Algorithms for unobservable planning: QBF

Translation into quantified Boolean formulae (QBF)

Why not by translation into propositional logic?

- We need to be able to say that *there is* a plan such that ...

  This is like the *satisfiability* problem in CPC: there is a valuation...

- We need to be able to say that *for all* executions ...

  This is like the *validity* problem in CPC: for all valuations...

Quantified Boolean formulae: definition

If ϕ is a propositional formula and σ is a sequence of $\exists p$ and $\forall p$, one for every $p \in P$, then $\sigma\phi$ is a QBF.

A formula $\exists x\phi$ is true if and only if $\phi[\top/x] \vee \phi[\bot/x]$ is true. (Equivalently, $\phi[\top/x]$ *is true or* $\phi[\bot/x]$ *is true*.)

A formula $\forall x\phi$ is true if and only if $\phi[\top/x] \wedge \phi[\bot/x]$ is true. (Equivalently, $\phi[\top/x]$ *is true and* $\phi[\bot/x]$ *is true*.)

This definition directly leads to an AND/OR tree traversal algorithm for evaluating QBF.

Quantified Boolean formulae: definition

The *evaluation problem of QBF* generalizes both the *satisfiability* and *validity/tautology problems* of the propositional logic. The latter are respectively NP-complete and co-NP-complete whereas the former is PSPACE-complete.

EXAMPLE.

The formulae $\forall x\exists y(x \leftrightarrow y)$ and $\exists x\exists y(x \wedge y)$ are true.

The formulae $\exists x\forall y(x \leftrightarrow y)$ and $\forall x\forall y(x \vee y)$ are false.

UO planning with QBF

There is a sequence of operators so that
all executions and initial states
reach a goal state.

$$\begin{array}{l}\exists o_1^0 \cdots o_m^0 \cdots o_1^{t-1} \cdots o_n^{t-1} \\ \forall p_1^0 \cdots p_n^0 a_1^0 \cdots a_k^0 \cdots a_1^{t-1} \cdots a_k^{t-1} \\ \exists p_1^1 \cdots p_n^1 \cdots p_1^t \cdots p_n^t \\ (I^0 \rightarrow (\mathcal{R}_3^0(P^0, P^1) \wedge \cdots \wedge \mathcal{R}_3^{t-1}(P^{t-1}, P^t) \wedge G^t))\end{array}$$

Variables $A^i = \{a_1^i, \ldots, a_k^i\}$ encode nondeterministic choices.

UO planning with QBF: nondeterminism

- We replace nondeterministic choice by dependence of the effects on values of "hidden" state variables a_j.
- Nondeterministic effect $e_1|e_2|\cdots|e_n$ roughly corresponds to a number of conditional effects:

$$(\phi_1 \rhd e_1) \wedge (\phi_2 \rhd e_2) \wedge \cdots \wedge (\phi_n \rhd e_n).$$

Formulae ϕ_i refer to valuations of a some unknown "hidden" state variables $a_1, \ldots, a_m$ (different at every time point).
For n choices we have $m = \lceil \log_2 n \rceil$ variables a_j.

UO planning with QBF: nondeterminism

$$o_7 = \langle A, (0.3(A \wedge (B \rhd D))|0.3(B \wedge C)|0.4C) \rangle$$

$\lceil \log_2 3 \rceil = 2$ auxiliary variables a_0, a_1 for 3 alternatives

valuation	effect
$\neg a_1 \wedge \neg a_0$	$A \wedge (B \rhd D)$
$\neg a_1 \wedge a_0$	$B \wedge C$
$a_1 \wedge \neg a_0$	C
$a_1 \wedge a_0$	C

UO planning with QBF: effects, precons

$$(o_7 \wedge \neg a_1 \wedge \neg a_0) \rightarrow A'$$
$$(o_7 \wedge \neg a_1 \wedge \neg a_0 \wedge B) \rightarrow D'$$

$$(o_7 \wedge \neg a_1 \wedge a_0) \rightarrow B'$$
$$(o_7 \wedge \neg a_1 \wedge a_0) \rightarrow C'$$

$$(o_7 \wedge a_1 \wedge \neg a_0) \rightarrow C'$$
$$(o_7 \wedge a_1 \wedge a_0) \rightarrow C'$$

$$o_7 \rightarrow A$$

UO planning with QBF: frame axioms

$$(\neg A \wedge A') \rightarrow ((o_7 \wedge \neg a_1 \wedge \neg a_0) \vee \cdots)$$
$$(A \wedge \neg A') \rightarrow \cdots$$
$$(\neg B \wedge B') \rightarrow ((o_7 \wedge \neg a_1 \wedge a_0) \vee \cdots)$$
$$(B \wedge \neg B') \rightarrow \cdots$$
$$(\neg C \wedge C') \rightarrow ((o_7 \wedge \neg a_1 \wedge a_0) \vee (o_7 \wedge a_1) \vee \cdots)$$
$$(C \wedge \neg C') \rightarrow \cdots$$
$$(\neg D \wedge D') \rightarrow ((o_7 \wedge B \wedge \neg a_1 \wedge \neg a_0) \vee \cdots)$$
$$(D \wedge \neg D') \rightarrow \cdots$$

UO planning with QBF: $\mathcal{R}_3^i(P_i, P_{i+1})$

The formula $\mathcal{R}_3^i(P_i, P_{i+1})$ is then the conjunction of all the formulae for

- operators' effects,
- operators' preconditions,
- frame axioms for all state variables,
- $\neg o \vee \neg o'$ for pairs of interfering operators o and o'

similarly to the encoding of deterministic planning.