

# Principles of Knowledge Representation and Reasoning

## Semantic Networks and Description Logics II: Description Logics – Terminology and Notation

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# Description Logics – Terminology and Notation

Introduction

Concept and Roles

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Outlook

# Motivation

- ▶ Main problem with **semantic networks** and **frames**
  - ▶ The lack of **formal semantics!**
  - ▶ Disadvantage of simple **inheritance networks**
  - ▶ Concepts are atomic and do not have any **structure**
- ↪ Brachman's **structural inheritance networks** (1977)

# Structural Inheritance Networks

- ▶ Concepts are *defined/described* using a small set of well-defined operators
- ▶ Distinction between *conceptual* and *object-related* knowledge
- ▶ Computation of *subconcept relation* and of *instance relation*
- ▶ *Strict inheritance* (of the entire structure of a concept)

# Systems and Applications

## ► Systems:

- **KL-ONE**: First implementation of the ideas (1978)
- ... then **NIKL**, **KL-TWO**, **KRYPTON**, **KANDOR**, **CLASSIC**, **BACK**, **KRIS**, **YAK**, **CRACK** ...
- ... currently **FaCT**, **DLP**, **RACER** 1998

## ► Applications:

- First, natural language understanding systems
- ... then configuration systems,
- ... information systems,
- ... currently, it is one tool for the *semantic web*
- **DAML+OIL**, now **OWL**

# Description Logics

- ▶ Previously also *KL-ONE-like languages*, *frame-based languages*, *terminological logics*, *concept languages*
- ▶ **Description Logics (DL)** allow us
  - to describe concepts using *complex descriptions*,
  - to introduce the terminology of an application and to structure it (**TBox**),
  - to introduce objects (**ABox**) and relate them to the introduced terminology,
  - and to *reason* about the terminology and the objects.

## Informal Example

Male is:	the opposite of female
A human is a kind of:	living entity
A woman is:	a human and a female
A man is:	a human and a male
A mother is:	a woman with at least one child that is a human
A father is:	a man with at least one child that is a human
A parent is:	a mother or a father
A grandmother is:	a woman, with at least one child that is a parent
A mother-wod is:	a mother with only male children

Elizabeth is a woman  
 Elizabeth has the child Charles  
 Charles is a man  
 Diana is a mother-wod  
 Diana has the child William

### Possible Questions:

Is a grandmother a parent?  
 Is Diana a parent?  
 Is William a man?  
 Is Elizabeth a mother-wod?

# Atomic Concepts and Roles

## ► **Concept names:**

- E.g., Grandmother, Male, ... (in the following usually *capitalized*)
- We will use **symbols** such as  $A, A_1, \dots$
- **Semantics:** Monadic predicates  $A(\cdot)$  or set-theoretically a subset of the universe  $A^{\mathcal{I}} \subseteq \mathcal{D}$ .

## ► **Role names:**

- In our example, e.g., child. Often we will use names such as has-child or something similar (in the following usually *lowercase*).
- Role names are *disjoint* from concept names
- **Symbolically:**  $t, t_1, \dots$
- **Semantics:** Dyadic predicates  $t(\cdot, \cdot)$  or set-theoretically  $t^{\mathcal{I}} \subseteq \mathcal{D} \times \mathcal{D}$ .



## Concept and Role Description

- ▶ Out of *concept* and *role names*, complex **descriptions** can be created
- ▶ In our example, e.g. “a Human and Female.”
- ▶ **Symbolically**:  $C$  for concept descriptions and  $r$  for role descriptions
- ▶ Which particular constructs are available depends on the chosen description logic
- ▶ **Predicate logic semantics**: A concept descriptions  $C$  corresponds to a formula  $C(x)$  with the free variable  $x$ . Similarly with  $r$ : It corresponds to formula  $r(x, y)$  with free variables  $x, y$ .
- ▶ **Set semantics**:

$$C^{\mathcal{I}} = \{d \mid C(d) \text{ “is true in” } \mathcal{I}\}$$

$$r^{\mathcal{I}} = \{(d, e) \mid r(d, e) \text{ “is true in” } \mathcal{I}\}$$

# Boolean Operators

- ▶ **Syntax:** let  $C$  and  $D$  be concept descriptions, then the following are also concept descriptions:
  - $C \sqcap D$  (**Concept conjunction**)
  - $C \sqcup D$  (**Concept disjunction**)
  - $\neg C$  (**Concept negation**)
- ▶ **Examples:**
  - Human  $\sqcap$  Female
  - Father  $\sqcup$  Mother
  - $\neg$  Female
- ▶ **Predicate logic semantics:**  $C(x) \wedge D(x)$ ,  $C(x) \vee D(x)$ ,  $\neg C(x)$
- ▶ **Set semantics:**  $C^{\mathcal{I}} \cap D^{\mathcal{I}}$ ,  $C^{\mathcal{I}} \cup D^{\mathcal{I}}$ ,  $\mathcal{D} - C^{\mathcal{I}}$

# Role Restrictions

## ► Motivation:

- Often we want to describe something by *restricting* the possible “fillers” of a role, e.g. Mother-wod.
- Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother

## ► Idea: Use quantifiers that range over the role-fillers

- $\text{Mother} \sqcap \forall \text{has-child.Man}$
- $\text{Woman} \sqcap \exists \text{has-child.Parent}$

## ► Predicate logic semantics:

$$\begin{aligned}
 (\exists r.C)(x) &= \exists y : (r(x,y) \wedge C(y)) \\
 (\forall r.C)(x) &= \forall y : (r(x,y) \rightarrow C(y))
 \end{aligned}$$

## Set semantics:

$$\begin{aligned}
 (\exists r.C)^{\mathcal{I}} &= \{d \mid \exists e : (d, e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\} \\
 (\forall r.C)^{\mathcal{I}} &= \{d \mid \forall e : (d, e) \in r^{\mathcal{I}} \rightarrow e \in C^{\mathcal{I}}\}
 \end{aligned}$$

# Cardinality Restriction

## ► Motivation:

- Often we want to describe something by *restricting the number* of possible “fillers” of a role, e.g., a Mother with at least 3 children or at most 2 children.

## ► Idea: We restrict the cardinality of the role filler sets:

- $\text{Mother} \sqcap (\geq 3 \text{ has-child})$
- $\text{Mother} \sqcap (\leq 2 \text{ has-child})$

## ► Predicate logic semantics:

$$(\geq n r)(x) = \exists y_1 \dots y_n : (r(x, y_1) \wedge \dots \wedge r(x, y_n) \wedge y_1 \neq y_2 \wedge \dots \wedge y_{n-1} \neq y_n)$$

$$(\leq n r)(x) = \neg(\geq n + 1 r)(x)$$

## ► Set semantics:

$$(\geq n r)^{\mathcal{I}} = \{d \mid |\{e \mid r^{\mathcal{I}}(d, e)\}| \geq n\}$$

$$(\leq n r)^{\mathcal{I}} = \mathcal{D} - (\geq n + 1 r)^{\mathcal{I}}$$

# Inverse Roles

▶ **Motivation:**

- How can we describe the concept “*children of rich parents*”?

▶ **Idea:** Define the “inverse” role for a given role (the **converse relation**)

- `has-child-1`

▶ **Application:**  $\exists \text{has-child}^{-1}.\text{Rich}$

▶ **Predicate logic semantics:**

$$r^{-1}(x, y) = r(y, x)$$

▶ **Set semantics:**

$$(r^{-1})^{\mathcal{I}} = \{(d, e) \mid (e, d) \in r^{\mathcal{I}}\}$$

# Role Composition

► **Motivation:**

- How can we define the role `has-grandchild` given the role `has-child`?

► **Idea:** Compose roles (as one can compose binary relations)

- `has-child` ◦ `has-child`

► **Predicate logic semantics:**

$$(r \circ s)(x, y) = \exists z : (r(x, z) \wedge s(z, y))$$

► **Set semantics:**

$$(r \circ s)^{\mathcal{I}} = \{(d, e) \mid \exists f : (d, f) \in r^{\mathcal{I}} \wedge (f, e) \in s^{\mathcal{I}}\}$$

# Role Value Maps

► **Motivation:**

- How do we express the concept “*women who know all the friends of their children*”

► **Idea:** Relate role filler sets to each other

- $\text{Woman} \sqcap (\text{has-child} \circ \text{has-friend} \sqsubseteq \text{knows})$

► **Predicate logic semantics:**

$$(r \sqsubseteq s)(x) = \forall y : (r(x, y) \rightarrow s(x, y))$$

► **Set semantics:** Let  $r^{\mathcal{I}}(d) = \{e \mid r^{\mathcal{I}}(d, e)\}$ .

$$(r \sqsubseteq s)^{\mathcal{I}} = \{d \mid r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d)\}$$

► **Note:** Role value maps lead to undecidability of satisfiability of concept descriptions!

# Terminology Box

- ▶ In order to *introduce* new terms, we use two kinds of **terminological axioms**:
  - $A \doteq C$
  - $A \sqsubseteq C$
 where  $A$  is a *concept name* and  $C$  is a *concept description*.
- ▶ A **terminology** or **TBox** is a finite set of such axioms with the following additional restrictions:
  - no multiple definitions of the same symbol such as  $A \doteq C$ ,  $A \sqsubseteq D$
  - no cyclic definitions (even not indirectly), such as  $A \doteq \forall r.B$ ,  $B \doteq \exists s.A$



# TBoxes: Semantics

- ▶ TBoxes restrict the set of possible interpretations.
- ▶ **Predicate logic semantics:**
  - $A \doteq C$  corresponds to  $\forall x : (A(x) \leftrightarrow C(x))$
  - $A \sqsubseteq C$  corresponds to  $\forall x : (A(x) \rightarrow C(x))$
- ▶ **Set semantics:**
  - $A \doteq C$  corresponds to  $A^{\mathcal{I}} = C^{\mathcal{I}}$
  - $A \sqsubseteq C$  corresponds to  $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- ▶ Non-empty interpretations which satisfy all terminological axioms are called **models** of the TBox.

# Assertional Box

- ▶ In order to state something about objects in the world, we use two forms of **assertions**:
  - $a : C$
  - $(a, b) : r$where  $a$  and  $b$  are **individual names** (e.g., ELIZABETH, PHILIP),  $C$  is a **concept description**, and  $r$  is a **role description**.
- ▶ An **ABox** is a finite set of assertions.

# ABoxes: Semantics

- ▶ **Individual names** are interpreted as elements of the universe under the **unique-name-assumption**, i.e., different names refer to different objects.
- ▶ **Assertions** express that an object is an instance of a concept or that two objects are related by a role.
- ▶ **Predicate logic semantics:**
  - $a : C$  corresponds to  $C(a)$
  - $(a, b) : r$  corresponds to  $r(a, b)$
- ▶ **Set semantics:**
  - $a^{\mathcal{I}} \in D$
  - $a : C$  corresponds to  $a^{\mathcal{I}} \in C^{\mathcal{I}}$
  - $(a, b) : r$  corresponds to  $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$
- ▶ **Models** of an ABox and of ABox+TBox can be defined analogously to models of a TBox.

# Example TBox

Male  $\doteq$   $\neg$ Female  
 Human  $\sqsubseteq$  Living\_entity  
 Woman  $\doteq$  Human  $\sqcap$  Female  
 Man  $\doteq$  Human  $\sqcap$  Male  
 Mother  $\doteq$  Woman  $\sqcap$   $\exists$ has-child.Human  
 Father  $\doteq$  Man  $\sqcap$   $\exists$ has-child.Human  
 Parent  $\doteq$  Father  $\sqcup$  Mother  
 Grandmother  $\doteq$  Woman  $\sqcap$   $\exists$ has-child.Parent  
 Mother-without-daughter  $\doteq$  Mother  $\sqcap$   $\forall$ has-child.Male  
 Mother-with-many-children  $\doteq$  Mother  $\sqcap$  ( $\geq 3$ has-child)

# Example ABox

CHARLES: Man

EDWARD: Man

ANDREW: Man

DIANA: Mother-without-daughter

(ELIZABETH, CHARLES): has-child

(ELIZABETH, EDWARD): has-child

(ELIZABETH, ANDREW): has-child

(DIANA, WILLIAM): has-child

(CHARLES, WILLIAM): has-child

DIANA: Woman

ELIZABETH: Woman

## Some Reasoning Services

- ▶ Does a description  $C$  make sense at all, i.e., is it **satisfiable**?
- ▶ A concept description  $C$  is satisfiable iff there exists an interpretation  $\mathcal{I}$  such that  $C^{\mathcal{I}} \neq \emptyset$ .
- ▶ Is one concept a specialization of another one, is it **subsumed**?
- ▶  $C$  is **subsumed by**  $D$ , in symbols  $C \sqsubseteq D$  iff we have for all interpretations  $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ .
- ▶ Is  $a$  an **instance** of a concept  $C$ ?
- ▶  $a$  is an instance of  $C$  iff for all interpretations, we have  $a^{\mathcal{I}} \in C^{\mathcal{I}}$ .
- ▶ **Note**: These questions can be posed with or without a TBox that restricts the possible interpretations.

# Outlook

- ▶ Can we **reduce** the reasoning services to perhaps just one problem?
- ▶ What could be **reasoning algorithms**?
- ▶ What about **complexity** and **decidability**?
- ▶ What has all that to do with **modal logics**?
- ▶ How can one build **efficient systems**?

# Literature



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## Summary: Concept Descriptions

Abstract	Concrete	Interpretation
$A$	$A$	$A^{\mathcal{I}}$
$C \sqcap D$	(and $C D$ )	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$C \sqcup D$	(or $C D$ )	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
$\neg C$	(not $C$ )	$\mathcal{D} - C^{\mathcal{I}}$
$\forall r.C$	(all $r C$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \subseteq C^{\mathcal{I}}\}$
$\exists r$	(some $r$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \neq \emptyset\}$
$\geq n r$	(atleast $n r$ )	$\{d \in \mathcal{D} :  r^{\mathcal{I}}(d)  \geq n\}$
$\leq n r$	(atmost $n r$ )	$\{d \in \mathcal{D} :  r^{\mathcal{I}}(d)  \leq n\}$
$\exists r.C$	(some $r C$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \cap C^{\mathcal{I}} \neq \emptyset\}$
$\geq n r.C$	(atleast $n r C$ )	$\{d \in \mathcal{D} :  r^{\mathcal{I}}(d) \cap C^{\mathcal{I}}  \geq n\}$
$\leq n r.C$	(atmost $n r C$ )	$\{d \in \mathcal{D} :  r^{\mathcal{I}}(d) \cap C^{\mathcal{I}}  \leq n\}$
$r \doteq s$	(eq $r s$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) = s^{\mathcal{I}}(d)\}$
$r \neq s$	(neq $r s$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \neq s^{\mathcal{I}}(d)\}$
$r \sqsubseteq s$	(subset $r s$ )	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d)\}$
$g \doteq h$	(eq $g h$ )	$\{d \in \mathcal{D} : g^{\mathcal{I}}(d) = h^{\mathcal{I}}(d) \neq \emptyset\}$
$g \neq h$	(neq $g h$ )	$\{d \in \mathcal{D} : \emptyset \neq g^{\mathcal{I}}(d) \neq h^{\mathcal{I}}(d) \neq \emptyset\}$
$\{i_1, i_2, \dots, i_n\}$	(oneof $i_1 \dots i_n$ )	$\{i_1^{\mathcal{I}}, i_2^{\mathcal{I}}, \dots, i_n^{\mathcal{I}}\}$

# Summary: Role Descriptions

Abstract	Concrete	Interpretation
$t$	$t$	$t^{\mathcal{I}}$
$f$	$f$	$f^{\mathcal{I}}$ , (functional role)
$r \sqcap s$	(and $r s$ )	$r^{\mathcal{I}} \cap s^{\mathcal{I}}$
$r \sqcup s$	(or $r s$ )	$r^{\mathcal{I}} \cup s^{\mathcal{I}}$
$\neg r$	(not $r$ )	$\mathcal{D} \times \mathcal{D} - r^{\mathcal{I}}$
$r^{-1}$	(inverse $r$ )	$\{(d, d') : (d', d) \in r^{\mathcal{I}}\}$
$r _C$	(restr $r C$ )	$\{(d, d') \in r^{\mathcal{I}} : d' \in C^{\mathcal{I}}\}$
$r^+$	(trans $r$ )	$(r^{\mathcal{I}})^+$
$r \circ s$	(compose $r s$ )	$r^{\mathcal{I}} \circ s^{\mathcal{I}}$
$\mathbf{1}$	self	$\{(d, d) : d \in \mathcal{D}\}$