Regression: BW2 with conditional effects

$$1 = \langle \top, \quad (\mathit{AonB} \wedge \mathit{Aclear}) \rhd (\mathit{AonT} \wedge \mathit{Bclear} \wedge \neg \mathit{AonB})\rangle$$
$$2 = \langle \top, \quad ((\mathit{BonA} \wedge \mathit{Bclear}) \rhd (\mathit{BonT} \wedge \mathit{Aclear} \wedge \neg \mathit{BonA})\rangle$$

$$\begin{array}{ll} goal & \mathit{AonT} \wedge \mathit{BonT} \\ regr1 & (\mathit{AonT} \vee (\mathit{AonB} \wedge \mathit{Aclear})) \wedge \mathit{BonT} \\ regr2 & (\mathit{AonT} \vee (\mathit{AonB} \wedge (\mathit{Aclear} \vee (\mathit{BonA} \wedge \mathit{Bclear})))) \\ & \wedge(\mathit{BonT} \vee (\mathit{BonA} \wedge \mathit{Bclear})) \end{array}$$

Regression: properties

LEMMA D: Let ϕ be a formula, o and operator with effect e, s any state and $s' = \mathrm{app}_o(s)$. Then $s \models \mathrm{regr}_e(\phi)$ if and only if $s' \models \phi$.

PROOF: The proof is by structural induction over subformulae ϕ' of ϕ. We show that the formula ϕ_r obtained from ϕ by replacing propositions $p \in P$ by $(p \wedge \neg \mathit{EPC}_{\neg p}(e)) \vee \mathit{EPC}_p(e)$ has the same truth-value in s as ϕ has in s'.

Induction hypothesis: $s \models \phi'_r$ if and only if $s' \models \phi'$.

Base cases 1 & 2, $\phi' = \top$ or $\phi' = \bot$: Trivial because $\phi'_r = \phi$.

Base case 3, $\phi' = p$ for some $p \in P$: Now $\phi'_r = (p \wedge \neg \mathit{EPC}_{\neg p}(e)) \vee \mathit{EPC}_p(e)$. By Lemma C $s \models \phi'_r$ if and only if $s' \models \phi'$.

Inductive case 1, $\phi' = \neg\psi$: By the induction hypothesis $s \models \psi_r$ iff $s' \models \psi$. Hence $s \models \phi'_r$ iff $s' \models \phi'$ by the truth-definition of $\neg$.

Inductive case 2, $\phi' = \psi \vee \psi'$: By the induction hypothesis $s \models \psi_r$ iff $s' \models \psi$, and $s \models \psi'_r$ iff $s' \models \psi'$. Hence $s \models \phi'_r$ iff $s' \models \phi'$ by the truth-definition of $\vee$.

Inductive case 3, $\phi' = \psi \wedge \psi'$: By the induction hypothesis $s \models \psi_r$ iff $s' \models \psi$, and $s \models \psi'_r$ iff $s' \models \psi'$. Hence $s \models \phi'_r$ iff $s' \models \phi'$ by the truth-definition of $\wedge$.

Regression: complexity issues

1. $\mathrm{regr}_{\langle a, \neg p\rangle}(p) = a \wedge \bot \equiv \bot$: the new set of states is empty.

   Testing that a formula $\mathrm{regr}_o(\phi)$ does not represent the empty set (= search is in a blind alley) is NP-hard.

2. $\mathrm{regr}_{\langle b, c\rangle}(a) = a \wedge b$: the new set of states is properly smaller.

   Testing that a regression step does not make the set of states smaller (= more difficult to reach) is NP-hard.

These tests would be useful in pruning the search tree.

Regression: complexity issues II

For a formula ϕ, $\text{regr}_1(\text{regr}_{o_2}(\cdots \text{regr}_{o_{n-1}}(\text{regr}_{o_n}(\phi))))$ may have size $\mathcal{O}(|\phi||o_1||o_2|\cdots|o_{n-1}||o_n|)$, i.e. the product of the sizes of ϕ and the operators.

Hence, the size can be in the worst case exponential in n ($\mathcal{O}(2^n)$).

Regression: complexity issues II

Split $\text{regr}_o(\phi)$ to $\phi_1, \ldots, \phi_n$ such that $\text{regr}_o(\phi) \equiv \phi_1 \vee \cdots \vee \phi_n$.

1. Transform $\text{regr}_o(\phi)$ to $\phi_1 \vee \cdots \vee \phi_n$ by suitable equivalences.

   Regression planners so far have transformed $\text{regr}_o(\phi)$ to disjunctive normal form $\phi_1 \vee \cdots \vee \phi_n$ where every ϕ_i is a conjunction of literals.

2. Choose state variable p, and define

$$\phi_1 = p \wedge \text{regr}_o(\phi) \quad \phi_2 = \neg p \wedge \text{regr}_o(\phi)$$

Plan search: search states for regression

For regression, the search state is represented as a sequence of operators and associated formulae.

$$\phi_n, o_n, \ldots, \phi_1, o_1, G$$

The neighbors of the state are those obtained by regression with respect to one of the operators or by dropping out some of the last actions and associated formulae:

1. $\text{regr}_o(\phi_n), o, \phi_n, o_n, \ldots, \phi_1, o_1, G$ for $o \in O$
2. $\phi_i, o_i, \ldots, \phi_1, o_1, G$ for $i < n$ (for local search only)

Plan search: distance estimates for regression

With progression we had for $I, o_1, s_1, o_2, s_2, \ldots, o_n, s_n$ the estimate

$$\delta_{s_n}(G)$$

With regression, we have for $\phi_n, o_n, \ldots, \phi_1, o_1, G$ the estimate

$$\delta_I(\phi_n)$$

Advantage: sets D_i for distance estimates are computed only once, because starting state stays the same.