Principles of Knowledge Representation and Reasoning

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Exercise Sheet 12
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Exercise 12.1 (Credulous extensions of inheritance networks, 2 + 4 marks)

In this exercise, we consider credulous reasoning in possibly unsatisfiable inheritance networks. We need the following definitions:

- A *path* is either a *positive path*, i.e., a sequence of one or more positive edges, or a negative path, i.e., a sequence of zero or more positive edges followed by a single negative edge.

- Let a be a node. A network is *a-connected* iff for every node x, there is a path (positive or negative) from a to x, and for every (positive or negative) edge (ν, x), there is a positive path from a to ν.

- A network is (potentially) *ambiguous* with respect to a if there is some node x such that there is both a positive and a negative path from a to x.

- A *credulous extension* of an inheritance network with respect to a node a is a maximal unambiguous a-connected subnetwork with respect to a.

Consider the following two collections of assertions:

- **Animals:**
    - A platypus is (typically) a furry animal.
    - A platypus is (typically) an egg layer.
    - A furry animal is (typically) a mammal.
    - An egg layer is (typically) not a mammal.
    - A mammal is (typically) a milk producer.

- **People:**
    - Republican Quakers are (typically) Republicans.
    - Republican Quakers are (typically) Quakers.
    - Quakers are (typically) pacifists.
    - Republicans are (typically) not pacifists.
    - Republicans are (typically) promilitary.
    - Pacifists are (typically) not promilitary.
    - Promilitary people are (typically) politically active.
    - Pacifists are (typically) politically active.

(a) Represent the assertions as inheritance networks.

(b) Give the sets of credulous extensions of the two networks with respect to the nodes "Platypus" and "RepublicanQuaker", respectively.

Exercise 12.2 (Formalization in Description Logic, 2 + 1 + 1 marks)

(a) Extend the TBox from the lecture by defining the following concepts:

- `Granddaughter`
- `Sister-in-Law`
- `Bachelor` (Unmarried man)
- `Bigamist` (Person married to at least two other persons)

Besides the roles and concepts used in the lecture, you may use the atomic role `married-to` and the (half-)sibling role `has-sibling` = (`has-child`$^{-1}$∘ `has-child`) ⊓ ¬1.

(b) Prove that in the given TBox the subsumption relation `Bachelor` ⊑ ¬`Bigamist` holds. To that end, translate the two concept expressions into their predicate logic semantics and give a proof in predicate logic.

(c) Prove that in the given TBox the subsumption relation `Sister-in-Law` ⊑ `Granddaughter` *does not* hold by giving a counterexample, i.e., a model of the TBox in which the subsumption does not hold.