Principles of AI Planning
Summer 2004

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Exercise 5

To be returned on Monday, June 14, 2004

Assignment 5.1

Nondeterministic actions can be represented as matrices (Boolean, with elements 0 and 1 only), propositional formulae, and operators. Consider the following actions on state variables A, B, C in one of the representations, and represent the action in the other two representations (in as simple form as you can).

1. $$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2. $$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

3. $\langle B \wedge C, (0.5A|0.5\neg A) \wedge (0.5C|0.5\neg C)\rangle$
4. $(A \leftrightarrow B) \wedge (A \leftrightarrow C')$

Assignment 5.2

Consider a system with two states s_1 and s_2 with transition probabilities expressed by the following matrix.

$$M = \begin{pmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{pmatrix}$$

So, probability of transition from s_1 to itself is 0.9 and to s_2 it is 0.1, and probability of transition from s_2 to s_1 is 0.8 and to s_2 itself is 0.2.

The initial state probabilities for s_1 and s_2 are expressed by the row vector

$$S = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}.$$

Compute the vectors $S \cdot M$, $(S \cdot M) \cdot M$ and $((S \cdot M) \cdot M) \cdot M$ representing the probabilities of the states after 1, 2 and 3 transitions.

The probabilities $P(s_1)$ and $P(s_2)$ of the system being respectively in states s_1 and s_2 asymptotically are

$$\begin{aligned} P(s_1) &= 0.9P(s_1) + 0.8P(s_2) \\ P(s_2) &= 0.1P(s_1) + 0.2P(s_2) \end{aligned}$$

(probabilities of the possible predecessor states multiplied by the probabilities of the transitions to the state in question.) Solve the above equations, i.e. determine what the asymptotic probabilities are. (Notice that in this case the asymptotic probabilities are independent of the initial probabilities but this is not the case for all transition systems.)

Can you see what is the connection between these probabilities and the sequence $S \cdot M$, $S \cdot M^2$, $S \cdot M^3$, $S \cdot M^4$,...?

You may work on these assignments and submit your results ***in groups of two students****. Make sure to clearly indicate both names on your work.* ***You may write your answers in English or German.***