Principles of Knowledge Representation and Reasoning

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Exercise Sheet 1

Due: April 29th, 2008

Exercise 1.1 (Propositional Logic, 1+1+3)

(a) Is the infinite set of clauses

$$S = \{A_1 \vee A_2, \neg A_2 \vee \neg A_3, A_3 \vee A_4, \neg A_4 \vee \neg A_5, \dots\}$$

satisfiable?

(b) Show that $(A \vee \neg(B \wedge A)) \wedge (C \vee (D \vee C))$ is logically equivalent to $(C \vee D)$ by applying the equivalences from the lecture.

(c) Prove that there is no polynomial time algorithm that transforms arbitrary propositional formulae to equivalent formulae in CNF.

Hint: Find a family of formulae in DNF with n variables for which *every* equivalent formula in CNF must consist of an exponential number of clauses. Be careful: It is not sufficient to show that the transformations that have been presented in the lecture lead to an exponential growth. (Why?)

Exercise 1.2 (Resolution and Horn Clauses, 2+3)

(a) Use resolution to show that

$$F = (\neg B \wedge \neg C \wedge D) \vee (\neg B \wedge \neg D) \vee (C \wedge D) \vee B$$

is a tautology (valid).

(b) Prove or disprove: For each propositional formula ϕ there is a Horn formula (a set of Horn clauses) that is logically equivalent to ϕ.

Hint: Prove first the following property of satisfiable Horn formulae: the interpretation that makes a variable true iff the variable is true in all models of the formula is also a model of the formula.