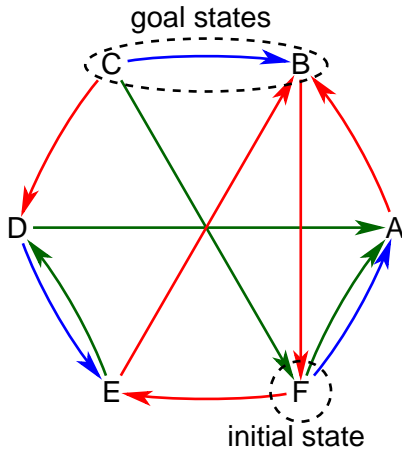


Transition systems



AI Planning

Transition systems

Definition

Example

Matrices

Reachability

Algorithm

Succinct TS

Transition systems

Formalization of the dynamics of the world/application

AI Planning

Definition

A **transition system** is $\langle S, I, \{a_1, \dots, a_n\}, G \rangle$ where

- S is a finite set of **states** (the **state space**),
- $I \subseteq S$ is a finite set of **initial states**,
- every action $a_i \subseteq S \times S$ is a binary relation on S ,
- $G \subseteq S$ is a finite set of **goal states**.

Definition

An action a is **applicable** in a state s if sas' for at least one state s' .

Transition systems

Definition

Example

Matrices

Reachability

Algorithm

Succinct TS

Transition systems

Deterministic transition systems

A transition system is **deterministic** if there is only **one initial state** and all **actions are deterministic**. Hence all future states of the world are completely predictable.

Definition

A **deterministic transition system** is $\langle S, I, O, G \rangle$ where

- S is a finite set of **states** (the **state space**),
- $I \in S$ is **a state**,
- actions $a \in O$ (with $a \subseteq S \times S$) are **partial functions**,
- $G \subseteq S$ is a finite set of **goal states**.

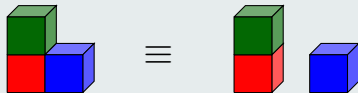
Successor state wrt. an action

Given a state s and an action A so that a is applicable in s , the **successor state** of s with respect to a is s' such that sas' , denoted by $s' = \text{app}_a(s)$.

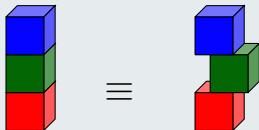
Blocks world

The rules of the game

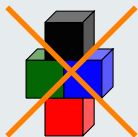
Location on the table does not matter



Location on a block does not matter



At most one block on/under a block is allowed



AI Planning

Transition
systems

Definition

Example

Matrices

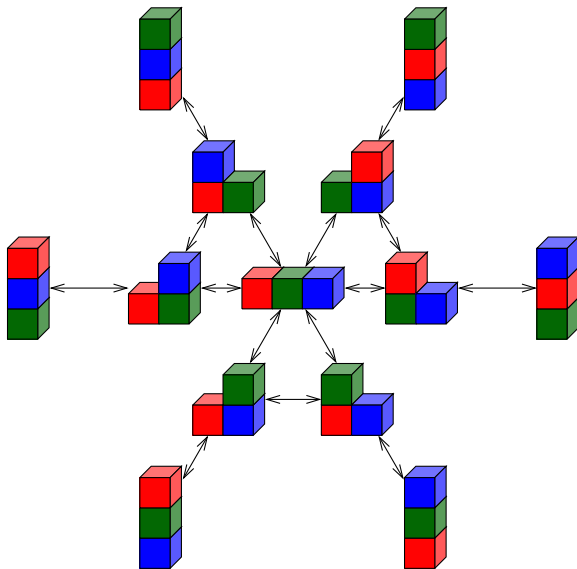
Reachability

Algorithm

Succinct TS

Blocks world

The transition graph for three blocks



AI Planning

Transition systems

Definition

Example

Matrices

Reachability

Algorithm

Succinct TS

Blocks world

Properties

blocks	states
1	1
2	3
3	13
4	73
5	501
6	4051
7	37633
8	394353
9	4596553
10	58941091

- 1 Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration)
- 2 Finding a shortest solution is NP-complete (for a compact description of the problem).

AI Planning

Transition
systems

Definition

Example

Matrices

Reachability

Algorithm

Succinct TS

Deterministic planning: plans

Definition

A **plan** for $\langle S, I, O, G \rangle$ is a sequence $\pi = o_1, \dots, o_n$ of operators such that $o_1, \dots, o_n \in O$ and s_0, \dots, s_n is a sequence of states (the **execution** of π) so that

- 1 $s_0 = I$,
- 2 $s_i = \text{app}_{o_i}(s_{i-1})$ for every $i \in \{1, \dots, n\}$, and
- 3 $s_n \in G$.

This can be equivalently expressed as

$$\text{app}_{o_n}(\text{app}_{o_{n-1}}(\dots \text{app}_{o_1}(I) \dots)) \in G$$

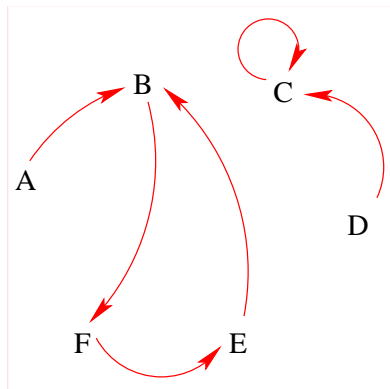
Transition relations as matrices

- 1 If there are n states, each action (a binary relation) corresponds to an $n \times n$ matrix: Element at row i and column j is 1 if the action maps state i to state j , and 0 otherwise.

For deterministic actions there is at most one non-zero element in each row.

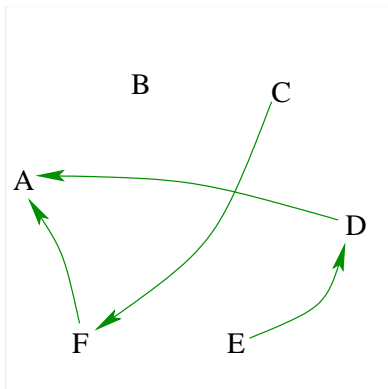
- 2 Matrix multiplication corresponds to **sequential composition**: taking action M_1 followed by action M_2 is the product $M_1 M_2$. (This also corresponds to the **join** of the relations.)
- 3 The unit matrix $I_{n \times n}$ is the NO-OP action: every state is mapped to itself.

Example



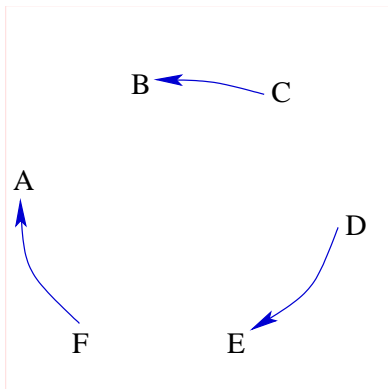
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	1	0	0	0	0
<i>B</i>	0	0	0	0	0	1
<i>C</i>	0	0	1	0	0	0
<i>D</i>	0	0	1	0	0	0
<i>E</i>	0	1	0	0	0	0
<i>F</i>	0	0	0	0	1	0

Example



	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	0	0	0	0	0
<i>B</i>	0	0	0	0	0	0
<i>C</i>	0	0	0	0	0	1
<i>D</i>	1	0	0	0	0	0
<i>E</i>	0	0	0	1	0	0
<i>F</i>	1	0	0	0	0	0

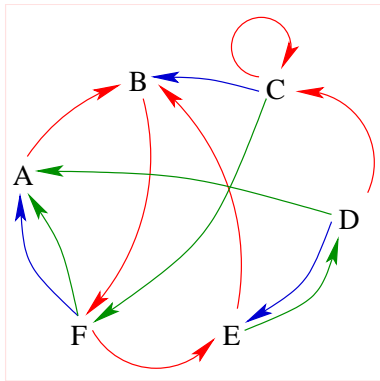
Example



	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	0	0	0	0	0
<i>B</i>	0	0	0	0	0	0
<i>C</i>	0	1	0	0	0	0
<i>D</i>	0	0	0	0	1	0
<i>E</i>	0	0	0	0	0	0
<i>F</i>	1	0	0	0	0	0

Sum matrix $M_R + M_G + M_B$

Representing one-step reachability by any of the component actions



	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	0	0	0	1
C	0	1	1	0	0	1
D	1	0	1	0	1	0
E	0	1	0	1	0	0
F	1	0	0	0	1	0

We use addition $0 + 0 = 0$ and $b + b' = 1$ if $b = 1$ or $b' = 1$.

Sequential composition as matrix multiplication

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ \hline 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times \left(\begin{array}{cccc|c|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & \mathbf{1} & 0 \end{array} \right) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

E is reachable from B by two actions
because

F is reachable from B by one action and
E is reachable from F by one action.

Reachability

Let M be the $n \times n$ matrix that is the (Boolean) sum of the matrices of the individual actions. Define

$$R_0 = I_{n \times n}$$

$$R_1 = I_{n \times n} + M$$

$$R_2 = I_{n \times n} + M + M^2$$

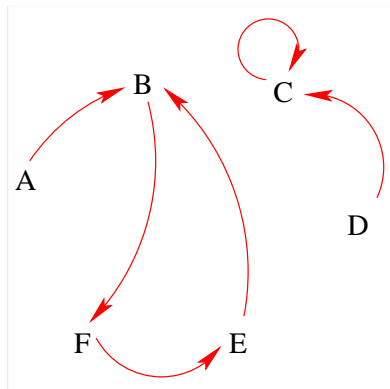
$$R_3 = I_{n \times n} + M + M^2 + M^3$$

$$\vdots$$

R_i represents reachability by i actions or less. If s' is reachable from s , then it is reachable with $\leq n - 1$ actions:

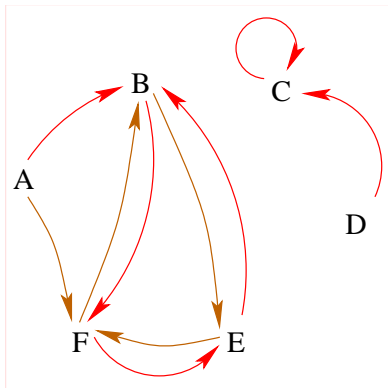
$$R_{n-1} = R_n.$$

Reachability: example, M_R



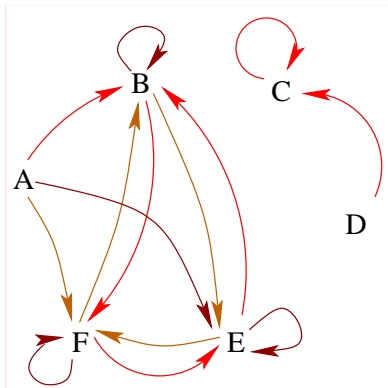
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>A</i>	0	1	0	0	0	0
<i>B</i>	0	0	0	0	0	1
<i>C</i>	0	0	1	0	0	0
<i>D</i>	0	0	1	0	0	0
<i>E</i>	0	1	0	0	0	0
<i>F</i>	0	0	0	0	1	0

Reachability: example, $M_R + M_R^2$



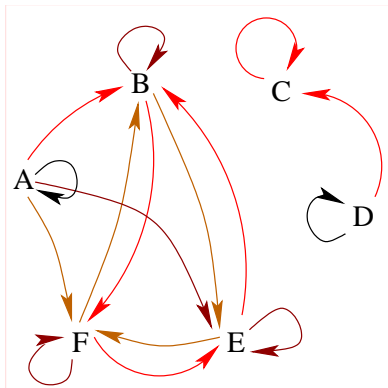
	A	B	C	D	E	F
A	0	1	0	0	0	1
B	0	0	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	0	1
F	0	1	0	0	1	0

Reachability: example, $M_R + M_R^2 + M_R^3$



	A	B	C	D	E	F
A	0	1	0	0	1	1
B	0	1	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	0	0	0
E	0	1	0	0	1	1
F	0	1	0	0	1	1

Reachability: example, $M_R + M_R^2 + M_R^3 + I_{6 \times 6}$



	A	B	C	D	E	F
A	1	1	0	0	1	1
B	0	1	0	0	1	1
C	0	0	1	0	0	0
D	0	0	1	1	0	0
E	0	1	0	0	1	1
F	0	1	0	0	1	1

Relations and sets as matrices

Row vectors as sets of states

Row vectors S represent sets.

SM is the set of states reachable from S by M .

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \times \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}^T$$

AI Planning

Transition
systems

Definition

Example

Matrices

Reachability

Algorithm

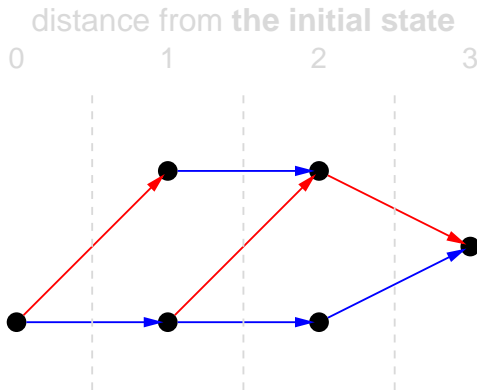
Succinct TS

A simple planning algorithm

- We next present a simple planning algorithm based on computing **distances** in the transition graph.
- The algorithm finds shortest paths less efficiently than Dijkstra's algorithm; we present the algorithm because we later will use it as a basis of an algorithm that is applicable to much bigger state spaces than Dijkstra's algorithm directly.

A simple planning algorithm

Idea



AI Planning

Transition systems

Definition

Example

Matrices

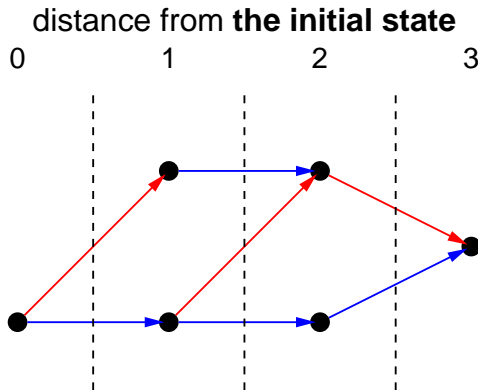
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

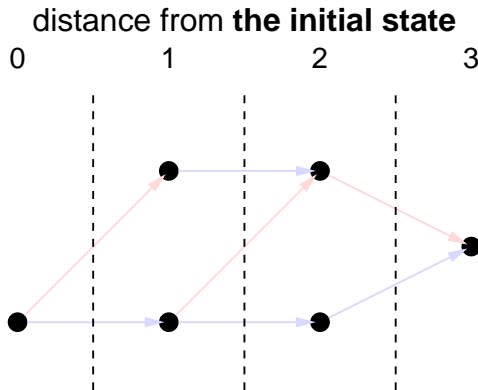
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

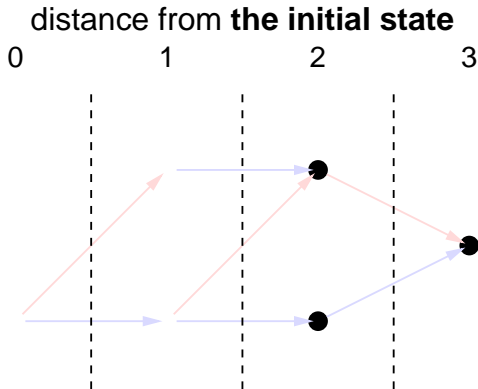
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

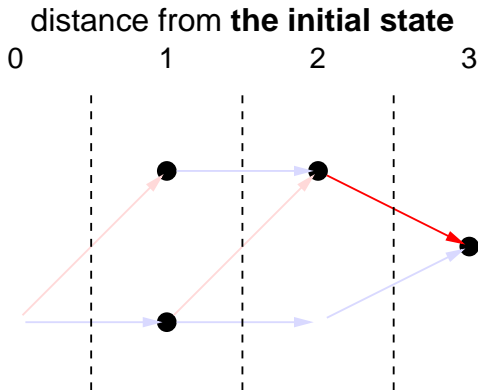
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

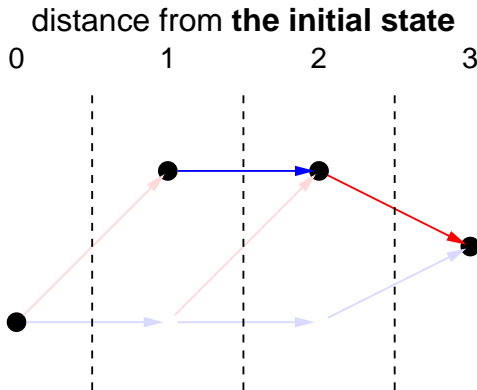
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

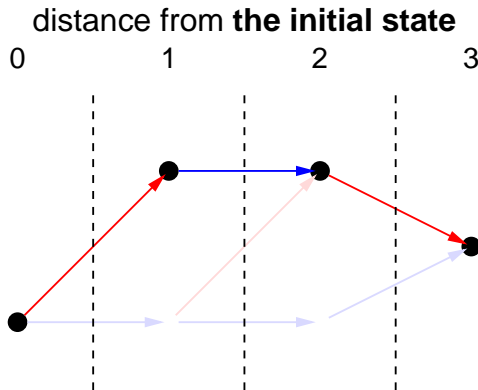
Reachability

Algorithm

Succinct TS

A simple planning algorithm

Idea



AI Planning

Transition
systems

Definition

Example

Matrices

Reachability

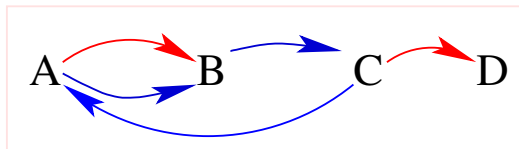
Algorithm

Succinct TS

A simple planning algorithm

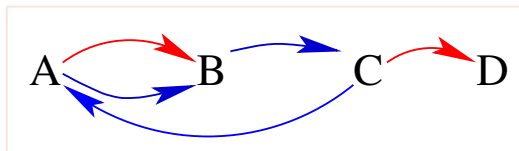
- 1 Compute the matrices $R_0, R_1, R_2, \dots, R_n$ representing reachability with $0, 1, 2, \dots, n$ steps with all actions.
- 2 Find the smallest i such that a goal state s_g is reachable from the initial state according to R_i .
- 3 Find an action (the last action of the plan) by which s_g is reached with one step from a state $s_{g'}$ that is reachable from the initial state according to R_{i-1} .
- 4 Repeat the last step, now viewing $s_{g'}$ as the goal state with distance $i - 1$.

Example



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array} + \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 0 \end{array} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}$$

Example



$$R_0 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 0 & 0 & 0 \\ B & 0 & 1 & 0 & 0 \\ C & 0 & 0 & 1 & 0 \\ D & 0 & 0 & 0 & 1 \end{array} \quad R_1 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 1 & 0 & 0 \\ B & 0 & 1 & 1 & 0 \\ C & 1 & 0 & 1 & 1 \\ D & 0 & 0 & 0 & 1 \end{array}$$

$$R_2 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 1 & 1 & 0 \\ B & 1 & 1 & 1 & 1 \\ C & 1 & 1 & 1 & 1 \\ D & 0 & 0 & 0 & 1 \end{array} \quad R_3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 1 & 1 & 1 \\ B & 1 & 1 & 1 & 1 \\ C & 1 & 1 & 1 & 1 \\ D & 0 & 0 & 0 & 1 \end{array}$$

Succinct representation of transition systems

- More **compact** representation of actions than as relations is often
 - ① **possible** because of symmetries and other regularities,
 - ② **unavoidable** because the relations are too big.
- Represent different aspects of the world in terms of different **state variables**. \implies A state is a **valuation of state variables**.
- Represent actions in terms of changes to the state variables.

State variables

- The state of the world is described in terms of a **finite set** of **finite-valued** state variables.

Example

HOUR : $\{0, \dots, 23\} = 13$

MINUTE : $\{0, \dots, 59\} = 55$

LOCATION : $\{51, 52, 82, 101, 102\} = 101$

WEATHER : $\{\text{sunny, cloudy, rainy}\} = \text{cloudy}$

HOLIDAY : $\{T, F\} = F$

- Any n -valued state variable can be replaced by $\lceil \log_2 n \rceil$ Boolean (2-valued) state variables.
- Actions change the values of the state variables.

Blocks world with state variables

State variables:

LOCATIONofA : {B, C, TABLE}

LOCATIONofB : {A, C, TABLE}

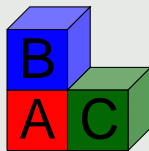
LOCATIONofC : {A, B, TABLE}

Example

$s(\text{LOCATIONofA}) = \text{TABLE}$

$s(\text{LOCATIONofB}) = A$

$s(\text{LOCATIONofC}) = \text{TABLE}$



Not all valuations correspond to an intended blocks world state, e.g. s such that $s(\text{LOCATIONofA}) = B$ and $s(\text{LOCATIONofB}) = A$.

Blocks world with Boolean state variables

AI Planning

Transition
systems

Succinct TS

State variables

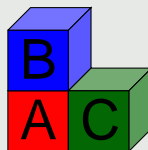
Logic

Operators

Schemata

Example

$s(\text{AonB})=0$	$s(\text{AonC})=0$	$s(\text{AonTABLE})=1$
$s(\text{BonA})=1$	$s(\text{BonC})=0$	$s(\text{BonTABLE})=0$
$s(\text{ConA})=0$	$s(\text{ConB})=0$	$s(\text{ConTABLE})=1$



Logical representations of state sets

- n state variables with m values induce a state space consisting of m^n states (2^n states for n Boolean state variables).
- A language for talking about *sets of states* (*valuations of state variables*) is the **propositional logic**.
- Logical connectives correspond to set-theoretical operations.
- Logical relations correspond to set-theoretical relations.

Propositional logic

Let A be a set of atomic propositions (\sim state variables.)

- 1 For all $a \in A$, a is a propositional formula.
- 2 If ϕ is a propositional formula, then so is $\neg\phi$.
- 3 If ϕ and ϕ' are propositional formulae, then so is $\phi \vee \phi'$.
- 4 If ϕ and ϕ' are propositional formulae, then so is $\phi \wedge \phi'$.
- 5 The symbols \perp and \top are propositional formulae.

The implication $\phi \rightarrow \phi'$ is an abbreviation for $\neg\phi \vee \phi'$.

The equivalence $\phi \leftrightarrow \phi'$ is an abbreviation for $(\phi \rightarrow \phi') \wedge (\phi' \rightarrow \phi)$.

Propositional logic

Valuations and truth

A **valuation** of A is a function $v : A \rightarrow \{0, 1\}$. Define the notation $v \models \phi$ for valuations v and formulae ϕ by

- 1 $v \models a$ if and only if $v(a) = 1$, for $a \in A$.
- 2 $v \models \neg\phi$ if and only if $v \not\models \phi$
- 3 $v \models \phi \vee \phi'$ if and only if $v \models \phi$ or $v \models \phi'$
- 4 $v \models \phi \wedge \phi'$ if and only if $v \models \phi$ and $v \models \phi'$
- 5 $v \models \top$
- 6 $v \not\models \perp$

Propositional logic

Some terminology

- A propositional formula ϕ is **satisfiable** if there is at least one valuation v so that $v \models \phi$. Otherwise it is **unsatisfiable**.
- A propositional formula ϕ is **valid** or a **tautology** if $v \models \phi$ for all valuations v . We write this as $\models \phi$.
- A propositional formula ϕ is a **logical consequence** of a propositional formula ϕ' , written $\phi' \models \phi$, if $v \models \phi$ for all valuations v such that $v \models \phi'$.
- A propositional formula that is a proposition a or a negated proposition $\neg a$ for some $a \in A$ is a **literal**.
- A formula that is a disjunction of literals is a **clause**.

Formulae vs. sets

sets	formulae
those $\frac{2^n}{2}$ states in which a is true	$a \in A$
$E \cup F$	$E \vee F$
$E \cap F$	$E \wedge F$
$E \setminus F$ (set difference)	$E \wedge \neg F$
\overline{E} (complement)	$\neg E$
the empty set \emptyset	\perp
the universal set	\top
question about sets	question about formulae
$E \subseteq F?$	$E \models F?$
$E \subset F?$	$E \models F$ and $F \not\models E?$
$E = F?$	$E \models F$ and $F \models E?$

Actions are represented as **operators** $\langle c, e \rangle$ where

- c (**the precondition**) is a propositional formula over A describing the set of states in which the action can be taken. (*States in which an arrow starts.*)
- e (**the effect**) describes the successor states of states in which the action can be taken. (*Where do the arrows go.*)

The description is procedural: how do the values of the state variable change?

Effects

For deterministic operators

AI Planning

Definition

Effects are then recursively defined as follows.

- 1 a and $\neg a$ for state variables $a \in A$ are effects.
- 2 $e_1 \wedge \dots \wedge e_n$ is an effect if e_1, \dots, e_n are effects (the special case with $n = 0$ is the empty conjunction \top .)
- 3 $c \triangleright e$ is an effect if c is a formula and e is an effect.

Atomic effects a and $\neg a$ are best understood respectively as assignments $a := 1$ and $a := 0$.

Transition
systems

Succinct TS

State variables

Logic

Operators

Schemata

Effects

Meaning of conditional effects \triangleright

$c \triangleright e$ means that change e takes place if c is true in the current state.

Example

Increment 4-bit numbers $b_3b_2b_1b_0$.

$$\begin{aligned} & (\neg b_0 \triangleright b_0) \wedge \\ & ((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \triangleright (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

AI Planning

Transition
systems

Succinct TS

State variables

Logic

Operators

Schemata

Example: operators for blocks world

For convenience we use also state variables *Aclear*, *Bclear*, and *Cclear* to denote that there is nothing on the block in question.

$$\langle \text{Aclear} \wedge \text{AonT} \wedge \text{Bclear}, \text{AonB} \wedge \neg \text{AonT} \wedge \neg \text{Bclear} \rangle$$
$$\langle \text{Aclear} \wedge \text{AonT} \wedge \text{Cclear}, \text{AonC} \wedge \neg \text{AonT} \wedge \neg \text{Cclear} \rangle$$

⋮

$$\langle \text{Aclear} \wedge \text{AonB}, \text{AonT} \wedge \neg \text{AonB} \wedge \neg \text{AonC} \rangle$$
$$\langle \text{Aclear} \wedge \text{AonC}, \text{AonT} \wedge \neg \text{AonB} \wedge \neg \text{AonC} \rangle$$
$$\langle \text{Bclear} \wedge \text{BonA}, \text{BonT} \wedge \neg \text{BonA} \wedge \text{Aclear} \rangle$$
$$\langle \text{Bclear} \wedge \text{BonC}, \text{BonT} \wedge \neg \text{BonC} \wedge \text{Cclear} \rangle$$

⋮

Operators: meaning

Changes caused by an operator

Assign each effect e and state s a set $[e]_s$ of literals as follows.

- 1 $[a]_s = \{a\}$ and $[\neg a]_s = \{\neg a\}$ for $a \in A$.
- 2 $[e_1 \wedge \dots \wedge e_n]_s = [e_1]_s \cup \dots \cup [e_n]_s$.
- 3 $[c \triangleright e]_s = [e]_s$ if $s \models c$ and $[c \triangleright e]_s = \emptyset$ otherwise.

Applicability of an operator

Operator $\langle c, e \rangle$ is **applicable in a state** s iff $s \models c$ and $[e]_s$ is consistent.

Operators: the successor state of a state

Definition (Successor state)

The **successor state** $app_o(s)$ of s with respect to operator $o = \langle c, e \rangle$ is obtained from s by making literals in $[e]_s$ true. This is defined only if o is applicable in s .

Example

Consider the operator $\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle$ and a state s such that $s \models a \wedge b \wedge c$.

The operator is applicable because $s \models a$ and

$[\neg a \wedge (\neg c \triangleright \neg b)]_s = \{\neg a\}$ is consistent.

Hence $app_{\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle}(s) \models \neg a \wedge b \wedge c$.

Operators

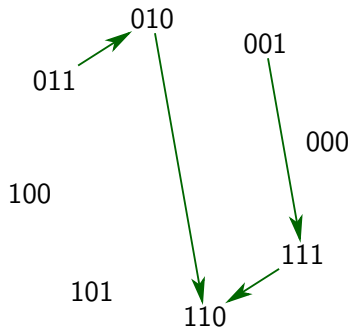
Example

State variables are

$$A = \{a, b, c\}.$$

An operator is

$$\langle (b \wedge c) \vee (\neg a \wedge b \wedge \neg c) \vee (\neg a \wedge c), \\ ((b \wedge c) \triangleright \neg c) \\ \wedge (\neg b \triangleright (a \wedge b)) \\ \wedge (\neg c \triangleright a) \rangle$$

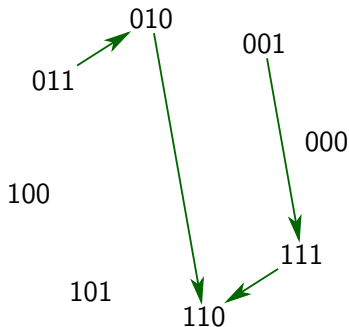


Operators

Example

The corresponding matrix is

	000	001	010	011	100	101	110	111
000	0	0	0	0	0	0	0	0
001	0	0	0	0	0	0	0	1
010	0	0	0	0	0	0	1	0
011	0	0	1	0	0	0	0	0
100	0	0	0	0	0	0	0	0
101	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	1	0



Succinct transition systems

Deterministic case

AI Planning

Transition
systems

Succinct TS

State variables

Logic

Operators

Schemata

Definition

A **succinct deterministic transition system** is

$\langle A, I, \{o_1, \dots, o_n\}, G \rangle$ where

- A is a finite set of **state variables**,
- I is an **initial state**,
- every o_i is an operator,
- G is a formula describing the **goal states**.

Mapping from succinct TS to TS

From every succinct transition system $\langle A, I, O, G \rangle$ we can produce a corresponding transition system $\langle S, I, O', G' \rangle$.

- 1 S is the set of all valuations of A ,
- 2 $O' = \{R(o) \mid o \in O\}$ where
 $R(o) = \{(s, s') \in S \times S \mid s' = \mathit{app}_o(s)\}$, and
- 3 $G' = \{s \in S \mid s \models G\}$.

Schematic operators

- Description of state variables and operators in terms of a given finite *set of objects*.
- Analogy: propositional logic vs. predicate logic
- Planners take input as schematic operators, and translate them into (**ground**) operators. This is called **grounding**.

Schematic operators: example

Schematic operator

$$x \in \{\text{car1}, \text{car2}\}$$

$$y_1 \in \{\text{Freiburg}, \text{Strassburg}\},$$

$$y_2 \in \{\text{Freiburg}, \text{Strassburg}\}, y_1 \neq y_2$$

$$\langle \text{in}(x, y_1), \text{in}(x, y_2) \wedge \neg \text{in}(x, y_1) \rangle$$

corresponds to the operators

$$\begin{aligned} &\langle \text{in}(\text{car1}, \text{Freiburg}), \text{in}(\text{car1}, \text{Strassburg}) \wedge \neg \text{in}(\text{car1}, \text{Freiburg}) \rangle, \\ &\langle \text{in}(\text{car1}, \text{Strassburg}), \text{in}(\text{car1}, \text{Freiburg}) \wedge \neg \text{in}(\text{car1}, \text{Strassburg}) \rangle, \\ &\langle \text{in}(\text{car2}, \text{Freiburg}), \text{in}(\text{car2}, \text{Strassburg}) \wedge \neg \text{in}(\text{car2}, \text{Freiburg}) \rangle, \\ &\langle \text{in}(\text{car2}, \text{Strassburg}), \text{in}(\text{car2}, \text{Freiburg}) \wedge \neg \text{in}(\text{car2}, \text{Strassburg}) \rangle \end{aligned}$$

Schematic operators: quantification

Existential quantification (for formulae only)

Finite disjunctions $\phi(a_1) \vee \dots \vee \phi(a_n)$ represented as
 $\exists x \in \{a_1, \dots, a_n\} \phi(x)$.

Universal quantification (for formulae and effects)

Finite conjunctions $\phi(a_1) \wedge \dots \wedge \phi(a_n)$ represented as
 $\forall x \in \{a_1, \dots, a_n\} \phi(x)$.

Example

$\exists x \in \{A, B, C\} \text{in}(x, \text{Freiburg})$ is a short-hand for
 $\text{in}(A, \text{Freiburg}) \vee \text{in}(B, \text{Freiburg}) \vee \text{in}(C, \text{Freiburg})$.

PDDL: the Planning Domain Description Language

- Used by almost all implemented systems for deterministic planning.
- Supports a language comparable to what we have defined above (including schematic operators and quantification)
- Syntax inspired by the Lisp programming language: e.g. prefix notation for formulae

```
(and (or (on A B) (on A C))  
      (or (on B A) (on B C))  
      (or (on C A) (on A B)))
```

A domain file consists of

- (define (domain DOMAINNAME))
- a :requirements definition (use :adl :typing by default)
- definitions of types (each parameter has a type)
- definitions of predicates
- definitions of operators

Example: blocks world in PDDL

```
(define (domain BLOCKS)
  (:requirements :adl :typing)
  (:types block - object
           blueblock smallblock - block)
  (:predicates (on ?x - smallblock ?y - block)
               (ontable ?x - block)
               (clear ?x - block)
               )
)
```

AI Planning

Transition
systems

Succinct TS

State variables

Logic

Operators

Schemata

PDDL: operator definition

- (:action OPERATORNAME
- list of parameters: (?x - type1 ?y - type2 ?z - type3)
- precondition: a formula

```
<schematic-state-var>  
(and <formula> ... <formula>)  
(or <formula> ... <formula>)  
(not <formula>)  
(forall (?x1 - type1 ... ?xn - typen) <formula>)  
(exists (?x1 - type1 ... ?xn - typen) <formula>)
```


- effect:

```
<schematic-state-var>  
(not <schematic-state-var>  
(and <effect> ... <effect>  
(when <formula> <effect>  
(forall (?x1 - type1 ... ?xn - typen) <effect>))
```

```
(:action fromtable
  :parameters (?x - smallblock ?y - block)
  :precondition (and (not (= ?x ?y))
                    (clear ?x)
                    (ontable ?x)
                    (clear ?y))
  :effect
    (and (not (ontable ?x))
         (not (clear ?y))
         (on ?x ?y)))
```

PDDL: problem files

A problem file consists of

- (define (problem PROBLEMNAME))
- declaration of which domain is needed for this problem
- definitions of objects belonging to each type
- definition of the initial state (list of state variables initially true)
- definition of goal states (a formula like operator precondition)

```
(define (problem blocks-10-0)
  (:domain BLOCKS)
  (:objects a b c - smallblock)
           d e - block
           f - blueblock)
  (:init (clear a) (clear b) (clear c) (clear d) (clear e)
         (ontable a) (ontable b) (ontable c)
         (ontable d) (ontable e) (ontable f))

  (:goal (and (on a d) (on b e) (on c f))))
)
```

Example run on the FF planner

```
edu/PS04> ./ff -o hamiltonian.pddl -f ham1.pddl
ff: parsing domain file, domain 'HAMILTONIAN-CYCLE' defined
ff: parsing problem file, problem 'HAM-1' defined
ff: found legal plan as follows
step      0: GO A B
          1: GO B D
          2: GO D F
          3: GO F C
          4: GO C E
          5: GO E A
0.01 seconds total time
```

AI Planning

Transition
systems

Succinct TS
State variables

Level

Operators

Schemata

Example: blocks world in PDDL

```
(define (domain BLOCKS)
  (:requirements :adl :typing)
  (:types block)
  (:predicates (on ?x - block ?y - block)
               (ontable ?x - block)
               (clear ?x - block)
               )
)
```

AI Planning

Transition
systems

Succinct TS

State variables

Logic

Operators

Schemata

```
(:action fromtable
  :parameters (?x - block ?y - block)
  :precondition (and (not (= ?x ?y))
                    (clear ?x)
                    (ontable ?x)
                    (clear ?y))
  :effect
    (and (not (ontable ?x))
         (not (clear ?y))
         (on ?x ?y)))
```

```
(:action totable
  :parameters (?x - block ?y - block)
  :precondition (and (clear ?x) (on ?x ?y))
  :effect
    (and (not (on ?x ?y))
          (clear ?y)
          (ontable ?x)))
```



```
(:action move
  :parameters (?x - block
               ?y - block
               ?z - block)
  :precondition (and (clear ?x) (clear ?z)
                    (on ?x ?y) (not (= ?x ?z)))
  :effect
    (and (not (clear ?z))
         (clear ?y)
         (not (on ?x ?y))
         (on ?x ?z)))
)
```

```
(define (problem blocks-10-0)
  (:domain BLOCKS)
  (:objects d a h g b j e i f c - block)
  (:init (clear c) (clear f)
         (ontable i) (ontable f)
         (on c e) (on e j) (on j b) (on b g)
         (on g h) (on h a) (on a d) (on d i))
  (:goal (and (on d c) (on c f) (on f j)
              (on j e) (on e h) (on h b)
              (on b a) (on a g) (on g i))))
)
```