Principles of Knowledge Representation and Reasoning

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Summer Semester 2008

Exercise Sheet 2

Due: May 6th, 2008

Exercise 2.1 (Propositional Logic, 1+2+2)

(a) What is the difference between $\phi \leftrightarrow \psi$ and $\phi \equiv \psi$?

(b) Let Δ be a *finite* set of clauses. Prove that one can derive all clauses D with $D \in R^*(\Delta)$ within a finite number of resolution steps.

(c) Show that for infinite unsatisfiable sets of clauses there can be infinite resolution trees that do not derive the empty clause $\square$.
Hint: There is such a set with only two literals in each clause.

Exercise 2.2 (Predicate Logic, 2+2+1)

(a) Classify the following expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. The usage of symbols complies with the convention introduced with the syntax of predicate logic.

(a) $P(x, y)$
(b) $f(a, b)$
(c) $\mathcal{I} \models P(a, f(b))$
(d) $\mathcal{I}, \alpha \models P(a, f(x))$
(e) $f(g(x), b)$
(f) $Q(x)$ is satisfiable.
(g) $\exists x(P(x, y) \wedge Q(x)) \vee P(y, x)$
(h) $\forall x(\exists y(P(x, y) \wedge Q(x)) \vee P(x, y))$
(i) $\forall x(\forall y(P(x, y) \wedge Q(x) \vee P(f(y), x)))$
(j) $Q(x) \vee P(x, y) \equiv P(x, y) \vee Q(x)$

(b) Consider the following theory:

$$\Theta = \left\{ \begin{array}{l} \forall x(\neg P(x, x)) \\ \forall x(\forall y(\forall z((P(x, y) \wedge P(y, z)) \rightarrow P(x, z)))) \\ \forall x(\forall y(P(x, y) \vee x = y \vee P(y, x))) \end{array} \right\}$$

Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models \Theta$. Why is it not necessary to specify a variable map to state a model of Θ?

(c) Are there also models of Θ with an infinite $\mathcal{D}$? Justify your answer.