Foundations of Artificial Intelligence

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Exercise Sheet 10

Due: Friday, July 11, 2008

Exercise 10.1 (Conditional probabilities)

Suppose you are given a bag containing n unbiased coins, out of which $n - 1$ are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

(b) Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?

Exercise 10.2 (Conditional independence)

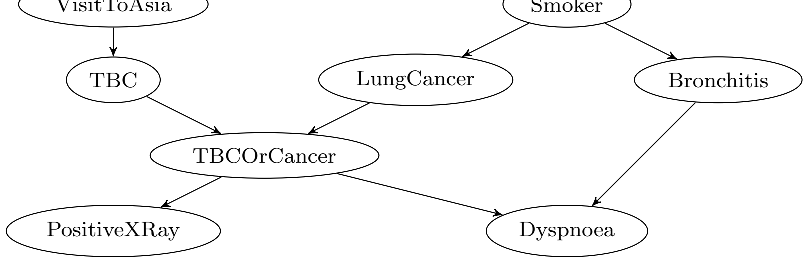
This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $\mathbf{P}(X|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?
   (i) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$
   (ii) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1, E_2|X)$
   (iii) $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$

(b) Suppose we know that $\mathbf{P}(E_1|X, E_2) = \mathbf{P}(E_1|X)$ for all values of X, E_1, and E_2. Now which of the three sets are sufficient?

Exercise 10.3 (Bayesian Networks)

Consider the following Bayesian network:



(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network ($Ind(U, V \mid W)$ denotes that U is conditionally independent of V given W, and $Ind(U, V)$ denotes unconditional independence of U and V):
   (i) $Ind(TBC, VisitToAsia)$
   (ii) $Ind(VisitToAsia, Smoker)$
   (iii) $Ind(VisitToAsia, PositiveXRay \mid TBCOrCancer)$
   (iv) $Ind(VisitToAsia, Dyspnoea \mid TBCOrCancer)$
   (v) $Ind(TBC, Smoker \mid PositiveXRay)$

(b) Compute $P(Dyspnoea|Smoker, \neg TBC)$. The relevant entries in the conditional probability tables are given below:

$$
\begin{aligned}
P(LungCancer|Smoker) &= 0.1 \\
P(LungCancer|\neg Smoker) &= 0.01 \\
P(Bronchitis|Smoker) &= 0.2 \\
P(Bronchitis|\neg Smoker) &= 0.1 \\
P(TBCOrCancer|TBC, LungCancer) &= 1 \\
P(TBCOrCancer|TBC, \neg LungCancer) &= 1 \\
P(TBCOrCancer|\neg TBC, LungCancer) &= 1 \\
P(TBCOrCancer|\neg TBC, \neg LungCancer) &= 0 \\
P(Dyspnoea|TBCOrCancer, Bronchitis) &= 0.9 \\
P(Dyspnoea|TBCOrCancer, \neg Bronchitis) &= 0.7 \\
P(Dyspnoea|\neg TBCOrCancer, Bronchitis) &= 0.6 \\
P(Dyspnoea|\neg TBCOrCancer, \neg Bronchitis) &= 0.05
\end{aligned}
$$

The exercise sheets may and should be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] `http://www.informatik.uni-freiburg.de/~ki/teaching/ss08/gki/coverSheet-english.pdf`