Foundations of AI

4. Informed Search Methods

Heuristics, Local Search Methods, Genetic Algorithms

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- Best-First Search
- A* and IDA*
- Local Search Methods
- Genetic Algorithms

Best-First Search

Search procedures differ in the way they determine the next node to expand.

Uninformed Search: Rigid procedure with no knowledge of the cost of a given node to the goal.

Informed Search: Knowledge of the cost of a given node to the goal is in the form of an *evaluation function* *f* or *h*, which assigns a real number to each node.

Best-First Search: Search procedure that expands the node with the "best" *f*- or *h*-value.

General Algorithm

```
function BEST-FIRST-SEARCH(problem, EVAL-FN) returns a solution sequence
  inputs: problem, a problem
          Eval-Fn, an evaluation function

  Queueing-Fn ← a function that orders nodes by EVAL-FN
  return GENERAL-SEARCH(problem, Queueing-Fn)
```

When *h* is always correct, we do not need to search!

Greedy Search

A possible way to judge the “worth” of a node is to estimate its distance to the goal.

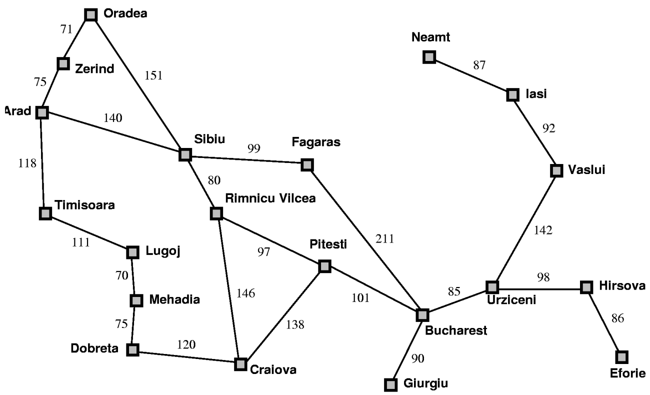
h(n) = estimated distance from n to the goal

The only real condition is that *h(n) = 0* if *n* is a goal.

A best-first search with this function is called a *greedy search.*

Route-finding problem: *h* = straight-line distance between two locations.

Greedy Search Example



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
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Pitesti	98
Rimnicu Vilcea	193
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Timisoara	329
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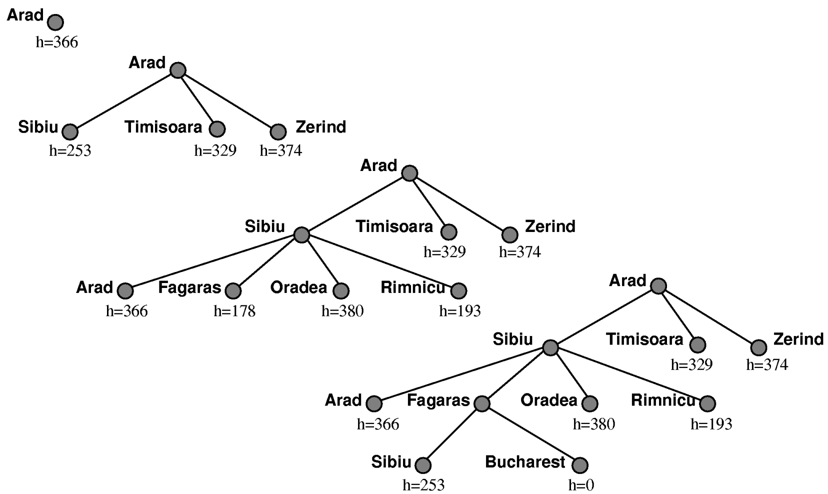
Vaslui	199
Zerind	374

Greedy Search from *Arad* to *Bucharest*



Heuristics

The evaluation function *h* in greedy searches is also called a *heuristic* function or simply a *heuristic.*

- The word *heuristic* is derived from the Greek word ευρισκειν (note also: ευρηκα!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and *focuses* the search!

A*: Minimization of the estimated path costs

A* combines the greedy search with the uniform-search strategy.

g(n) = actual cost from the initial state to *n*.

h(n) = estimated cost from *n* to the next goal.

f(n) = g(n) + h(n), the estimated cost of the cheapest solution through *n*.

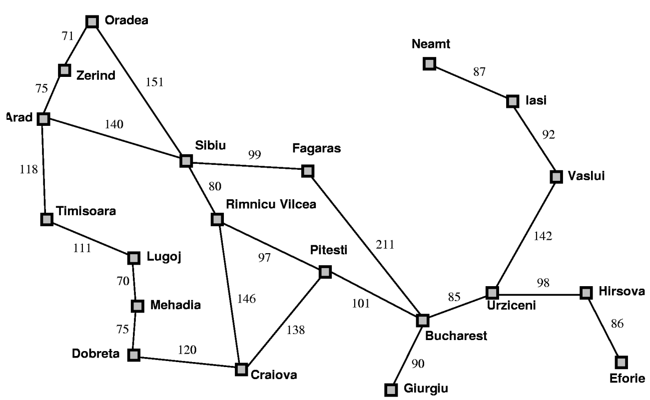
Let h*(n) be the actual cost of the optimal path from *n* to the next goal.

h is *admissible* if the following holds for all *n* :

$$h(n) \leq h^*(n)$$

We require that for A*, *h* is admissible (straight-line distance is admissible).

A* Search Example



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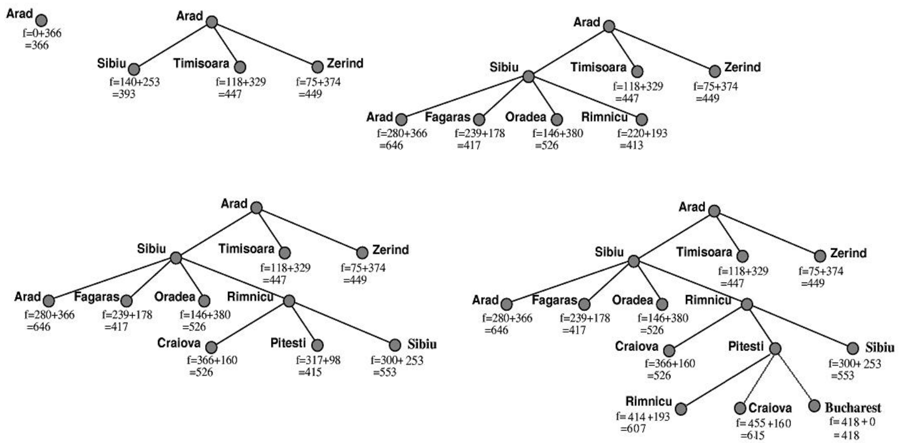
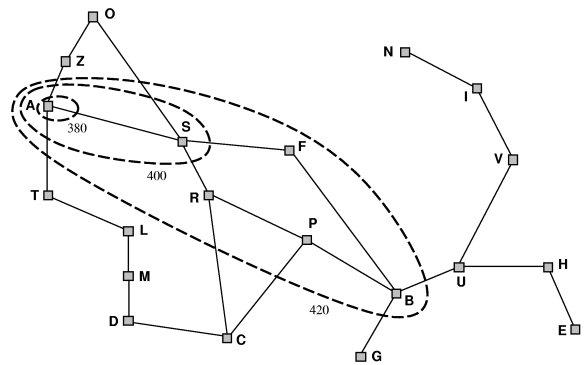
A* Search from *Arad* to *Bucharest*



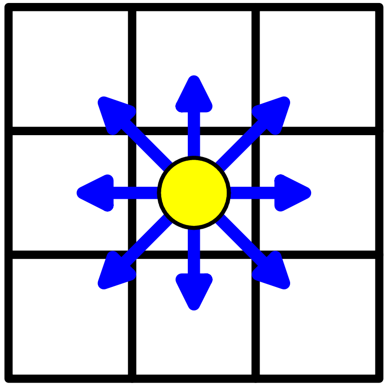
Contours in A*

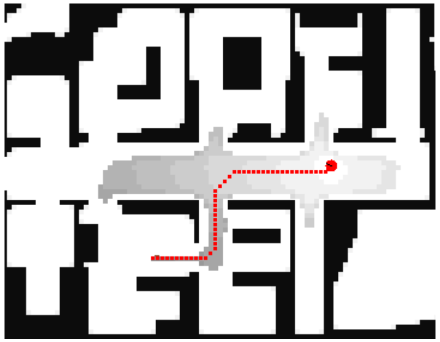
Within the search space, contours arise in which for the given *f*-value all nodes are expanded.



Contours at f = 380, 400, 420

Example: Path Planning for Robots in a Grid-World

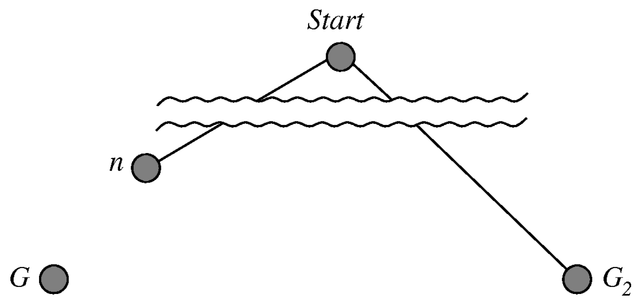




Optimality of A*

Claim: The first solution found has the minimum path cost.

Proof: Suppose there exists a goal node G with optimal path cost f^*, but A* has found another node G_2 with $g(G_2) > f^*$.



Let n be a node on the path from the start to G that has not yet been expanded. Since h is admissible, we have

$$f(n) \leq f^*.$$

Since n was not expanded before G_2, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$

It follows from $h(G_2) = 0$ that

$$g(G_2) \leq f^*.$$

→ Contradicts the assumption!

Completeness and Complexity

Completeness:

If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant δ such that every operator has at least cost δ.

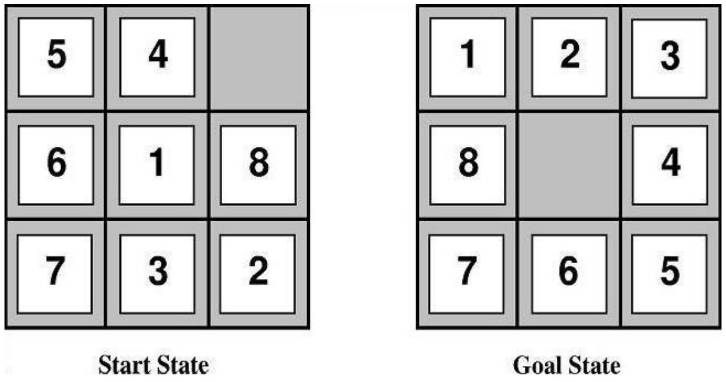
→ Only a finite number of nodes n with $f(n) \leq f^*$.

Complexity:

In the case where $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only a sub-exponential number of nodes will be expanded.

Normally, growth is exponential because the error is proportional to the path costs.

Heuristic Function Example



h_1 = the number of tiles in the wrong position
h_2 = the sum of the distances of the tiles from their goal positions (*Manhatten distance)*

Empirical Evaluation

- *d* = distance from goal
- Average over 100 instances

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Iterative Deepening A* Search (IDA*)

Idea: A combination of IDS and A*. All nodes inside a contour are searched.

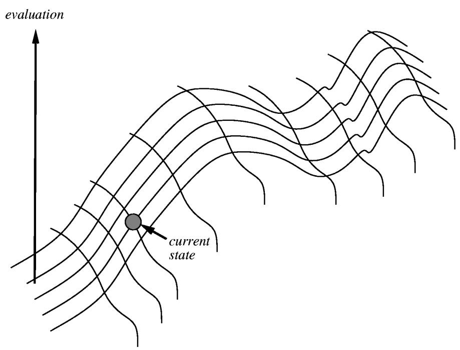
```
function IDA*(problem) returns a solution sequence
  inputs: problem, a problem
  static: f-limit, the current f-COST limit
          root, a node

  root ← MAKE-NODE(INITIAL-STATE[problem])
  f-limit ← f-COST(root)
  loop do
      solution, f-limit ← DFS-CONTOUR(root, f-limit)
      if solution is non-null then return solution
      if f-limit = ∞ then return failure; end
---------------------------------------------------------------
function DFS-CONTOUR(node, f-limit) returns a solution sequence and a new f-COST limit
  inputs: node, a node
          f-limit, the current f-COST limit
  static: next-f, the f-COST limit for the next contour, initially ∞

  if f-COST[node] > f-limit then return null, f-COST[node]
  if GOAL-TEST[problem](STATE[node]) then return node, f-limit
  for each node s in SUCCESSORS(node) do
      solution, new-f ← DFS-CONTOUR(s, f-limit)
      if solution is non-null then return solution, f-limit
      next-f ← MIN(next-f, new-f); end
  return null, next-f
```


Local Search Methods

In many problems, it is unimportant how the goal is reached – only the goal itself matters (8-queens problem, VLSI Layout, TSP).

If in addition a quality measure for states is given, a **local search** can be used to find solutions.

Idea: Begin with a randomly-chosen configuration and improve on it stepwise → **Hill Climbing**.

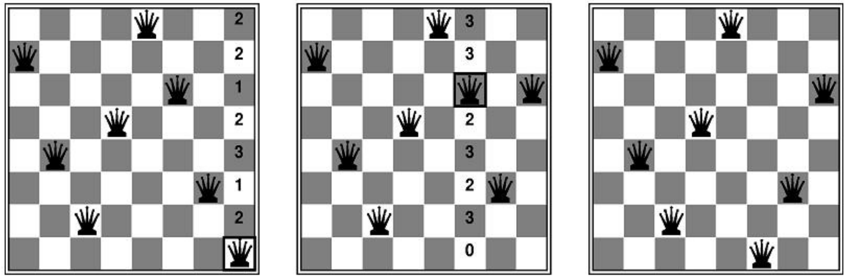


Hill Climbing

```
function HILL-CLIMBING(problem) returns a solution state
  inputs: problem, a problem
  static: current, a node
          next, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
     next ← a highest-valued successor of current
     if VALUE[next] < VALUE[current] then return current
     current ← next
  end
```


Example: 8-Queens Problem

Selects a column and moves the queen to the square with the fewest conflicts.



Problems with Local Search Methods

- *Local maxima*: The algorithm finds a sub-optimal solution.
- *Plateaus*: Here, the algorithm can only explore at random.
- Ridges: Similar to plateaus.

Solutions:

- *Start over* when no progress is being made.
- "Inject smoke" → random walk
- Tabu search: Do not apply the last *n* operators.

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.

Simulated Annealing

In the simulated annealing algorithm, "smoke" is injected systematically: first a lot, then gradually less.

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"
  static: current, a node
          next, a node
          T, a "temperature" controlling the probability of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
     T ← schedule[t]
     if T=0 then return current
     next ← a randomly selected successor of current
     ΔE ← VALUE[next] – VALUE[current]
     if ΔE > 0 then current ← next
     else current ← next only with probability e^(ΔE/T)
```

Has been used since the early 80's for VSLI layout and other optimization problems.

Genetic Algorithms

Evolution appears to be very successful at finding good solutions.

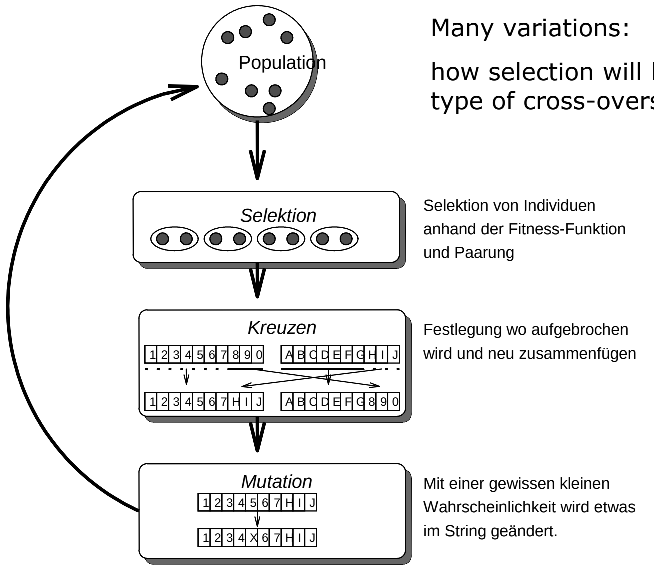
Idea: Similar to evolution, we search for solutions by "crossing", "mutating", and "selecting" successful solutions.

Ingredients:

- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

Example: 8-queens problem as a chain of 8 numbers. Fitness is judged by the number of non-attacks. The population consists of a set of arrangements of queens.

Selection, Mutation, and Crossing



Many variations:

how selection will be applied, what type of cross-overs will be used, etc.

Summary

- Heuristics focus the search
- Best-first search expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal h we obtain a greedy search.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e., h^* is never overestimated, we obtain the A* search, which is complete and optimal.
- IDA* is a combination of the iterative-deepening and A* searches.
- Local search methods only ever work on one state, attempting to improve it step-wise.
- Genetic algorithms imitate evolution by combining good solutions.