Principles of Knowledge Representation and Reasoning

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Exercise Sheet 7

Due: June 17, 2008

Exercise 7.1 (Cumulative Logics, 5 marks)

Consider the set of plausible consequences

$$K = \{n \mathrel{|\!\sim} v, v \mathrel{|\!\sim} u, w \mathrel{|\!\sim} \neg u, n \mathrel{|\!\sim} w\}.$$

Examine whether in the system **C** the following statements follow from K by stating a proof (i.e. a derivation) or a counter example.

- $n \mathrel{|\!\sim} u \vee w$
- $n \mathrel{|\!\sim} \neg p$
- $n \mathrel{|\!\sim} u \vee \neg u$
- $n \wedge v \mathrel{|\!\sim} u$

Exercise 7.2 (Cumulative Logics and Set Theory, 5 marks)

There is a connection between set theory and nonmonotonic reasoning: We call a rule (e.g. left logical equivalence or right weakening) *set theoretically plausible* if the following holds for arbitrary finite sets I of interpretations:
If all propositional premises of the rule are fulfilled and for each premise of the form $\alpha \mathrel{|\!\sim} \beta$ holds: At least 99% of the interpretations in I that fulfill α, also fulfill β, **then** it holds for each conclusion of the form $\alpha' \mathrel{|\!\sim} \beta'$ that: At least 95% of the interpretations in I that fulfill α' also fulfill β'.

Decide whether the following rules are set theoretically plausible:

(a) Right Weakening

$$\frac{\models \alpha \rightarrow \beta,\ \gamma \mathrel{|\!\sim} \alpha}{\gamma \mathrel{|\!\sim} \beta}$$

(b) Cut

$$\frac{\alpha \mathrel{|\!\sim} \beta,\ \alpha \wedge \beta \mathrel{|\!\sim} \gamma}{\alpha \mathrel{|\!\sim} \gamma}$$

(c) Contraposition

$$\frac{\alpha \mathrel{|\!\sim} \beta}{\neg\alpha \mathrel{|\!\sim} \neg\beta}$$

(d) Transitivity

$$\frac{\alpha \mathrel{|\!\sim} \beta,\ \beta \mathrel{|\!\sim} \gamma}{\alpha \mathrel{|\!\sim} \gamma}$$