

Exercise 6

To be returned on Monday, June 21, 2004

Assignment 6.1

Consider the following situation:

While hiking in the local desert, you lose your water bottle. You know where the nearest oasis is located, and thanks to your high-tech GPS equipment, you can also determine your current location. However, because of limited battery charge, you can only activate your GPS system every ten minutes, after you have walked 1 km in some direction, and sometimes you lose your way and deviate from the intended path. The desert is surrounded by a mountain ridge on all four sides, and you never lose your sense of direction near the mountains because they serve as a landmark.



Assuming that you dislike going into any other direction than the four cardinals (north, east, south, west), this can be modelled by the non-deterministic planning task $\langle P, I, O, G \rangle$ defined as follows:

$$\begin{aligned}
 P &= \{x_0, x_1, x_2, x_3, x_4, y_0, y_1, y_2, y_3, y_4\} \\
 I &= x_0 \wedge \neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4 \wedge \neg y_0 \wedge y_1 \wedge \neg y_2 \wedge \neg y_3 \wedge \neg y_4, \\
 O &= \{ N_{ij} \mid i \in \{0, 1, 2, 3, 4\}, j \in \{0, 1, 2, 3\} \} \\
 &\quad \cup \{ S_{ij} \mid i \in \{0, 1, 2, 3, 4\}, j \in \{1, 2, 3, 4\} \} \\
 &\quad \cup \{ W_{ij} \mid i \in \{1, 2, 3, 4\}, j \in \{0, 1, 2, 3, 4\} \} \\
 &\quad \cup \{ E_{ij} \mid i \in \{0, 1, 2, 3\}, j \in \{0, 1, 2, 3, 4\} \} \\
 G &= x_2 \wedge y_2
 \end{aligned}$$

Modelling movements in sight range of the mountains, for $i, j \in \{0, 1, 2, 3, 4\}$ and $\{i, j\} \cap \{0, 4\} \neq \emptyset$, we have

$$\begin{aligned}
 N_{ij} &= \langle x_i \wedge y_j, \neg y_j \wedge y_{j+1} \rangle \text{ if } j \neq 4 & S_{ij} &= \langle x_i \wedge y_j, \neg y_j \wedge y_{j-1} \rangle \text{ if } j \neq 0 \\
 W_{ij} &= \langle x_i \wedge y_j, \neg x_i \wedge x_{i-1} \rangle \text{ if } i \neq 0 & E_{ij} &= \langle x_i \wedge y_j, \neg x_i \wedge x_{i+1} \rangle \text{ if } i \neq 4
 \end{aligned}$$

Modelling movement in the heart of the desert, for $i, j \in \{1, 2, 3\}$ we have

$$\begin{aligned}
 N_{ij} &= \langle x_i \wedge y_j, (0.5(\neg y_j \wedge y_{j+1})|0.5\top) \wedge (0.5\top|0.5(\neg x_i \wedge (0.5x_{i-1}|0.5x_{i+1}))) \rangle \\
 S_{ij} &= \langle x_i \wedge y_j, (0.5(\neg y_j \wedge y_{j-1})|0.5\top) \wedge (0.5\top|0.5(\neg x_i \wedge (0.5x_{i-1}|0.5x_{i+1}))) \rangle \\
 E_{ij} &= \langle x_i \wedge y_j, (0.5(\neg x_i \wedge x_{i+1})|0.5\top) \wedge (0.5\top|0.5(\neg y_j \wedge (0.5y_{j-1}|0.5y_{j+1}))) \rangle \\
 W_{ij} &= \langle x_i \wedge y_j, (0.5(\neg x_i \wedge x_{i-1})|0.5\top) \wedge (0.5\top|0.5(\neg y_j \wedge (0.5y_{j-1}|0.5y_{j+1}))) \rangle
 \end{aligned}$$

1. Precisely describe the precondition and effect of operator S_{13} in words (don't be more verbose than necessary).
2. Which of the operators are in normal form I? (Give a very short explanation for your answer.)

3. Translate operator N_{12} into normal form II. You only need to specify the end result of the translation.
4. Find a conditional plan that guarantees that you reach the oasis after at most one hour (i.e., after at most six steps) with probability at least 0.5. Describe the plan informally but unambiguously and prove that it has the required property.

Assignment 6.2

Consider the transition relation $\{(01, 00), (01, 01), (10, 10)\}$ represented by the formula

$$((A \leftrightarrow \neg B) \wedge (A \leftrightarrow A') \wedge (B \leftrightarrow B')) \vee (\neg A \wedge B \wedge \neg A' \wedge \neg B')$$

and the set $\{01, 10\}$ of states represented by $A \leftrightarrow \neg B$.

Compute the *strong preimage* of the set of states with respect to the transition relation by means of universal and existential abstraction as presented in the lecture. Simplify the resulting formula as much as possible.

You may work on these assignments and submit your results in groups of two students. Make sure to clearly indicate both names on your work. You may write your answers in English or German.