Principles of Knowledge Representation and Reasoning

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Exercise Sheet 3

Due: May 20, 2008

Exercise 3.1 (Complexity Theory, 1+2)

(a) Prove that P is closed under complement, in particular $\mathrm{P} \subseteq \mathrm{NP} \cap \text{co-NP}$.

(b) Prove: Each problem in NP is Turing-reducible to a problem in co-NP and vice versa.

Exercise 3.2 (Modal Logic, 3+4)

(a) Prove the following statements:

(a) If $\mathcal{F}$ is reflexive and Euclidean, then $\mathcal{F}$ is symmetric and transitive.

(b) If $\mathcal{F}$ is symmetric and transitive, then $\mathcal{F}$ is Euclidean.

(c) The following statements are equivalent:

i. $\mathcal{F}$ is reflexive, symmetric, and transitive.

ii. $\mathcal{F}$ is symmetric, transitive, and serial.

iii. $\mathcal{F}$ is reflexive and Euclidean.

(b) Examine which of the following formulae are valid in **S4** and which are valid in **S5**. Provide a semantic proof or a model that is a counter example. In addition, please provide an overview in table form.

(a) $\Box\Diamond p \rightarrow \Diamond\Box p$

(b) $\Diamond\Box p \rightarrow \Box\Diamond p$

(c) $\Box(\Box p \rightarrow p) \rightarrow \Box p$

(d) $\Box(\Box p \rightarrow q) \vee \Box(\Box q \rightarrow p)$

(e) $p \rightarrow \Box(\Diamond p \rightarrow p)$

(f) $\Diamond(p \wedge \Box q) \rightarrow \Box(p \vee \Diamond q)$