Foundations of Artificial Intelligence

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Exercise Sheet 7

Due: Friday, June 15, 2007

Exercise 7.1 (Semantics of First Order Predicate Logic)
Consider the interpretation $\mathcal{I} = \langle D, \cdot^{\mathcal{I}} \rangle$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D$, $plus^{\mathcal{I}}(a,b) = (a+b) \mod 4$

and the variable assignment $\alpha = \{(x,0), (y,1)\}$.
Decide for the following formulae ϕ_i if $\mathcal{I}$ is a model for ϕ_i under α, i.e. if $\mathcal{I}, \alpha \models \phi_i$. Explain your answer.

(a) $\phi_1 = odd(y) \wedge even(two)$
(b) $\phi_2 = \forall x (even(x) \vee odd(x))$
(c) $\phi_3 = \forall x \exists y \, lessThan(x,y)$
(d) $\phi_4 = \forall x (even(x) \to \exists y \, lessThan(x,y))$
(e) $\phi_5 = \forall x (odd(x) \to even(plus(x,y)))$

Exercise 7.2 (Skolem Normal Form, Herbrand Expansion)

(a) Transform the following formula into Skolem normal form:

$$\neg\forall x Q(x) \wedge \forall y \exists z \, (\neg\forall x \exists y \forall t \neg R(x,y,z,t) \wedge P(g(z),y))$$

(b) Write down the ten smalles terms of the Herbrand universe and the ten smallest formulae of the Herbrand expansion of the following formula:

$$\forall x \forall y P(c, f(x,b), g(y))$$

Exercise 7.3 (Substitution, Unification, Resolution)

(a) Can the following formulae be unified? If so, specify the most general unifier and the resulting formula. If not, why?

(i) $P(f(y), w, g(z,y))$ und $P(x, x, g(z, A))$
(ii) $F(x, g(f(a), u))$ und $F(g(u,v), x)$
(iii) $Q(y,y)$ und $Q(x, f(x))$

(b) Let $KB = \{\forall x \forall y (P(x,y) \wedge P(f(x),y) \to Q(y)), \forall x \forall y (P(x,y) \to P(f(x),y))\}$. Use first order resolution to show (graphically) that $KB \models \forall y (\exists x P(x,y) \to Q(y))$ holds. Indicate clearly the substitutions you made and the unifiers you used.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.