Towards Cognitive Adequacy of Topological Spatial Relations

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Abstract. Qualitative spatial reasoning is often considered to be akin to human reasoning. This, however, is mostly based on the intuition of researchers rather than on empirical data. In this paper we continue our effort in empirically studying the cognitive adequacy of systems of topological relations. As compared to our previous empirical investigation [KRR97], we partially lifted constraints on the shape of regions in configurations that we presented subjects in a grouping task. With a high level of agreement, subjects distinguished between different possibilities of how spatial regions can touch each other. Based on the results of our investigation, we propose to develop a new system of topological relations on a finer level of granularity than previously considered.

1 Introduction

Reasoning about spatial information is an important part not only of human cognition but also of industrial applications such as geographical information systems (GIS). While in most existing GIS spatial information is represented in a quantitative way, which is certainly useful from an engineering point of view, new approaches to GIS try to come closer to the way spatial information is communicated by natural language and, thus, to the way human cognition is considered to represent spatial information, namely, by representing the relationships between spatial entities. This has the advantage of being more user-friendly—queries to a spatial information system or space related instructions to an autonomous agent, for instance, can be formulated in natural language and immediately be translated into the internal representation.

Qualitative spatial reasoning (QSR), a sub-discipline of artificial intelligence, is devoted to this way of representing and reasoning about spatial information.

Many different aspects of space can be treated in a qualitative way, e.g., distance, size, direction, shape, or topology. But, as Cohn [Coh97, page 22] wrote in his overview article on qualitative spatial reasoning, "An issue which has not been much addressed yet in the QSR literature is the issue of cognitive validity – claims are often made that qualitative reasoning is akin to human reasoning, but with little or no empirical justification". In this paper we will continue our effort in studying the cognitive validity, also referred to as cognitive adequacy (we will define this term later), of existing approaches to QSR based on empirical investigations. Similar to our previous work [KRR97], we will perform a grouping task where all presented items show two different colored spatial regions related in a different way. In [KRR97], all objects were circles. Although this does not prevent subjects from grouping items according to size, distance, direction, or topology, other aspects of space cannot be distinguished. In the investigation presented in this paper we weaken this restriction on the shape of objects and use regular polygons instead. Unlike circles, these polygons can touch each other not only at one point but also at more than one point or along a line. This simple extension of the shape of regions led to surprising results that encourage the development of a new and extended topological calculus for qualitative spatial reasoning.

The remainder of the paper is structured as follows. In Section 2 we define the term cognitive adequacy and give an overview of the psychological methodology used to study cognitive adequacy. In Section 3 we introduce topological spatial relations and calculi based on these relations. Section 4 summarizes our previous investigation on topological spatial relations. In Section 5 we describe an empirical investigation and present the obtained results. Finally, in Section 6 we discuss our results, propose developing a new system of topological relations, and give links to possible future work.

2 Psychological Background

In determining the cognitive adequacy of a QSR formalism, we previously argued that this has to be broken down into at least two sub-aspects, namely the representational and the inferential aspect of the calculus. We termed these two sub-concepts of cognitive adequacy conceptual adequacy and inferential adequacy [KRS95,KRR97]. Conceptual adequacy refers to the degree in which a set of entities and relations correspond to the mental set of a person when he/she conceptualizes space and spatial configurations. Inferential adequacy refers to the degree in which the computational inference algorithm (e.g. constraint satisfaction, theorem proving, ...) conforms to the human mental reasoning process. A cognitively adequate qualitative reasoning system would therefore be based on a set of spatial relations that humans also use quite naturally, and would draw inferences in a way people do with respect to certain criteria (type of inferences; order of inferences; preferred, easy and hard inferences; in the extreme maybe also in accordance with processing latencies). The determination of the cognitive adequacy would be of interest to basic research in the field of the psy-

chology of knowledge and the psychology of thinking on the one hand, and for computer science and artificial intelligence on the other. But mostly, applied research like cognitive ergonomics and human-computer-interaction (HCI) research would benefit from these results, since these could then justify a decision for one candidate out of a set of concurrent spatial calculi in spatial information systems. In the following we concentrate on the problem of conceptual adequacy of spatial relations and its empirical determination.

Looking for empirical methods to determine the conceptual adequacy, many results can be found in psychological research on conceptual structure. But most theories are about the conceptual structure of entities like natural kind terms or artifacts (see for example the overviews given in Komatsu [Kom92], and Rips [Rip95]). Relational concepts, however, were not investigated very heavily, and therefore psychological methods are tuned to the assessment of the conceptual structure of entities, but not for relations. Other research on spatial relations comes from (psycho-)linguistics where mainly acceptability ratings of natural language expressions like "above", "below", "left", "right" and so on, were obtained (see for example [HT95, VR98]). These results are heavily dependent on the investigated language and may, therefore, not reveal everything about people's conceptualization of space; this is due to the fact that different languages may provide different means to express the conceptualizations, and linguistic expressions are "cut" differently to cover spatial configurations, as one can easily verify when comparing English prepositions with those of other languages, even with those of close relatives like German. Therefore, language independent methods are the most promising approach to determine the human conceptualization of space. Mark et al. [MCE⁺95] mentioned some of them, like the graphic examples task, the graphic comparison task, and the grouping task. In particular, the grouping task seems to be one prominent empirical method that is easy to establish and to communicate to the participant. The subject is given a sample of spatial configurations that she/he is prompted to group together. The basic idea is that the conceptualization of space guides the grouping of items into categories. These observable categories give important hints on the subjects' conceptualization and may serve to exclude certain sets of relations as cognitively adequate. Subsequent intensional descriptions of the groupings may provide additional hints (1) what informational content was used by the subject to group the items, and (2) to lower the risk of choosing the wrong intension of extension-equivalent categories.

3 Topological Relations for Spatial Reasoning

Relationships that are invariant under certain transformations of the underlying space are called topological relationships. This property makes them very interesting for both GIS and QSR where representation of and reasoning about topological relationships is often used. When representing direction, the question of the underlying frame of reference is ubiquitous. Whether using an intrinsic, extrinsic, or deictic frame of reference is a matter of the represented situation.

Looking at a spatial situation from different angles often changes the direction of objects with respect to a certain frame of reference. When representing distance, a typical problem is that of how many times "close" becomes "far", e.g., given a sequence of n objects that are all subsequently close to each other, for which number n is an object far from the first object of the sequence. Difficulties like these do not occur for topological relations, although reasoning with topological relations can also become extremely hard (see, e.g., Grigni et al. [GPP95]).

Systems of topological relations were developed in the area of GIS and QSR independently of each other. In GIS, Egenhofer [Ege91] classified the relationship between two spatial entities according to the nine possible intersections of their interior, exterior, and boundaries, hence called the 9-intersection-model. When looking only at whether these intersections are empty or non-empty and imposing certain constraints on the nature of the considered spatial entities (two-dimensional, one-piece, non-holed), this results in eight different binary topological relationships (see Figure 1). In QSR, Randell et al. [RCC92] studied the different possibilities for connecting spatial regions, in their approach regular subsets of a topological space (i.e., arbitrary but equal dimension, multiple pieces and holes possible). Among the many possibilities, they selected a set of eight different topological relationships with respect to connectedness which they called Region Connection Calculus RCC-8 (see Figure 1). The names of the relations are abbreviations of their meanings: DisConnected (DC), Externally Connected (EC), Partially Overlapping (PO), EQual (EQ), Tangential Proper Part (TPP), Non-Tangential Proper Part (NTPP), and their converses TPP⁻¹ and $NTPP^{-1}$.

Surprisingly, these two completely different approaches to topological relationships lead to exactly (apart from the different constraints on regions) the same set of topological relations. Thus, there seems to be a natural agreement about what is a reasonable level of granularity of topological relations. Within both approaches systems of five topological relations on a coarser level of granularity were developed that are slightly different from each other. They are referred to as medium resolution in Egenhofer's case (as opposed to the set of eight relations referred to as high resolution), and RCC-5 in Randell et al.'s case. The differences between the two systems were compared in [KRR97] and are shown in Figure 1. All systems of relations consist of jointly exhaustive and pairwise disjoint relations, i.e., between any two regions (that behave according to the given constraints) exactly one of these relations holds. Note that in the following we only refer to the Region Connection Calculus when referring to topological relations. This is only for increasing readability. All statements we make about the RCC-8 relations are of course equally true for the eight relations defined by Egenhofer.

Apart from these systems of topological relations, there is also work on topological relations on a finer level of granularity where more relations can be distinguished. Gotts [Got94,Got96], for instance, identified a large number of different topological relations that can be distinguished by their connectedness (the "C"

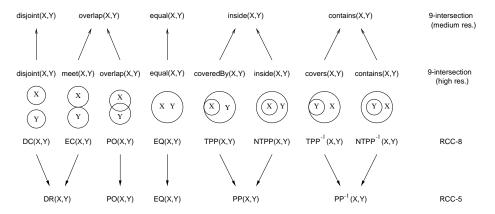


Fig. 1. Comparison of different systems of topological relations: the Region Connection Calculi RCC-8 and RCC-5 and the high and medium resolution relations of Egenhofer's 9-intersection model [KRR97].

relation). Another example of more expressive topological relations is given by Papadimitriou et al. [PSV96].

4 Previous Empirical Studies on Topological Relations

In this section we summarize the results of our previous empirical investigation [KRR97] on the cognitive adequacy of topological relations. The goal of that investigation was to find first evidence of whether the systems of topological relations developed by Egenhofer [Ege91] and by Randell et al. [RCC92] (see previous section) were cognitively adequate and which level of granularity is more adequate. For this investigation, 20 subjects had to group a set of 96 different items, each showing two different colored circles with differing centers and radii, such that items of the same group are most similar to each other. Examples of these items are shown in Figure 2. For each of the eight RCC-8 relations, 12 ± 3 items were chosen randomly as instances of these relations. After the grouping

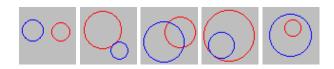


Fig. 2. Items with two different colored circles as used in [KRR97].

phase, subjects had to describe their groupings in a few words. This was done

in order to obtain further information on the criteria that guided the grouping behavior.

The cluster analysis of the groupings showed that all instances of the RCC-8 relations were clustered immediately. After a few clustering steps, instances of the relations TPP and TPP⁻¹ and of NTPP and NTPP⁻¹ were clustered which is due to disregarding the colors of the circles. At no stage of the cluster analysis the RCC-5 relations or Egenhofer's medium resolution relations were present which can be regarded as an empirical evidence that these systems of relations are conceptually inadequate.

The evaluation of the verbal descriptions of the groupings which was done by two independent raters (who agreed in more than 97% of their ratings) showed that for more than 62% of the descriptions only topological terms were used, more than 33% used topological and orientational or topological and metrical terms. Descriptions based on purely orientation or metric terms did not occur.

These results support the claim that the systems of topological relations developed by Egenhofer and Randell et al. might be regarded as cognitively adequate. We use this term very carefully and consider our previous investigation only as a first step towards proving cognitive adequacy. In order to finally prove cognitive adequacy our investigation was not general enough, since all used regions were circles. This does not prevent subjects from grouping items according to distance, size, orientation, or topology. However, for a general assessment of cognitive adequacy of the topological relations, we have to lift the restriction of the shape of regions we were using in our previous investigation. In the following section we present our new empirical investigation where we partially lift this restriction of the shape of regions.

5 The Empirical Investigation

In our new investigation we partially lift the constraints on the shape of the regions. In the former investigation all regions were circles (see Figure 2), whereas in our new investigation the regions are regular polygons such as squares, pentagons or hexagons. The main difference to using circles is that when circles externally connect each other or are tangential proper part of each other, their boundaries always touch each other at a single point, whereas regular polygons with these properties can touch each other at one or more points, along a line, or even at both. Some of the different possibilities of how regular polygons can touch each other are shown in Figure 3.

5.1 Subjects, Method and Procedure

19 students (10 female, 9 male) of the University of Freiburg participated for course credit. Subjects had to accomplish a grouping task (on a Sun Workstation) that consisted of 200 items showing a configuration of a red and a blue regular polygon as shown in Figure 3. For the sample of 200 items we randomly selected 25 ± 5 items for each of the eight RCC-8 relations with the following restrictions.

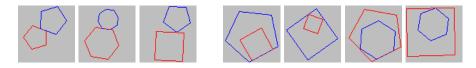


Fig. 3. Different possibilities of external and internal connection of regular polygons.

We generated regular polygons with four to eight corners, and made sure that there were at least some items for each possibility of how two regions can connect to each other.

After solving some practice trials to get acquainted with the procedure subjects saw the color screen as depicted in Figure 4. The screen was split into two different sections. The left section contained the items and the right section the groupings subjects had to form. The items had to be moved with a mouse from the left to the right. In order not to bias the number of groupings, only one group was displayed on the right in the beginning, and subjects had the possibility to add as many groupings as they wanted. The 200 items in the left section were arbitrarily split into four different windows such that subjects could switch between the four windows at any time.

All subjects had to judge the same 200 items because the aggregation of data had to be done across subjects per item. After the grouping phase, subjects had to describe their groupings by natural language descriptions (in German). We did this to obtain further information on the criteria that guided subjects' grouping behavior. We applied this (unexpected) phase at the end of the empirical session in order to avoid influences of verbalizations on the pure grouping behavior itself. For each subject the complete procedure was stored in a log-file and we developed a tool for tracing the complete procedure afterwards.

5.2 Results of the Investigation

Subjects needed about 45 minutes to group items and to describe their groupings. By aggregating grouping answers over all 19 subjects, we obtained a 200×200 matrix of Hamming distances between items that was the basis of a cluster analysis using the method of average linkage between groups. Unlike the results of [KRR97] where only some subjects did not distinguish between TPP and TPP⁻¹ and between NTPP and NTPP⁻¹, this time the result is more unique. As it can be seen in the dendrogram in Figure 5, there is no distinction between NTPP and NTPP⁻¹ and between TPP and TPP⁻¹ at all. As in our previous study, the clustering of TPP and NTPP with their converses can be attributed to disregarding the relationship of a reference object (RO) with a to-be-localized object (LO). However, there are sub-clusters of TPP or TPP⁻¹ items and of EC items. By looking at the items of the different sub-clusters, we found that these sub-clusters belong exactly to the different ways two regular polygons can be externally or internally connected to each other, either at a single point, at two

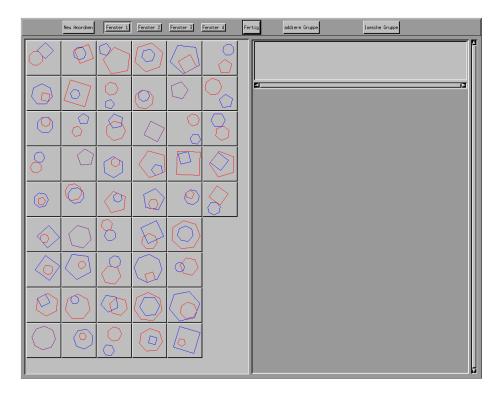


Fig. 4. Screen-shot of the monitor at the beginning of the grouping task.

points, or along a line. The clustering of groups on a higher level of the cluster analysis is due to idiosyncratic categorizations or mis-groupings of single items by a few subjects (one subject wrote that he used one group as a clipboard and forgot to assign these items in the end).

Furthermore, the verbalizations of subjects' groupings were analyzed by categorizing them according to whether they included purely topological (T), orientational (O), and metrical information (M). Additionally, we introduced two new categories: for the degree of contact of the boundaries (C), and for mentioning of pure shape (S). We also considered all possible combinations of these five factors. In Table 1, we present the results of the categorization of subjects' verbalizations, as completed by one independent rater.

Table 1 clearly shows that the informational content of nearly all verbalizations incorporated topological information alone or in combination with other information (42.1% + 14.6% + 36.6% + 1.2% = 94.5%). Metrical and orientational information was only used in combination with topological information. The category "other" in Table 1 consists of verbalizations that do not belong to any combination of T, O, M, C, or S, but were mostly mis-interpretations of the EQ items. The most surprising result, however, is that a substantial num-

Rescaled Distance Cluster Combine

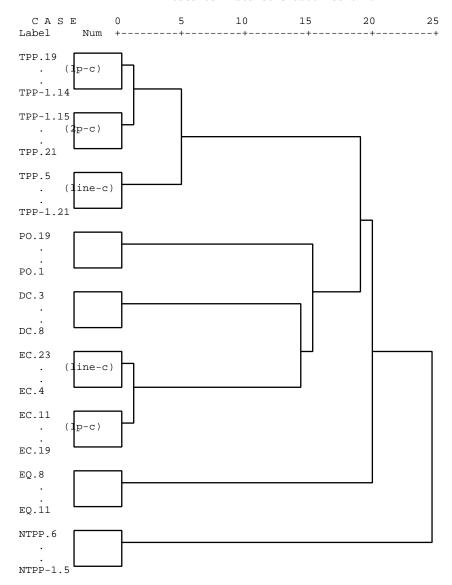


Fig. 5. Dendrogram of the cluster analysis of 200 items based on Hamming distances between items. Note that the labels given on the left are only for identification purposes. The three sub-clusters of the TPP and TPP⁻¹ items have the property of one-point contact (1p-c), two-point contact (2p-c), or line-contact (line-c), respectively. For the EC items there are two sub-clusters of items with line contact and one-point contact.

Informational content	${f Percentage}$
Topology (T)	42.1%
Orientation (O)	_
Metric (M)	_
Contact (C)	1.8%
Shape (S)	=
T + O	14.6%
T + C	36.6%
T + M	1.2%
Other	3.4%

Table 1. Categorization of subjects' verbalizations of their groupings (n=164 verbalizations).

ber of verbalizations mentioned the degree of contact as an additional modifier of topological relations. This also confirms the findings of the cluster analysis, where EC , TPP and TPP^{-1} configurations had sub-clusters that differed with respect to the degree of contact.

This observation was confirmed by evaluating the log-files we collected during the experiments. The "trace" tool enabled us to recover the final screen the subjects produced when finishing the grouping and the verbalization tasks (see Figure 6). We found that except for one subject, who grouped the items belonging to TPP and NTPP and to TPP⁻¹ and NTPP⁻¹ together and who distinguished items showing disconnected regions according to their direction, all other subjects did not distinguish the color of the regions presented on the objects, i.e., they did not distinguish between NTPP and NTPP⁻¹ and between TPP and TPP⁻¹. Table 2 shows the detailed evaluation of the final screens of the subjects. By RCC-8* we denote the set of RCC-8 relations without distinction of the reference object and the to-be-localized object. Figure 6 gives an example

#of subjects	description of their groupings
1	TPP and NTPP grouped together, DC plus direction
	RCC-8*
1	RCC-8* plus directions
1	RCC-8* plus size of overlapping area
	RCC-8* plus different connections for TPP
	RCC-8* plus different connections for EC and TPP
	RCC-8* plus different connections for EC and TPP and PO
1	RCC-8* plus different connections for EC and TPP,
	and different directions for PO and DC

Table 2. Evaluation of the subjects' final screen.

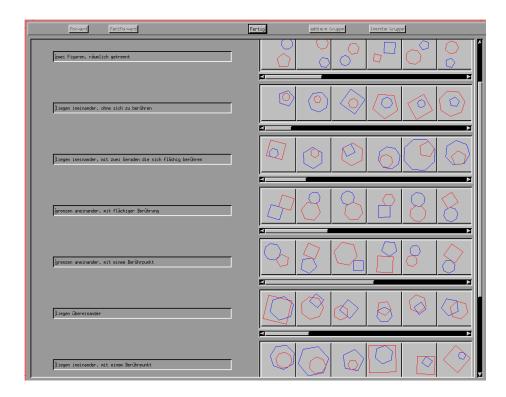


Fig. 6. Screen-shot of the recovered final screen of one of the subjects.

of the most common result, where the subject distinguished different kinds of external and internal connection.

6 Discussion and Future Work

For the investigation presented in this paper, we increased the different possibilities of how subjects can group items by partially lifting the constraints on the shape of the regions we used. Except for one subject, all others grouped the items according to the distinctions made by the RCC-8 relations. Even those subjects who also took direction or size into account only did this to further refine the RCC-8 relations but never used orientation or shape independently of the topological relations. This result gives further evidence that topological relations in general are an important part of human spatial cognition, and that the level of granularity given by the RCC-8 relations is particularly important. Topological distinctions on a coarser level of granularity do not seem to be cognitively adequate. This confirms the observations made in our previous investigation [KRR97].

Many subjects also distinguished items according to a finer level of granularity than given by the RCC-8 relations. 11 out of 19 subjects distinguished between the different possibilities of how two regions can touch each other from the outside or from the inside. These distinctions can all be made on a purely topological level without taking into account other spatial aspects such as direction, distance, or shape. Thus, it appears that topological relationships are much more important in human spatial cognition than any other relationships between spatial regions. If one wants to develop a new system of topological relations based on the results of our investigation, we propose to refine the relations EC, TPP, and TPP⁻¹ into different kinds of connection. Some possibilities are to distinguish connections at a single point, at more than one point, or along a line. It follows from Gotts's work [Got94,Got96] that all these relations can be defined in terms of the connected relation which is also used to define the RCC-8 relations.

Although almost all subjects did not distinguish between TPP and TPP⁻¹ and between NTPP and NTPP⁻¹, we do not consider the set of six relations we called RCC-8* as cognitively adequate. As already explained, this is due to disregarding the relationship of a reference object (RO) with a to-be-localized object (LO). We believe that when this distinction is explicitly emphasized in the instructions, subjects will group items accordingly. Furthermore, it follows from [RN99] that reasoning with RCC-8* relations is NP-hard whereas reasoning with the eight RCC-8 relations is tractable [Neb95].

Future work includes further lifting the restrictions on the shape of the presented regions. One question is whether subjects continue to distinguish the number of connections and up to which number. In our investigation some subjects distinguished between connections at a single point, at two points, and along a line. It will be interesting to see whether subjects distinguish connections at $1, 2, 3, \ldots, n$ points, and more than n points and which number n turns out to be the most commonly chosen (maybe it is the magical number 7 ± 2 , the capacity of the human short term memory [Mil56]). For arbitrary shapes of regions it is also possible that two regions connect along two or more lines. Another interesting question is how subjects group items when regions are allowed to consist of multiple pieces or to have holes. Answering this question will also allow judging the adequacy of the different restrictions on regions as given by Egenhofer [Ege91] and by Randell et al. [RCC92] (cf. Section 3). So far we have only investigated the conceptual cognitive adequacy of systems of topological relations. It will be interesting to see how humans reason about these relations, and to investigate the inferential cognitive adequacy of the reasoning methods used in qualitative spatial reasoning (cf. [KRS95]).

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