# Planning with partial observability





- If not all can be observed, plans cannot be defined as mappings from states to actions because current state is not in general known.
- If something can be observed, plans cannot be defined as sequences of actions because different observations suggest different actions.
- A more general notion of plans is needed that generalizes both action sequences and state-to-action mappings.

# Conditional plans





- A conditional plan is essentially a finite automaton (a graph).
- The nodes in the graph represent all the relevant information from earlier observations.
- For the reachability and maintenance objectives this information could just as well be represented by the belief state, and plans could be in principle defined also as mappings from belief states to actions. (This is, however, not sufficient for some more general objectives.)

## Definition

Let $\Pi = \langle A, I, O, G, V \rangle$ be a succinct transition system. A plan for $\Pi$ is a triple $\langle N, b, l \rangle$ where

1. $N$ is a finite set of nodes,
2. $b \subseteq \mathcal{L} \times N$ maps initial states to starting nodes, and
3. $l : N \rightarrow O \times 2^{\mathcal{L} \times N}$ is a function that assigns each node $n$ an operator and a set of pairs $\langle \phi, n' \rangle$ where $\phi$ is a formula over the observable state variables $V$ and $n' \in N$ is a successor node.

   Nodes $n$ with $l(n) = \langle o, \emptyset \rangle$ for some $o \in O$ are terminal nodes.

# Conditional plans
## Execution



1. Plan execution starts from a node $n \in N$ and state $s$ such that $\langle \phi, n \rangle \in b$ and $s \models I \wedge \phi$.
2. Execution in node $n$ with $l(n) = \langle o, B \rangle$:
    1. Execute $o$.
    2. If $\phi$ is true in all possible current states for some $\langle \phi, n' \rangle \in B$ then continue execution from $n'$.
       At most one $\phi$ may be true for this to be well-defined.
       Plan execution ends when none of the branch labels matches the current state. In a terminal node plan execution necessarily ends.
3. Definition of execution graphs has to take into account the plan node: the nodes of the execution graphs are pairs $(s, n)$ where $s$ is a state of the transition system and $n$ is a plan node.
4. There is an edge from $(s, n)$ to $(s', n')$ if executing the action for plan node $n$ in $s$ may lead to state $s'$ and node $n'$.

# Conditional plans
## Example

# Execution graphs of conditional plans



**Definition (Execution graph of a conditional plan)**

Let $\langle A, I, O, G, V\rangle$ be a transition system and $\pi = \langle N, b, l\rangle$ a plan. The execution graph of $\pi$ is $\langle M, E\rangle$ where

1. $M = S \times (N \cup \{\bot\})$ where $S$ is all valuations of $A$,
2. $E \subseteq M \times M$ with an edge from $\langle s, n\rangle$ to $\langle s', n'\rangle$ iff
   1. $l(n) = \langle o, B\rangle$ and
   2. for some $\langle \phi, n'\rangle \in B$ $s' \in \mathit{img}_o(s)$ and $s' \models \phi$.

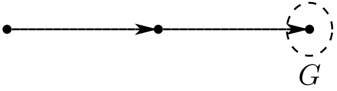
   There is an edge from $\langle s, n\rangle$ to $\langle s', \bot\rangle$ iff
   1. $l(n) = \langle o, B\rangle$,
   2. $s' \in \mathit{img}_o(s)$, and
   3. there is no $\langle \phi, n'\rangle \in B$ such that $s' \models \phi$.

The initial nodes are $\langle s, n\rangle$ such that $s \models I$ and $s \models \phi$ for some $\langle \phi, n\rangle \in b$.
The goal nodes are $\langle s, n\rangle$ such that $s \models G$.

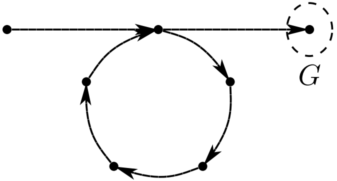
# Summary of objectives



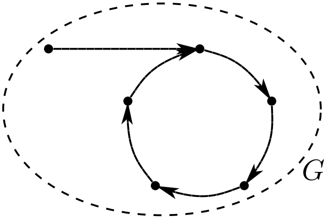
Bounded Reachability




Unbounded Reachability




Maintenance



# Mapping memoryless plans to conditional plans



## Definition

Let $\langle A, I, O, G, V \rangle$ be a transition system. Let $S$ be the set of all states on $A$. Let $\pi : S \rightarrow O$ be a memoryless plan. Define $C(\pi) = \langle N, b, l \rangle$ where

1. $N = O$,
2. $b = \{\langle \mathit{FMA}(\{s \in S | \pi(s) = o\}), o\rangle | o \in O\}$, and
3. $l(o) = (o, \{\langle \mathit{FMA}(\{s \in S | \pi(s) = o\}), o'\rangle | o' \in O\})$ for all $o \in O$.

Above $\mathit{FMA}(T)$ is a formula $\phi$ such that $T = \{s \in S | s \models \phi\}$. The memoryless plan $\pi$ corresponds the conditional plan $C(\pi)$ in the sense that the subgraphs induced by the initial nodes are isomorphic, and this isomorphism preserves both initial and goal nodes.

# Sufficiency of memoryless plans for full observability

- **Heuristic (forward) search with AND-OR trees.**
  AO$*$ (plans without loops), LAO$*$ (with loops)
- **Dynamic programming (backward search)**
  Start from the set of goal states.
  Find state sets from which already generated state sets can be reached by actions and branching.
- **Reduction to full observability**
  For reachability and maintenance goals the planning problem is in principle solvable by reduction to fully observable planning in the belief space. But this is impractical because there are $2^n$ belief states for $n$ states, and $2^{2^m}$ belief states for $m$ state variables.

# Conditional planning with and-or search



## AND-OR trees for conditional planning

**OR nodes 1** Choice of action

**OR nodes 2** Choice of observations

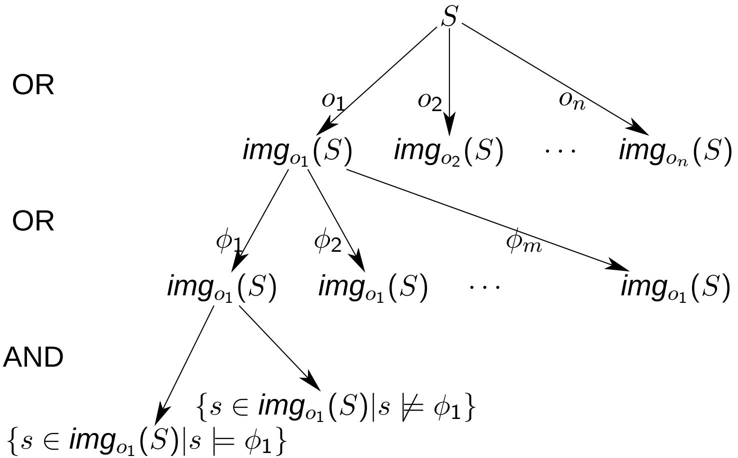
**AND nodes** Nondeterminism (actions / observations)

## Binary branching vs. general branching

Conditional plans can be defined with binary branching (IF-THEN-ELSE) or with $n$-ary branching (CASE/SWITCH). Latter can always be reduced to former.

# Conditional planning with and-or search
## Example






For $n$ observable state variables there are $m = \frac{2^n - 2}{2}$ non-trivial formulae $\phi_i$ to consider for binary branching.

# Conditional planning with and-or search



- Conflict between plan size and branching:
  1. If all observations are always used, plans become very big.
  2. If not all observations are used it may be impossible to find a plan.

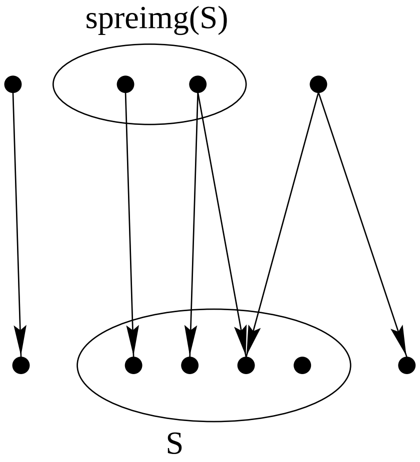
  Trying out all possible ways to branch is not feasible. No good general solutions to this problem exist.
- AND-OR search algorithms use heuristics for making branching decisions.

# Backward search algorithms



- Flavor similar to the backward algorithms for fully observable problems.
- Backward steps with operator applications: strong preimages.
- Backward steps with branching: strong preimage of the union of belief states from different observational classes.

# Regression/preimages

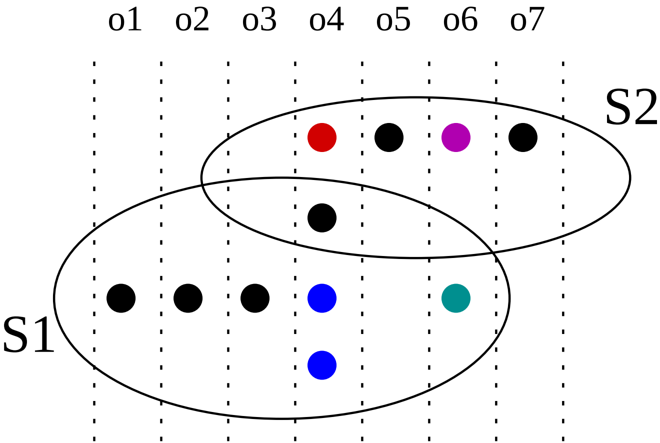


# Branching in backward search



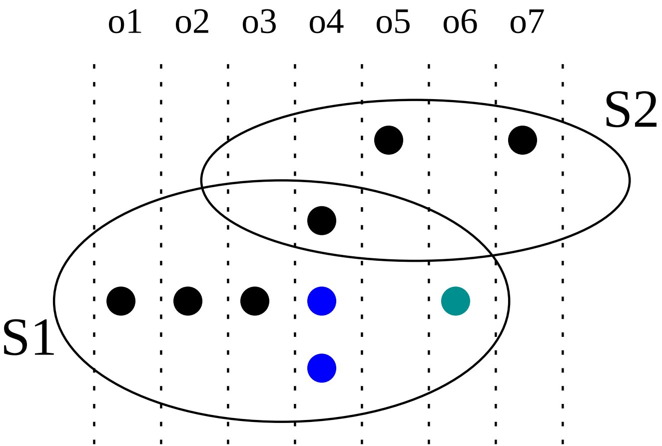
- Let the observational classes be $C_1, \ldots, C_n$.
- Let $B_1, B_2, \ldots, B_n$ be sets of states with plans so that for all $i, j$ such that $i \neq j$ there is no observational class $C \in \{C_1, \ldots, C_n\}$, such that $S_i \cap C \neq \emptyset$ and $S_j \cap C \neq \emptyset$. Now they can be combined to $B = B_1 \cup \cdots \cup B_n$ that has a plan starting with a branch.
- We may pick exactly one belief $B_i$ state from every observational class.

# Branching



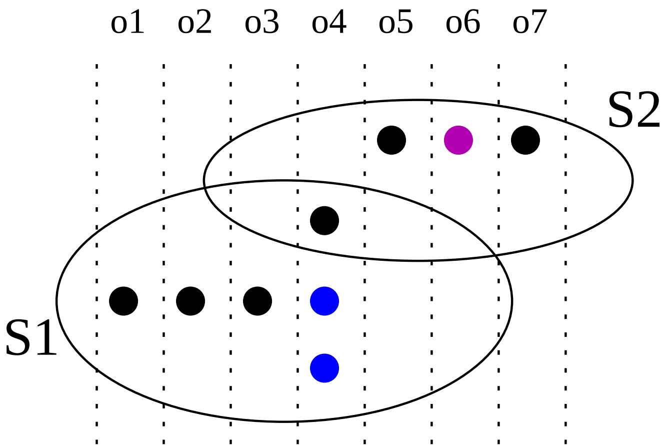
# Combination 11

4 observational classes with choice between plan plan for $S1$ or $S_2$



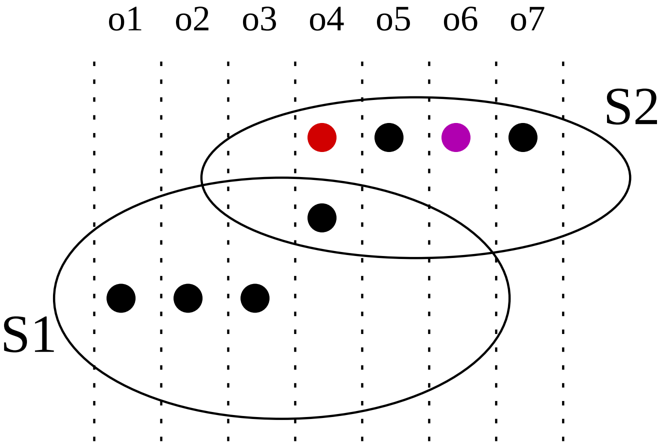
# Combination 12

4 observational classes with choice between plan plan for $S1$ or $S_2$



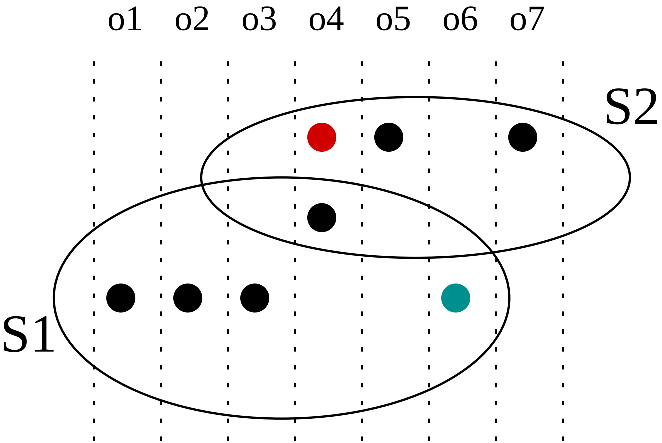
# Combination 22

4 observational classes with choice between plan plan for $S1$ or $S_2$



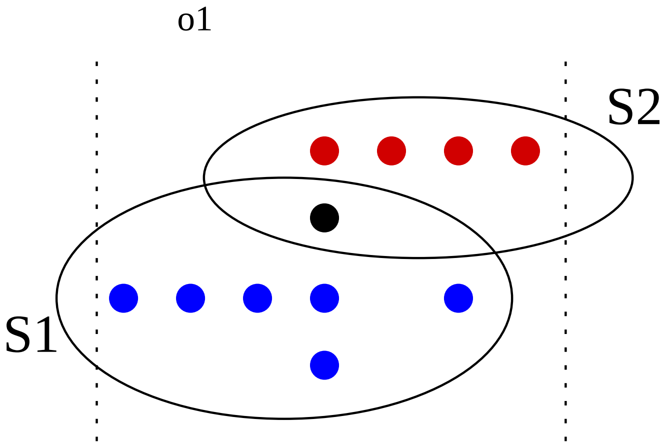
# Combination 21

4 observational classes with choice between plan plan for $S1$ or $S_2$



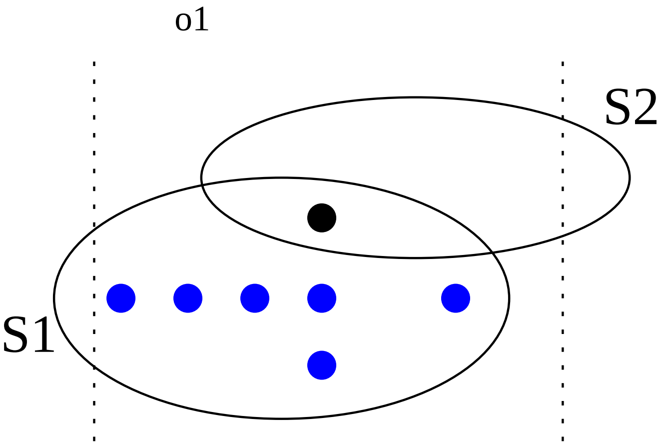
# No observability ⇒ No branching

Only one observational class: no choice between subplans



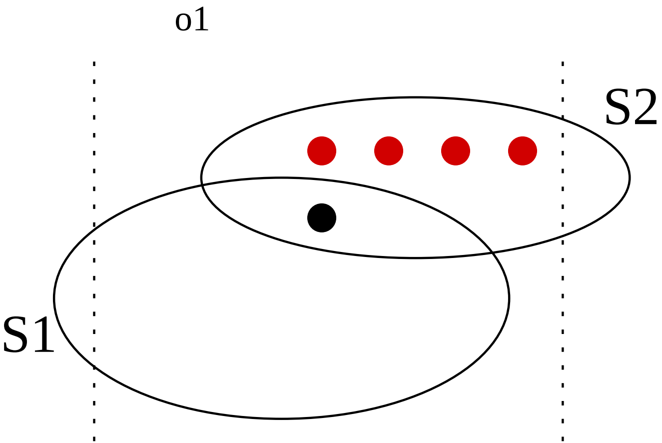
# Combination 1

No choice between subplans during execution



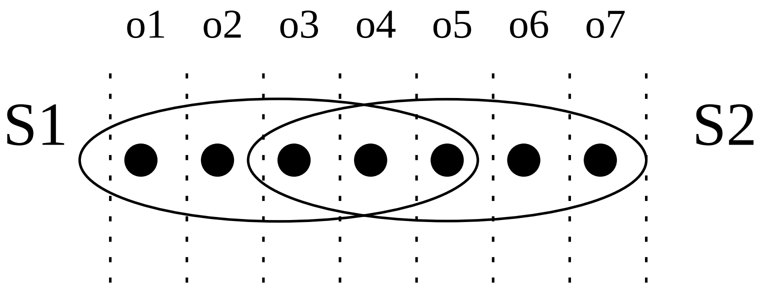
# Combination 2

No choice between subplans during execution



# Combination with full observability

Different plan can be used for every state



# One step in the backward algorithm



1. Pick from each observational class one belief state.
2. Compute the strong preimage of their union w.r.t. operator $o$.
3. Split the resulting set of states to belief states for different observational classes.

# Algorithm idea: construction of plans





If plans for belief states $Z_1, \ldots, Z_n$, respectively corresponding to observational classes $C_1, \ldots, C_n$, are $\pi_1, \ldots, \pi_n$, then a plan for a new belief state is

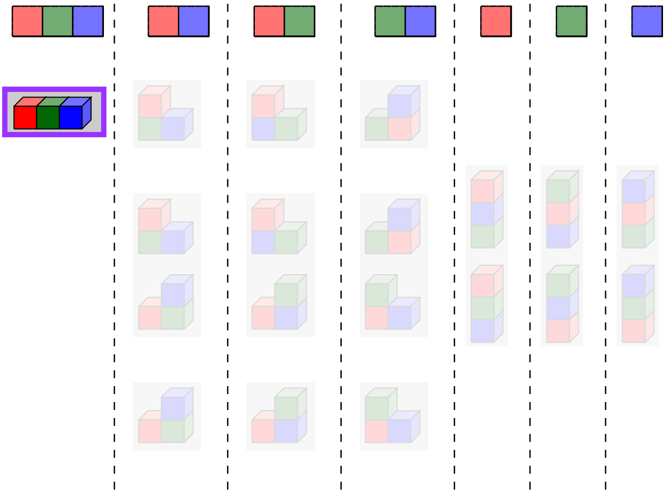
1. Apply $o$.
2. If new current state is in $C_i$ for $i \in \{1, \ldots, n\}$, continue with $\pi_i$.

# Backward search in the belief space
## Example



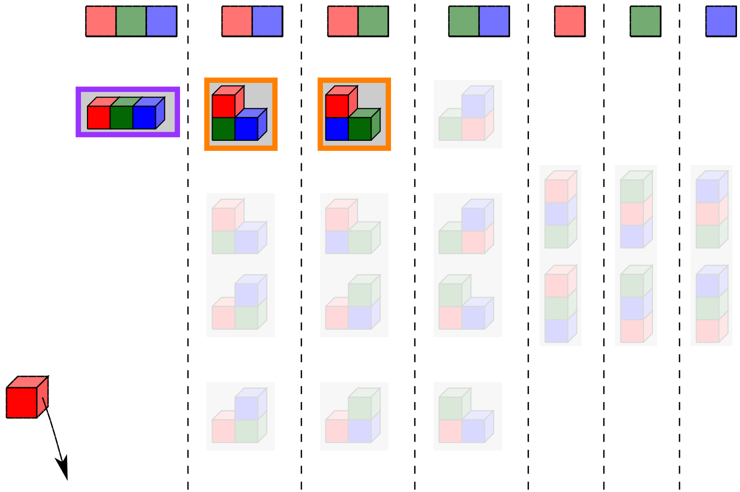
- Blocks world with 3 blocks
- Goal: all blocks are on the table
- Only the variables clear(X) are observable.
- A block can be moved onto the table if the block is clear.
- 8 observational classes corresponding to the 8 valuations of {clear(A), clear(B), clear(C)} (one of the valuations does not correspond to a blocks world state.)

# Plan construction by backward search
## Example: goal belief state

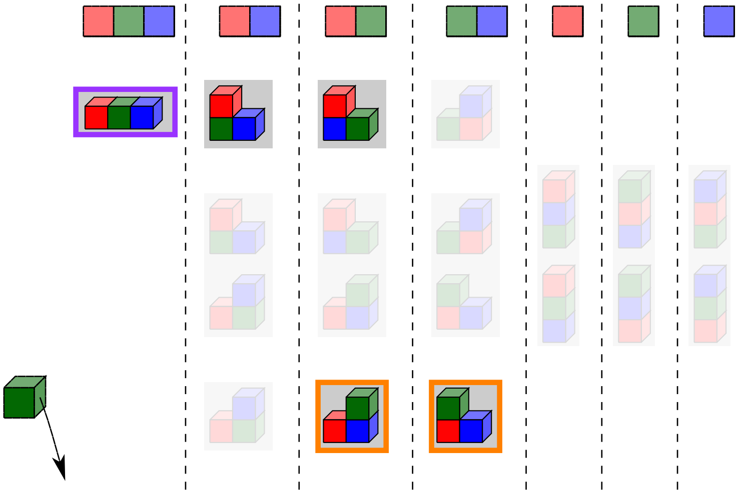


# Plan construction by backward search

## Example: backward step with red-block-onto-table



# Plan construction by backward search
## Example: backward step with green-block-onto-table



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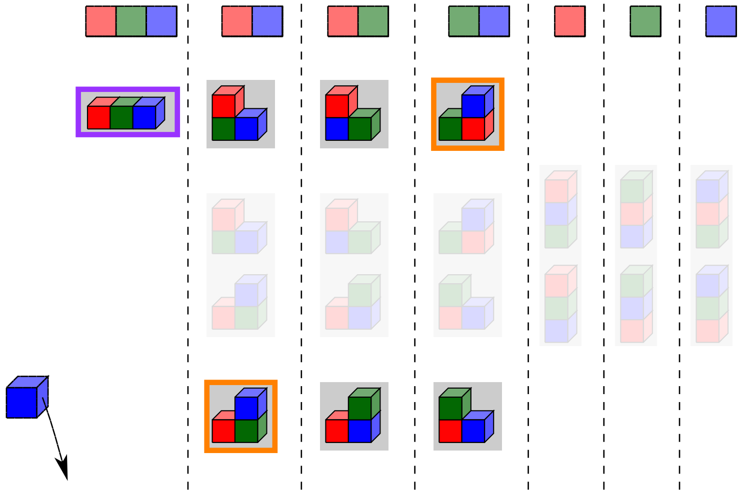

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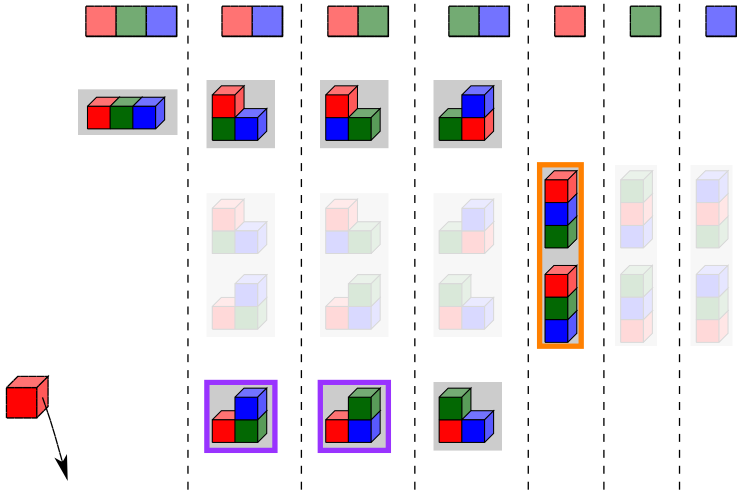

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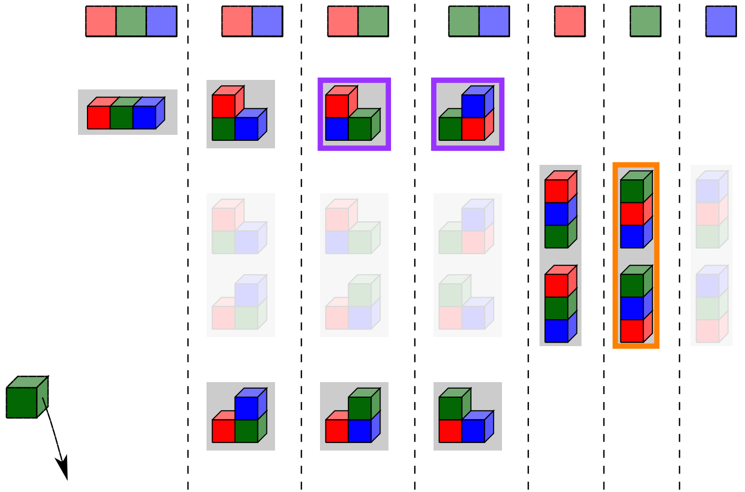

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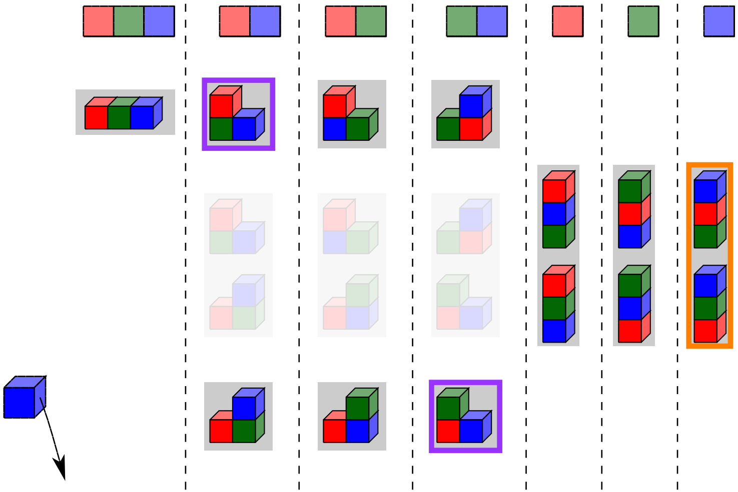

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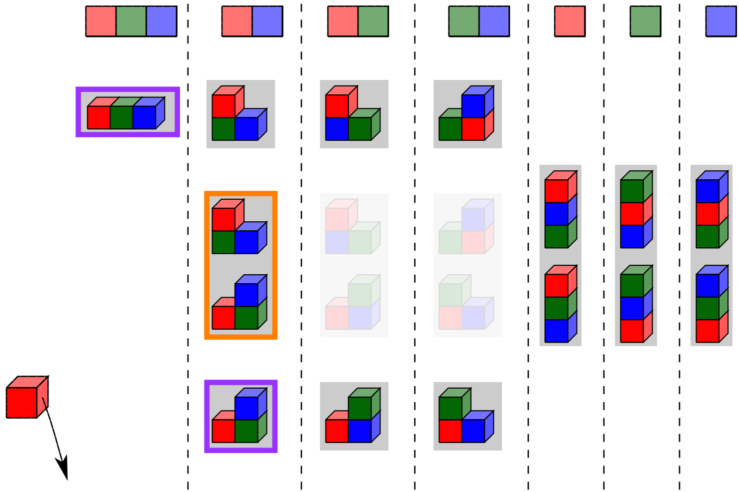
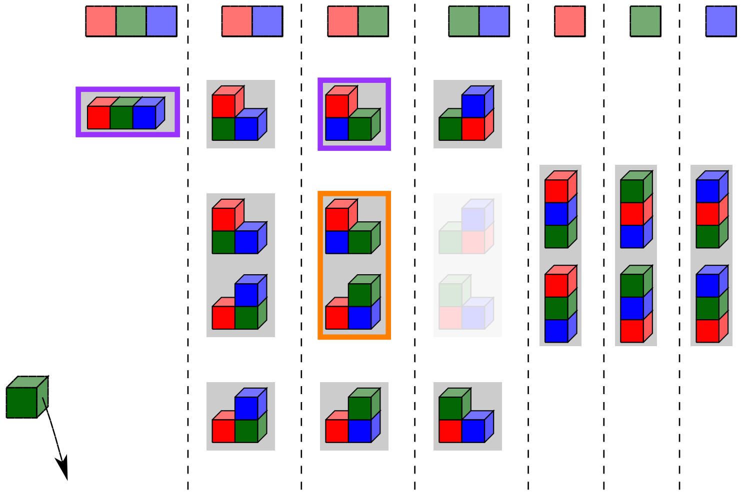


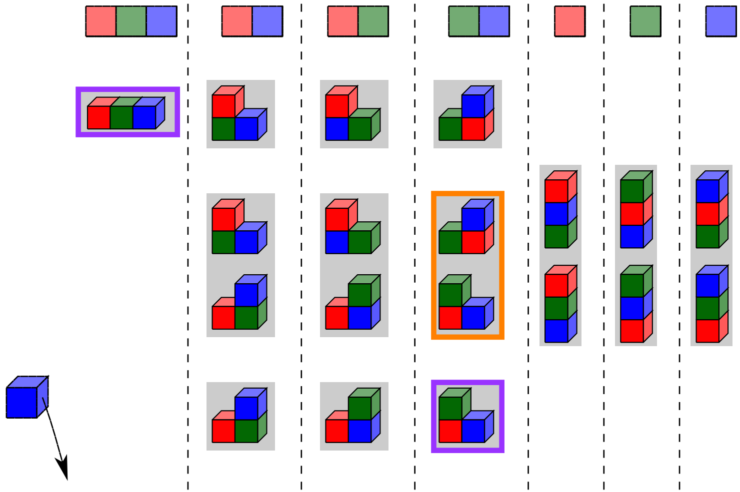
# Plan construction by backward search

## Example: backward step with green-block-onto-table



# Plan construction by backward search

## Example: backward step with blue-block-onto-table



# Data structure



**Definition (Belief space)**

Let $P = (C_1, \ldots, C_n)$ be a partition of the set of all states. Then *a belief space* is an $n$-tuple $\langle G_1, \ldots, G_n \rangle$ where $G_i \subseteq 2^{C_i}$ for all $i \in \{1, \ldots, n\}$ and $B \not\subset B'$ for all $i \in \{1, \ldots, n\}$ and $\{B, B'\} \subseteq G_i$.

A belief space is a set of belief states partitioned to subsets corresponding to the observational classes.
The simplest belief spaces are obtained from sets $B$ of states as $\mathcal{F}(B) = \langle \{C_1 \cap B\}, \ldots, \{C_n \cap B\} \rangle$.

# Data structure



A belief space is extended as follows.

**Definition (Extension)**

Let $P = (C_1, \ldots, C_n)$ be the partition of all states, $G = \langle G_1, \ldots, G_n \rangle$ a belief space, and $T$ a set of states. Define $G \oplus T$ as $\langle G_1 \Cup (T \cap C_1), \ldots, G_n \Cup (T \cap C_n) \rangle$ where the operation $\Cup$ adds the latter set of states to the former set of sets of states and eliminates sets that are not set-inclusion maximal, defined as $U \Cup V = \{R \in U \cup \{V\} | R \not\subset K \text{ for all } K \in U \cup \{V\}\}$.

# Data structure



A factored belief space $G = \langle G_1, \ldots, G_n \rangle$ can be viewed as representing the set of sets of states

$$\text{flat}(G) = \{s_1 \cup \cdots \cup s_n | s_i \in G_i \text{ for all } i \in \{1, \ldots, n\}\}.$$

The cardinality of flat$(G)$ is $|G_1| \cdot |G_2| \cdot \ldots \cdot |G_n|$.

# Data structure: membership test



**Theorem**

*For belief spaces $G$ and state sets $B$, testing whether there is $B' \in$ flat$(G)$ such that $B \subseteq B'$, and computing $G \oplus B$ takes polynomial time.*

**Proof.**

This is simply by testing whether for all $i \in \{1, \ldots, n\}$ there is $B' \in G_i$ such that $B \cap C_i \subseteq B'$. □

# Complexity of finding new belief states



The basic backward step in algorithms for partial observability is computationally difficult.

## Theorem

*Testing if for belief space $G$ there is $R \in$ flat$(G)$ such that preimg$_o(R) \not\subseteq R'$ for all $R' \in$ flat$(G)$ is NP-complete. This holds even for deterministic actions $o$.*

## Proof

Membership in NP is easy: nondeterministically choose $s_i \in G_i$ for every $i \in \{1, \ldots, n\}$, compute the preimage $r$ of $s_1 \cup \cdots \cup s_n$, verify that $r \cap C_i$ for some $C_i$ is not in $G_i$.

# Complexity of finding new belief states

## Proof continues.

NP-hardness is by reduction from SAT. We illustrate the proof by an example. Let $T = \{A \vee B \vee C, \neg A \vee B, \neg C\}$. Construct a belief space so that $T$ is satisfiable iff strong preimage of $o(x) = x_0$ is not in the FBS: clause is mapped to the set of literals not in it; satisfying valuation = a new belief state.

$$\langle \{\{\widehat{A}, \widehat{B}, \widehat{C}\}, \{A, \widehat{B}, C, \widehat{C}\}, \{A, \widehat{A}, B, \widehat{B}, C\}\},$$
$$\{\{A_0\}, \{\widehat{A}_0\}\},$$
$$\{\{B_0\}, \{\widehat{B}_0\}\},$$
$$\{\{C_0\}, \{\widehat{C}_0\}\}\rangle$$

□

# An algorithm for finding new belief states



A new belief state can be find by the following algorithm that runs in worst-case exponential time.

*PROCEDURE* findnew($o$,$A$,$F$,$H$);<br>
*IF* $F = \langle\rangle$ *AND* *preimg*$_o(A) \not\subseteq B$ for all $B \in$ flat$(H)$<br>
*THEN RETURN* $A$; *(* New belief state was found *)*<br>
*IF* $F = \langle\rangle$ *THEN RETURN* $\emptyset$;<br>
$F$ is $\langle\{f_1, \ldots, f_m\}, F_2, \ldots, F_k\rangle$ for some $k \geq 1$;<br>
*FOR* $i$ := 1 *TO* $m$ *DO*<br>
    $B$ := findnew($o$,$A \cup f_i$,$\langle F_2, \ldots, F_k\rangle$,$H$);<br>
    *IF* $B \neq \emptyset$ *THEN RETURN* $B$;<br>
*END*;<br>
*RETURN* $\emptyset$;

# A planning algorithm: plan($I$,$O$,$G$);

*PROCEDURE* plan($I$,$O$,$G$);
$H$ := $\mathcal{F}(G)$;
progress := true;
*WHILE* progress and $I \not\subseteq I'$ for all $I' \in$ flat$(H)$ *DO*
    progress := false;
    *FOR EACH* $o \in O$ *DO*
        $B$ := findnew($o$,$\emptyset$,$H$,$H$);
        *IF* $B \neq \emptyset$ *THEN* *(* New belief state was found *)*
            *BEGIN*
                $H$ := $H \oplus$ *preimg*$_o(B)$; *(* Add it to belief space *)*
                progress := true;
            *END*;
    *END*;
*END*;
*IF* $I \subseteq I'$ for some $I' \in$ flat$(H)$ *THEN RETURN* true
*ELSE RETURN* false;

# Summary



- Planning with partial observabilty in general requires a definition of plans that generalizes plans respectively required in the fully observable and unobservable special cases: mappings state$\longrightarrow$action and sequences of actions.
- Main algorithms:
  1. Reduction to full observability by viewing belief states as states.
  2. AND-OR search forward.
  3. Generation of belief states backward starting from goal belief states.