueing-Fn ← a function that orders nodes by EVAL-FN
   return GENERAL-SEARCH(problem, Queueing-Fn)
```

When *h* is always correct, we do not need to search!

Greedy Search

A possible way to judge the “worth” of a node is to estimate its distance to the goal.

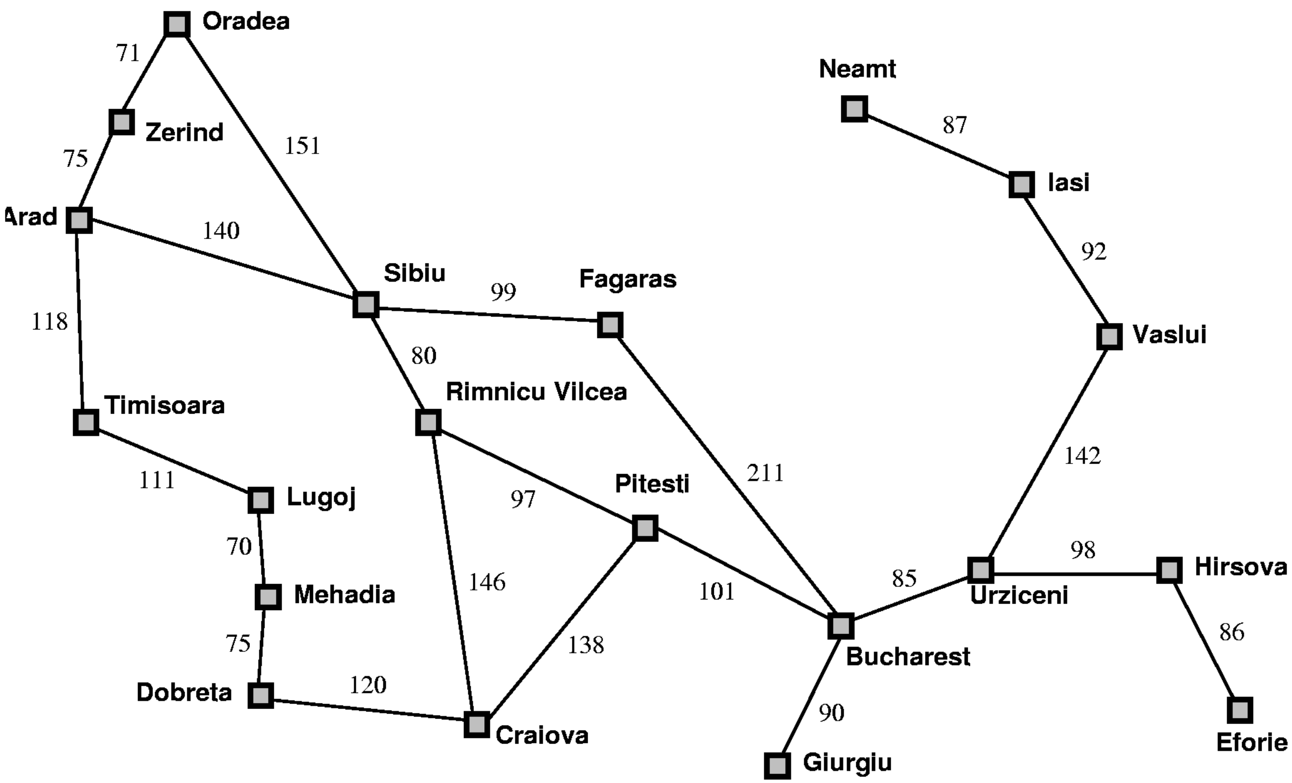
$$h(n) = \textit{estimated distance from } n \textit{ to the goal}$$

The only real condition is that $h(n) = 0$ if n is a goal.

A best-first search with this function is called a *greedy search.*

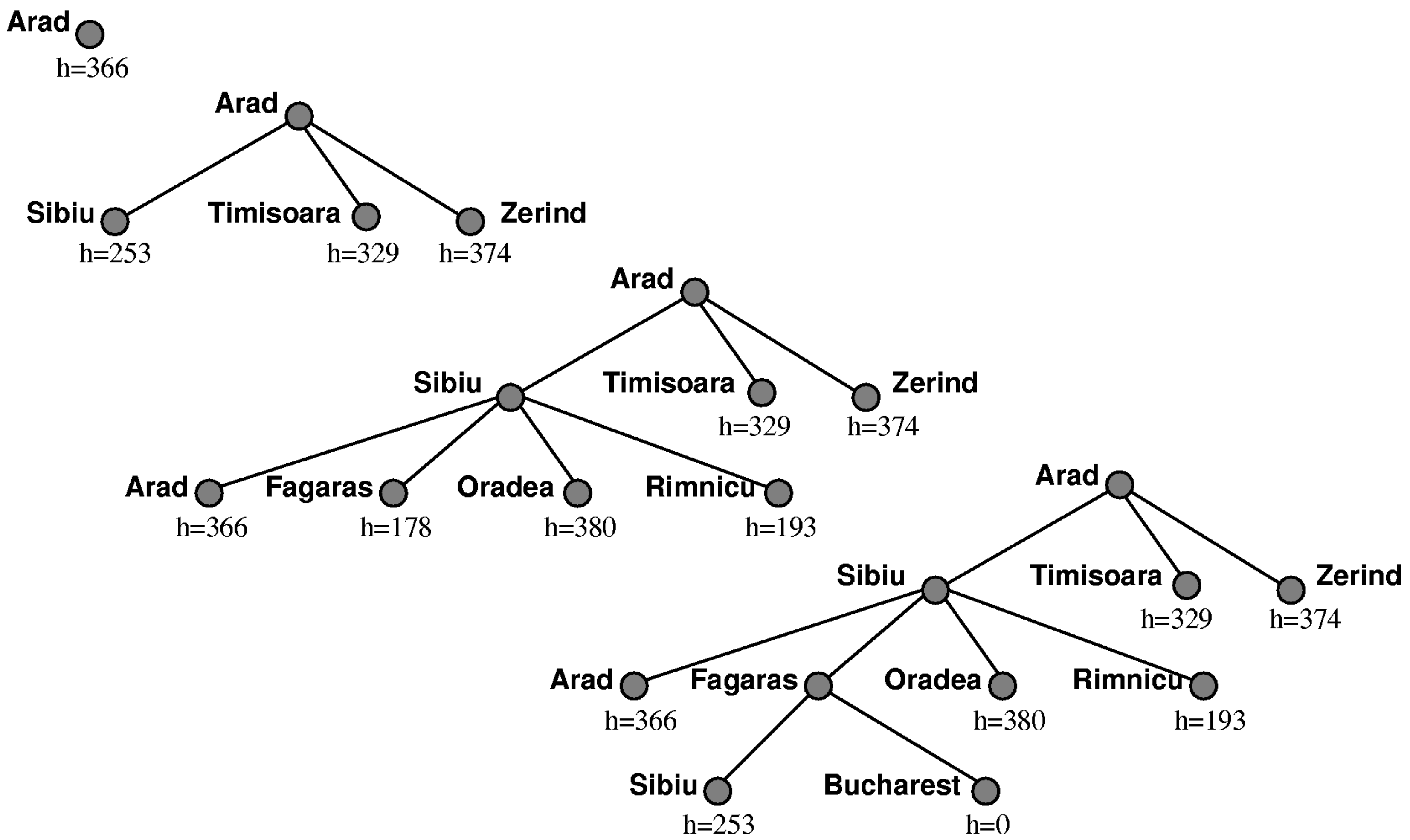
Route-finding problem: h = straight-line distance between two locations.

Greedy Search Example



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search from *Arad* to *Bucharest*



Heuristics

The evaluation function *h* in greedy searches is also called a *heuristic* function or simply a *heuristic.*

- The word *heuristic* is derived from the Greek word ευρισκειν (note also: ευρηκα!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and *focuses* the search!

A*: Minimization of the estimated path costs

A* combines the greedy search with the uniform-search strategy.

g(n) = actual cost from the initial state to *n*.

h(n) = estimated cost from *n* to the next goal.

f(n) = *g(n)* + *h(n)*, the estimated cost of the cheapest solution through *n*.

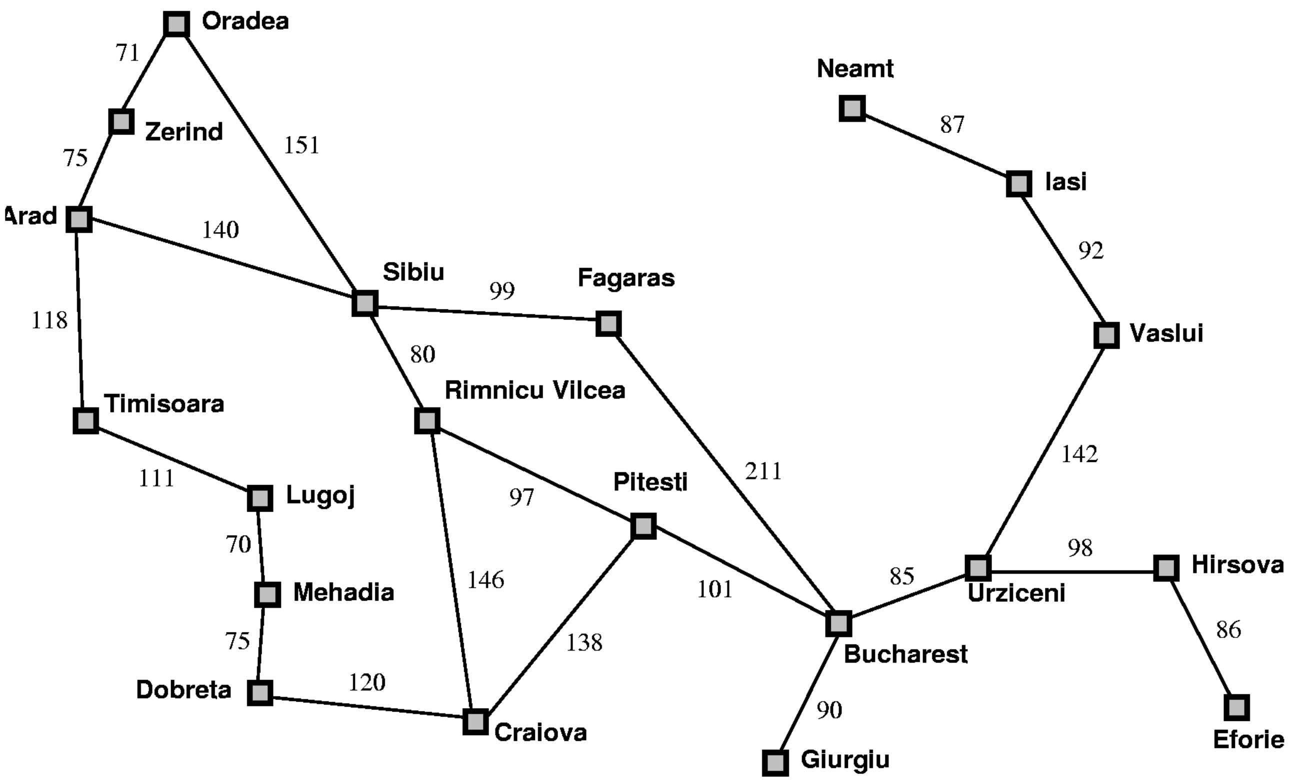
Let h*(n) be the actual cost of the optimal path from *n* to the next goal.

h is *admissible* if the following holds for all *n* :

$$h(n) \leq h^*(n)$$

We require that for A*, *h* is admissible (straight-line distance is admissible).

A* Search Example



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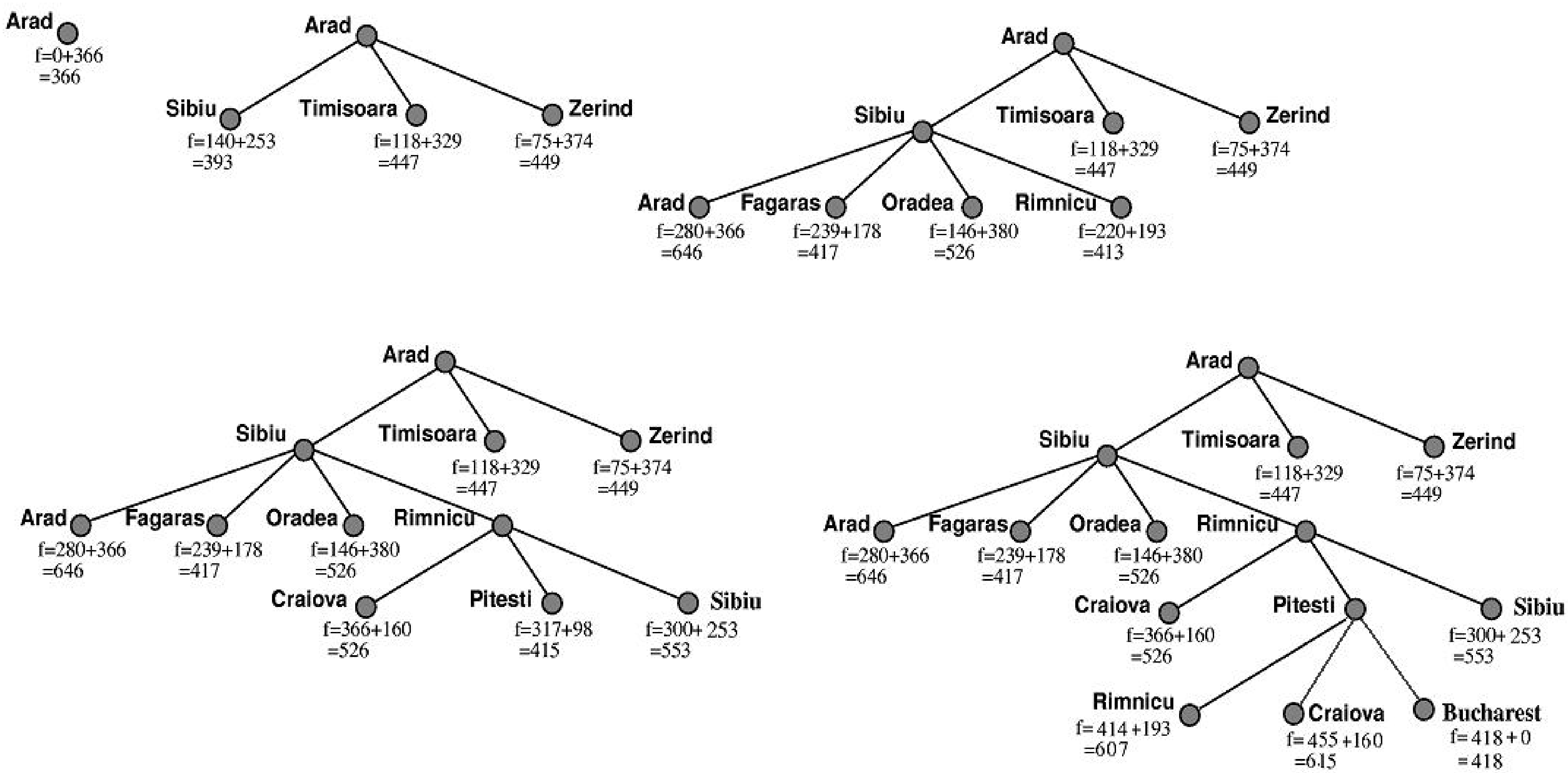
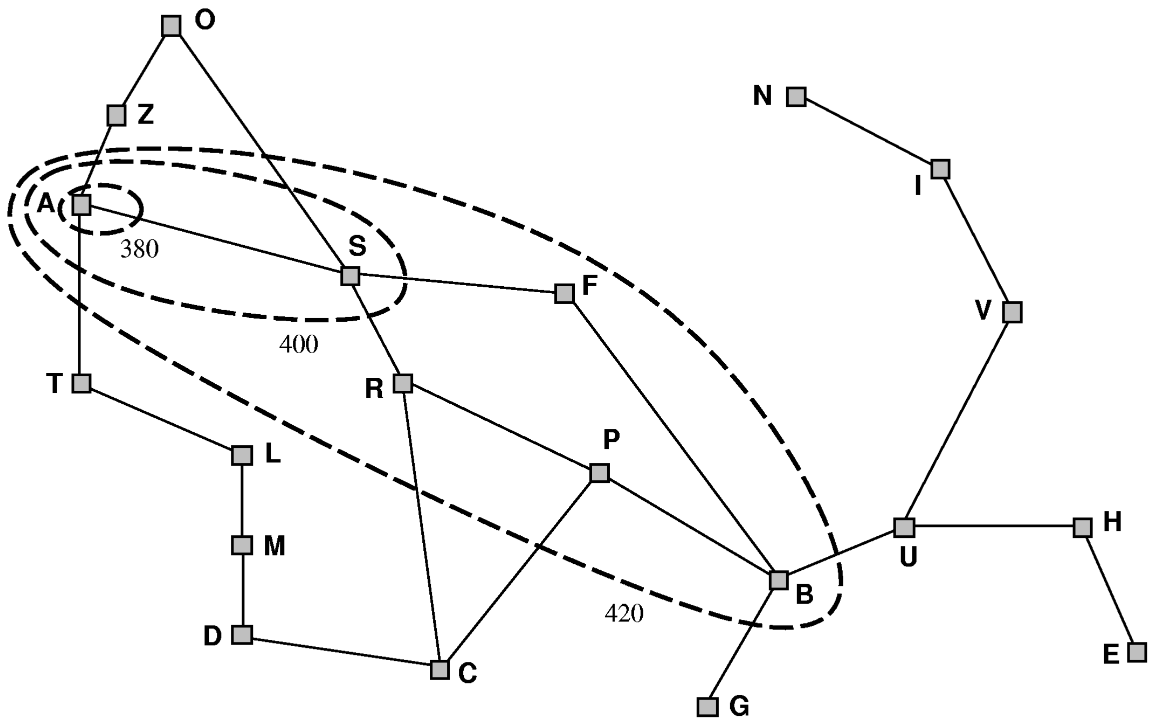
A* Search from *Arad* to *Bucharest*



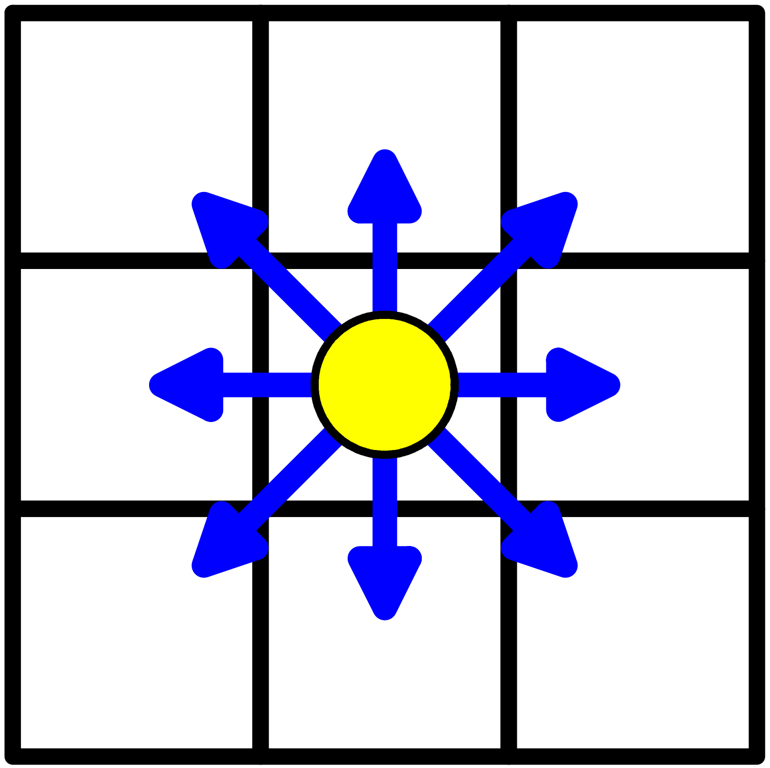
Contours in A*

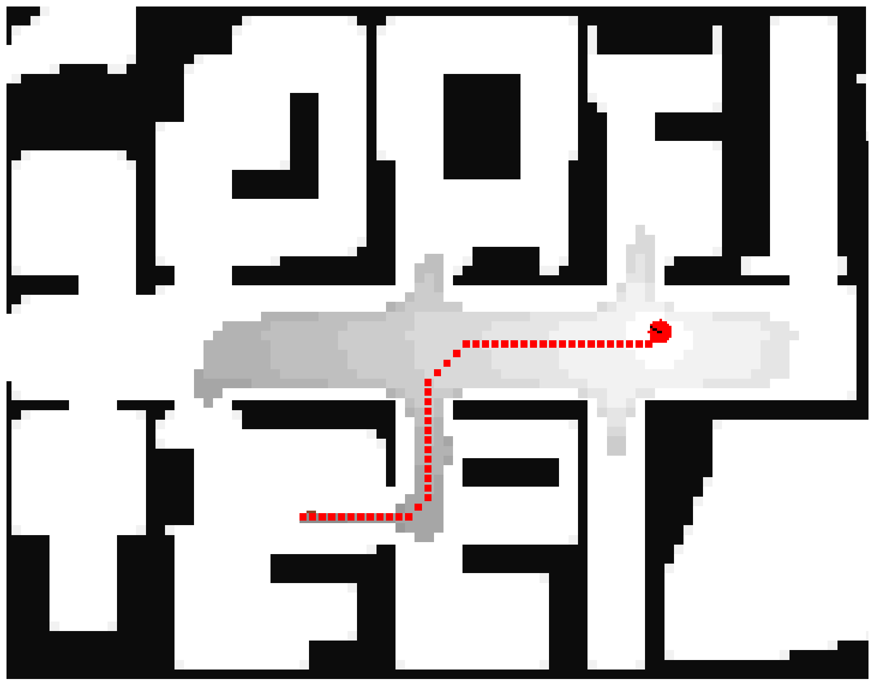
Within the search space, contours arise in which for the given *f*-value all nodes are expanded.



Contours at f = 380, 400, 420

Example: Path Planning for Robots in a Grid-World

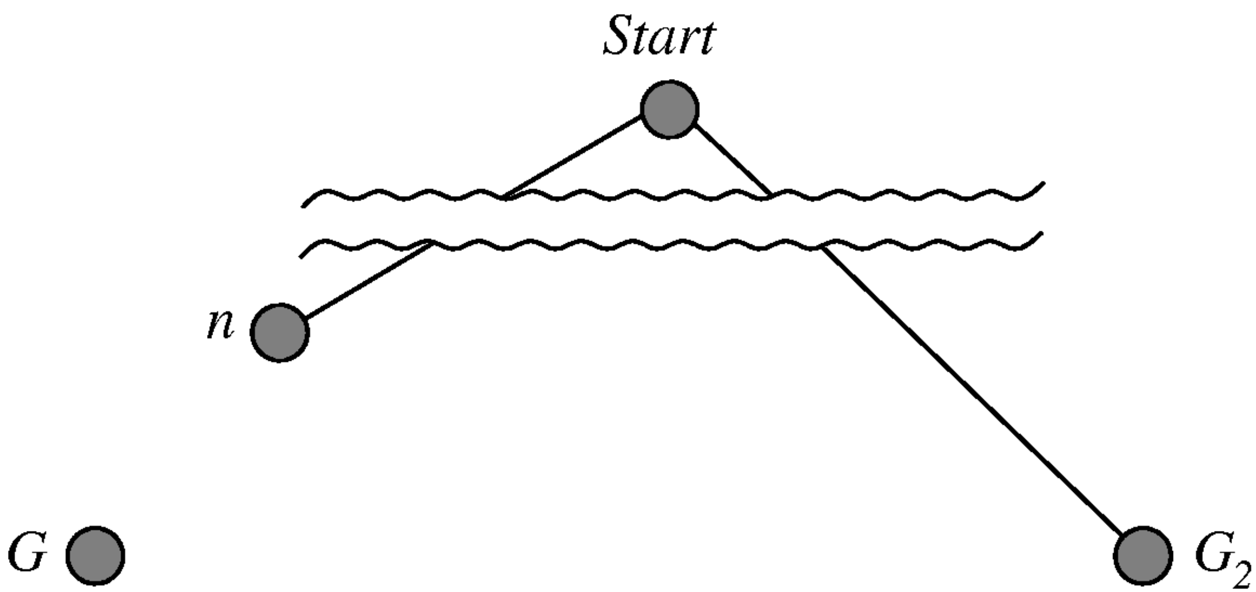




Optimality of A*

Claim: The first solution found has the minimum path cost.

Proof: Suppose there exists a goal node G with optimal path cost *f**, but A* has found another node G_2 with $g(G_2) > f^*$.



Let n be a node on the path from the start to G that has not yet been expanded. Since h is admissible, we have

$$f(n) \leq f^*.$$

Since n was not expanded before G_2, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$

It follows from $h(G_2) = 0$ *that*

$$g(G_2) \leq f^*.$$

→ Contradicts the assumption!

Completeness and Complexity

Completeness:

If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant δ such that every operator has at least cost δ.

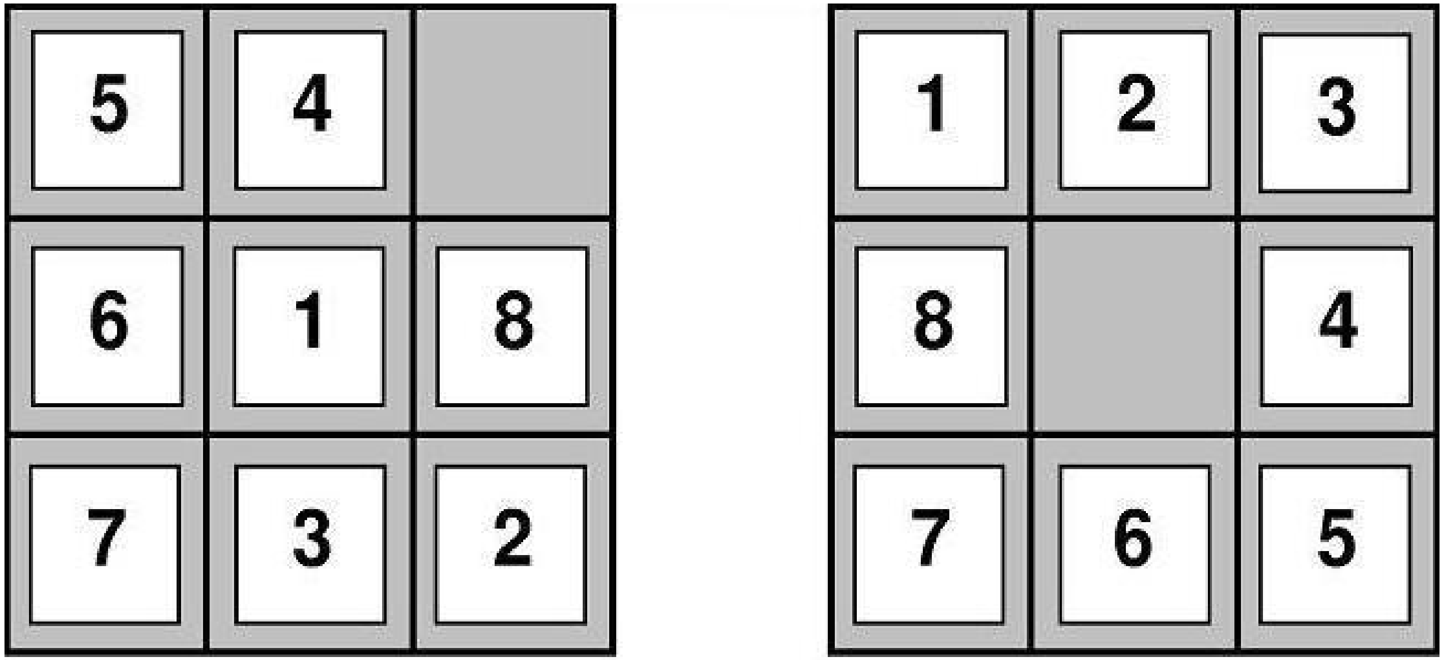
→ Only a finite number of nodes *n* with $f(n) \leq f^*$.

Complexity:

In the case where $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only a sub-exponential number of nodes will be expanded – provided the search space is a tree and there is only one goal state. This, however, is a quite unrealistic assumption [Helmert & Roeger, 2008] (best AAAI paper 2008)

Normally, growth is exponential because the error is proportional to the path costs.

Heuristic Function Example



Start State Goal State

h_1 = the number of tiles in the wrong position

h_2 = the sum of the distances of the tiles from their goal positions (*Manhatten distance)*

Empirical Evaluation

- d = distance from goal
- Average over 100 instances

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Iterative Deepening A* Search (IDA*)

Idea: A combination of IDS and A*. All nodes inside a contour are searched.

```
function IDA*(problem) returns a solution sequence
  inputs: problem, a problem
  static: f-limit, the current f- COST limit
          root, a node

  root ← MAKE-NODE(INITIAL-STATE[problem])
  f-limit ← f- COST(root)
  loop do
      solution, f-limit ← DFS-CONTOUR(root, f-limit)
      if solution is non-null then return solution
      if f-limit = ∞ then return failure; end
```

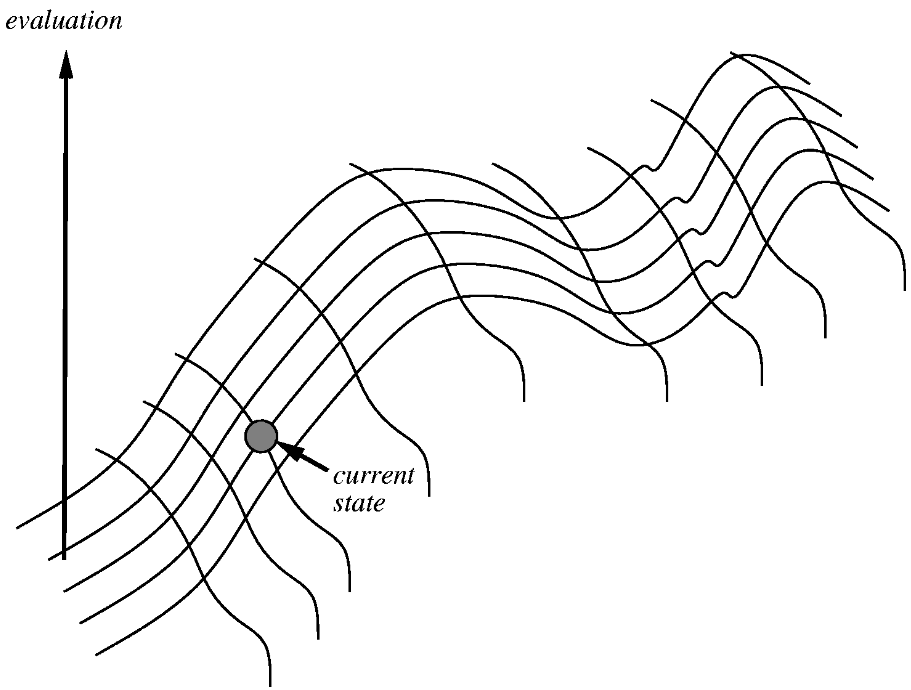
```
function DFS-CONTOUR(node, f-limit) returns a solution sequence and a new f- COST limit
  inputs: node, a node
          f-limit, the current f- COST limit
  static: next-f, the f- COST limit for the next contour, initially ∞

  if f- COST[node] > f-limit then return null, f- COST[node]
  if GOAL-TEST[problem](STATE[node]) then return node, f-limit
  for each node s in SUCCESSORS(node) do
      solution, new-f ← DFS-CONTOUR(s, f-limit)
      if solution is non-null then return solution, f-limit
      next-f ← MIN(next-f, new-f); end
  return null, next-f
```

Local Search Methods

In many problems, it is unimportant how the goal is reached – only the goal itself matters (8-queens problem, VLSI Layout, TSP).

If in addition a quality measure for states is given, a **local search** can be used to find solutions.

Idea: Begin with a randomly-chosen configuration and improve on it stepwise → **Hill Climbing**.

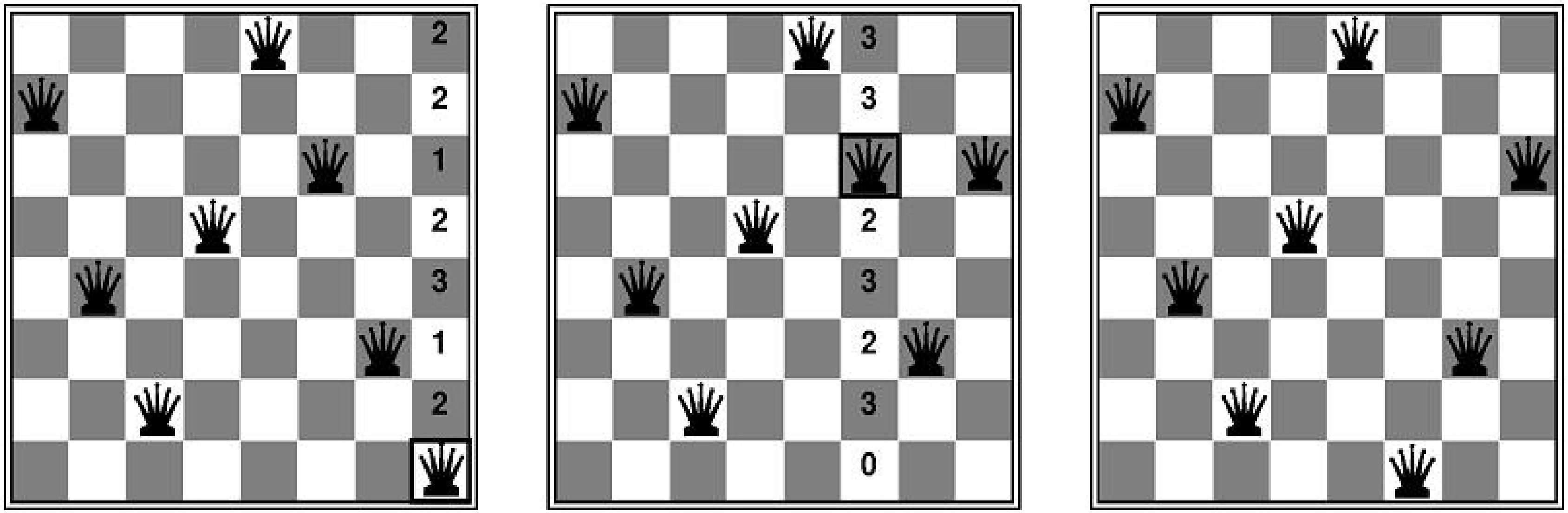


Hill Climbing

```
function HILL-CLIMBING(problem) returns a solution state
   inputs: problem, a problem
   static: current, a node
           next, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
       next ← a highest-valued successor of current
       if VALUE[next] < VALUE[current] then return current
       current ← next
   end
```

Example: 8-Queens Problem

Selects a column and moves the queen to the square with the fewest conflicts.



Problems with Local Search Methods

- *Local maxima*: The algorithm finds a sub-optimal solution.
- *Plateaus*: Here, the algorithm can only explore at random.
- Ridges: Similar to plateaus.

Solutions:

- *Start over* when no progress is being made.
- "Inject smoke" → random walk
- Tabu search: Do not apply the last *n* operators.

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.

Simulated Annealing

In the simulated annealing algorithm, "smoke" is injected systematically: first a lot, then gradually less.

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"
  static: current, a node
          next, a node
          T, a "temperature" controlling the probability of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
     T ← schedule[t]
     if T=0 then return current
     next ← a randomly selected successor of current
     ΔE ← VALUE[next] – VALUE[current]
     if ΔE > 0 then current ← next
     else current ← next only with probability e^(ΔE/T)
```

Has been used since the early 80's for VSLI layout and other optimization problems.

Genetic Algorithms

Evolution appears to be very successful at finding good solutions.

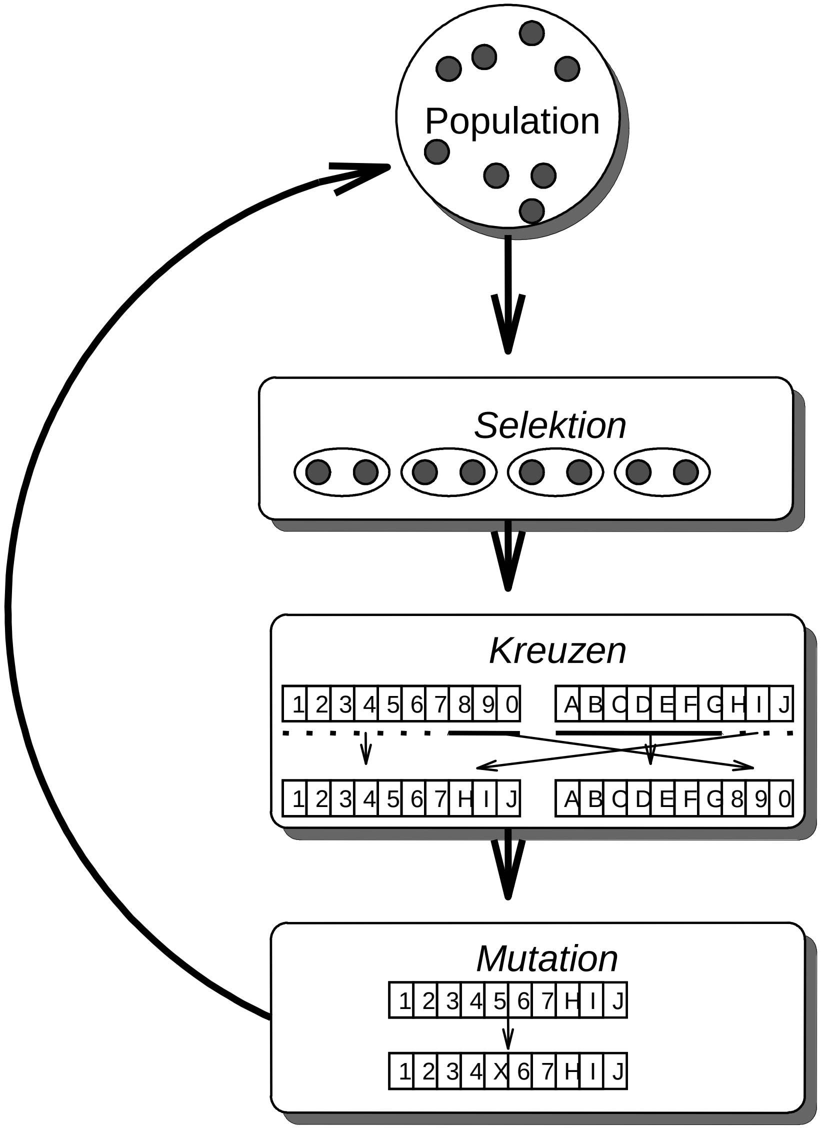
Idea: Similar to evolution, we search for solutions by “crossing”, “mutating”, and “selecting” successful solutions.

Ingredients:

- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

Example: 8-queens problem as a chain of 8 numbers. Fitness is judged by the number of non-attacks. The population consists of a set of arrangements of queens.

Selection, Mutation, and Crossing



Many variations:

how selection will be applied, what type of cross-overs will be used, etc.

Selektion von Individuen anhand der Fitness-Funktion und Paarung

Festlegung wo aufgebrochen wird und neu zusammenfügen

Mit einer gewissen kleinen Wahrscheinlichkeit wird etwas im String geändert.

Summary

- Heuristics focus the search
- Best-first search expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal *h* we obtain a greedy search.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e., h^* is never overestimated, we obtain the A* search, which is complete and optimal.
- IDA* is a combination of the iterative-deepening and A* searches.
- Local search methods only ever work on one state, attempting to improve it step-wise.
- Genetic algorithms imitate evolution by combining good solutions.