

# Principles of Knowledge Representation and Reasoning

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## Exercise Sheet 4

Due: May 27, 2008

### Exercise 4.1 (Modal Logic, 5 marks)

State for each of the following frame properties an axiom schema that defines it. Prove that your schema is valid in each frame that has the property. Furthermore, state an appropriate frame that does not have the property and in which the axiom is false.

- (a)  $uRv$  and  $uRw$  implies  $v = w$ .
- (b) For each  $w$  there exists exactly one  $v$  with  $wRv$ .
- (c) For each  $u$  and  $w$  with  $uRw$  exists a  $v$  with  $uRv$  and  $vRw$ .

### Exercise 4.2 (Tableaux Rules, 5 marks)

Use tableaux to prove the following statements:

- (a)  $\Box\Diamond p \leftrightarrow \Box\Diamond\Box\Diamond p$  is S4-valid.
- (b)  $\Diamond\Box p \rightarrow \Box\Diamond p$  is S5-valid.
- (c)  $\Box(\Box p \rightarrow q) \vee \Box(\Box q \rightarrow p)$  is S5-valid.
- (d)  $\Diamond(p \wedge \Box q) \rightarrow \Box(p \vee \Diamond q)$  is S5-valid.
- (e)  $\Box(p \vee \Box q) \leftrightarrow (\Box p \vee \Box q)$  is S5-valid.