Foundations of Artificial Intelligence

Prof. Dr. B. Nebel, Prof. Dr. W. Burgard — University of Freiburg
C. Plagemann, P. Pfaff, D. Zhang, R. Mattmüller — Department of Computer Science
Summer Term 2007

Exercise Sheet 5

Due: Friday, May 25, 2007

Exercise 5.1 (Truth Tables, Models)

(a) Use truth tables to prove the validity of the following equivalences:

(i) $(\alpha \rightarrow \beta) \equiv (\neg\alpha \vee \beta)$
(ii) $(\alpha \rightarrow \beta) \equiv (\neg\beta \rightarrow \neg\alpha)$
(iii) $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$
(iv) $((\alpha \wedge \beta) \vee \gamma) \equiv ((\alpha \vee \gamma) \wedge (\beta \vee \gamma))$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(i) $(A \wedge B) \vee (B \wedge C)$
(ii) $A \vee B$
(iii) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 5.2 (Modeling, Proofs)

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?
If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Exercise 5.3 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show that $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 5.4 (Functional Completeness, Negation Normal Form)

In this assignment, we only consider propositional logic *without* the implication and biimplication operators $\rightarrow$ and $\leftrightarrow$. A propositional formula is in *Negation Normal Form* if negation symbols $\neg$ only appear immediately in front of atoms.

(a) Show: Each propositional formula can be transformed into an equivalent formula only containing the operators $\neg$ and $\vee$.

(b) Show: Each propositional formula can be transformed into an equivalent formula in Negation Normal Form.

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.