A Computational Model for Spatial Reasoning with Mental Models

Marco Ragni (ragni@informatik.uni-freiburg.de)
Department of Computer Science, Georges-Köhler-Allee
79110 Freiburg, Germany
Markus Knauff (markus.knauff@cognition.iig.uni-freiburg.de)
Center for Cognitive Science, Friedrichstraße 50
79098 Freiburg, Germany
Bernhard Nebel (nebel@informatik.uni-freiburg.de)
Department of Computer Science, Georges-Köhler-Allee
79110 Freiburg, Germany

Abstract

We propose a computational model for spatial reasoning by means of mental models. Our SRM model (**S**patial **R**easoning by **M**odels) maps spatial working memory to a two-dimensional array and uses a spatial focus that places objects in the array, manipulates the position of objects, and inspects the array to find spatial relations that are not given in the premises. The SRM model results in a computational complexity measure that relies on the number of operations in the array and the number of relations that must be handled. The performance of the SRM model is compared to the performance of human subjects reported in the literature and in our own study.

Introduction

People make extensive use of binary spatial relations which locate one object with respect to another, for example relations such as "to the left of", "in front of". Individuals are also able to reason with such relations; that is, to infer relations not explicitly given from the ones already known. Take, for instance, the following reasoning problem:

The hammer is to the right of the pliers.
The screwdriver is to the left of the pliers.
The wrench is in front of the screwdriver.
The saw is in front of the pliers.
Which relation holds between the wrench and the saw?

The four sentences are called *premises*, the tools are the *terms*, and the question refers to a possible *conclusion* that follows from the premises. There are basically two possible ways to make such inferences: by applying formal rules of inference to the linguistic representation of the premises or by constructing and inspecting a spatial array that represents the state of affairs described in the premises. This paper is based on the latter approach. The account was introduced by Huttenlocher (1968) and was further elaborated in the mental models theory (MMT) of reasoning (Johnson-Laird & Byrne, 1991; Johnson-Laird, 2001). According to the MMT, linguistic processes are only relevant to transfer the information from the premises into a spatial array and back again, but the reasoning process itself totally relies on non-linguistic processes for the construction and inspection of spatial mental models.

The aim of the following paper is to suggest a detailed computational model for human reasoning with spatial relations based on mental models. We formally describe the main assumptions that underlie our computational theory. Then we describe how the SRM model (**S**patial **R**easoning by **M**odels) constructs mental models from the premises and uses a spatial focus to manipulate the position of objects and to inspect the array to find spatial relations that are not given in the premises. In the third part, we describe a complexity measure that immediately results from the SRM model. It simply relies on the number of operations in the model and the number of relations that must be handled. We show that our approach accounts for many experimental findings.

Basic assumptions for the computational model

The aim of this section is to define in formal terms the main tenets underlying our SRM model. First, a model is defined in the usual logical sense as a structure in which the premises are true. Psychologically, a model is an internal representation of objects and relations in spatial working memory that matches the state of affairs given in the premises of the reasoning task. From the *representational view*, the model could account for *metrical* or *relational* information. The former is the more constraining (i.e., stronger) and usually identified with visual mental images. The latter is less constraining (i.e., weaker) and typically identified with spatial representations (Berendt & Schlieder, 1998). Following the principle of representational parsimony, our account is based on relational information alone. Thus, models are spatial representations that are more abstract than visual images. They avoid excessive visual detail to bring out salient information for inferences (Knauff & Johnson-Laird, 2002). In the SRM model, spatial working memory is conceptualized as a *spatial array*. From the different ways in which spatial relations can be represented, we choose the most parsimonious, namely to represent only that one object is to the left of, to the right of, in front of, or behind the other object. Each of these binary relations is defined as a triple (X, r, Y) in which

X is the referent
r is a binary local relation, and
Y is the relatum.

The referent X is the "to be located object" (LO), and the relatum Y is the reference object (RO) (Miller & Johnson-Laird, 1976). Thus, in a typical case the RO must be integrated into the model first followed by the LO. One exception is the first premise. Here, we assume that reasoners prefer to switch the roles of RO and LO in favor of an incremental model construction. An incremental model construction saves working memory capacities because each bit of information is immediately processed and integrated into the model (Johnson-Laid & Byrne, 1991). The SRM model does not account for the problems related to the ambiguity of spatial relations (Knauff, 1999). We simply assume that "left" means that the LO is to the left of the RO and exactly in the same line. It can be adjacent to the RO or there can be other cells (empty or filled) in between. The relation "in front of" means that the LO is in a cell in front of the RO and exactly in the same line. It can also be adjacent to the RO, or there can be other cells in between. "Right" and "behind" are defined equivalently.

The reasoning process in the SRM model is realized as a move of a spatial focus. This focus can place an element into the model or inspect the model to find new information (Schaeken et. al., 1996). We assume that the reasoning process proceeds in three steps. In the *construction phase*, reasoners construct a mental model that reflects the information from the premises. For the preceding example, they, for instance, construct the following model:

screwdriver	pliers	hammer
wrench	saw	

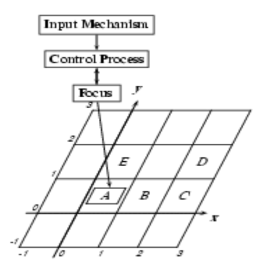
In agreement with many experimental findings, we assume that if new information is encountered during the reading of the premises it is immediately used in the construction of the model (Johnson-Laird & Byrne, 1991). In the *inspection phase*, this model is inspected to find new information that is not explicitly given in the premises. From this model it follows: *the wrench is to the left of the saw*. In the *variation phase* alternative models are constructed from the premises that refute this putative conclusion. In our example no such model exists and thus the conclusion is valid. Although this phase lies in the heart of reasoning, it is still unclear how it exactly works. The orthodox view is an iteration of the first two phases in which alternative models are generated and inspected in turn (Johnson-Laird & Byrne, 1991). In the following, however, we refer to our own account saying that there is no iteration process but rather a process that starts from the preferred mental model (PMM) and then *varies* this model to find alternative interpretations of the premises (Rauh, Hagen, Knauff, Kuß, Schlieder, & Strube, 2005). The term PMM refers to a phenomenon encountered during reasoning with multiple-model problems. In problems in which more than one model is consistent with the premises (so called indeterminate problems) reasoners often construct only one single model – the PMM. This model is the one that is easier to construct and to maintain in working memory than other possible models (Knauff, Rauh, Schlieder, & Strube, 1998). From many studies it is known that indeterminate problems are more difficult than determinate ones, and the PMM frequently lead to incorrect conclusions because other possible models are ignored (Rauh et al., 2005).

The SRM model

We are now in the position to formally define our SRM model as a quintuple (*I*, *O*, *A*, *F*, *C*), with

- *I the input mechanism*. This process reads the premises from an external device (The comprehension of the meaning of the premises is not part of our model. We simply assume that there is an external "parser" that supplies the correct meaning to the model).
- *O* being a set of object names.
- *A* being a spatial array.
- *F* being the focus that is on the spatial array, initially on position (0, 0), is able to move right, left, front, behind, but can also perform a no-move operation. In addition it has a grouping function, and a shift operator for groups.
- *C* being a control process that is responsible for controlling the focus and other executive functions.

At each time of the inference process the SRM model has a problem *input* (a set of premises) where the relational problem is stated, a control process to generate a mental model from the problem input, a two-dimensional spatial array, and a focus which can be used to inspect the model or to place an element into the model. According to findings from Vandierendonck et al. (2004), the focus is also able to write annotations to objects (see below). These annotations are only in use if the model detects the indeterminacy of the premises. In this case, the relation that holds between the RO and the LO is attached to the LO (see below). An illustration of the SRM model is given in Figure 1.



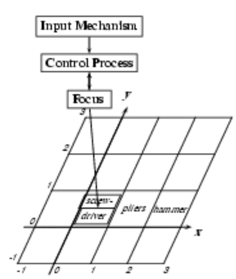
How does the SRM model implement the three phases of the inference? At the beginning of the construction phase, the focus is at position (0, 0) and there are 5 directions in which the focus can be moved: right, left, front, behind, and no-move. Now four types of premises must be distinguished: (1) premises in which no object has been in the array so far(that is the first premise), (2) premises in which one object from the preceding premise appears (a middle term that connects the premise to the previous one) and the next object must be added to the array, (3) the type of premises in which no object of the previous premise appears. This is typically the case, when the second premise of a discontinuous premise order must be processed, i.e. D r C, A r B, B r C, and (4) premise in which an object appears that connects two formally separate models. This is the case when the third premise of a discontinuous premise order must be processed. The SRM model now works on an input I in the following way:

(1) Initially the SRM model receives a premise of type 1.
(2) The SRM model inserts the first object of the first premise in cell (0, 0). Then it uses this object as RO and adds the second object into the next adjacent cell according to the local relation.
(3) The "parser" reads the next premise
(4) The SRM model decides on the type of premise:
    - The premise is from type 2: the focus moves to the RO, from there it inserts into the next cell according to the relation the LO. If there is already an object in the cell the machine moves back to the reference object and makes an annotation. Then it moves then to the next free cell according to the relation and inserts the object into the next free position (according to the relation to RO).
    - The premise is of type 3: a new spatial array is generated and both objects are inserted as for premises of type 1 (see Step 2), (Schaeken et al., 1996).
    - The premise is of type 4: the focus groups one model and inserts it into the other model (Bara et al., 2001)

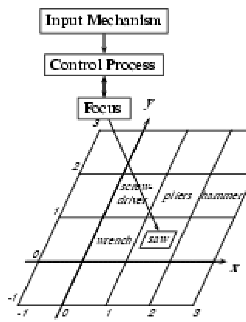
When the model construction is finished, the *inspection phase* works for our example in the following way: a conclusion must be generated that defines the relation that holds between the wrench and the saw. So the focus moves to the wrench (RO) and then inspects the model to find the saw (LO). In previous studies, we were able to determine how this inspection process works (Knauff et al., 1998). After constructing the mental model, the focus is positioned on the last end-term of the last premise and this should also be the starting point for the scanning of the RO. In our model, then the scanning for the LO proceeds in the same direction as before when it found the RO. This saves the costs of re-focusing (see below). If the LO cannot be found in this direction the focus changes its direction and preceeds until it has found the LO. It is important that in our model, the focus only checks the cells of the array in which an object is. Empty cells are not scanned. In other words, the system "knows" which cells are occupied but not which object is in the cell. The current model does not make any assumptions about how this is realized (although it is easy to imagine that filled cells are more activated in the array). If the LO is found from the scan direction the relation between the two objects is known (the meaning is again provided by the external module). What happens if a possible conclusion must be verified? This is the case when the question for the relation is replaced by a conclusion that must be verified. Assume that the model must check whether the conclusion "The wrench is to the left of the saw" is valid. In this case, the focus moves to the saw (RO) and then scans the array to the left to find the wrench (LO). Since the conclusion is valid the model generates the output "valid conclusion" (also not part of our model). It is important to notice that in the SRM model no variation of the model is assumed if a conclusion is generated. The SRM model stops when it has found just one model – which often leads to errors. Model variation only comes into play if a conclusion must be verified or if more than one model can be constructed from the premises. We are still working on the exact details of the *variation phase*, but we definitely assume that there is no iteration of the first two phases in which alternative models are generated and inspected in turn (Johnson-Laird & Byrne, 1991). Instead, the current version of the SRM model starts from the PMM and then successively generates alternative models by modifying the PMM with minimal changes (Rauh et al. 2005). The minimal changes follow the principle of "conceptual neighborhood" which we have empirically determined in recent studies (Rauh et al. 2005). The principle says that alternative models are generated by local transformations, i.e. moving one object in the model. To find the next alternative model, the SRM model starts from the RO of the conclusion and first checks if the next objects have annotations with respect to the LO. As already mentioned, this annotation basically stores the relation that must hold between RO and LO. If so (this is always the case in indeterminate problems because the premises itself are forgotten) the SRM model starts to change the position of the objects as long as the constraint from the annotation is satisfied. This takes it stepwise to alternative models but also has the consequence that models which are difficult to reach are thus more likely to be neglected than models which are only minor revisions of the PMM. This phenomenon we reported in recent experiments (Rauh et al., 2005).

A Processing Example

In the following, we describe how the SRM model works with the above mentioned example. The *construction* phase: The SRM model has received the first premise: *The hammer is to the right of the pliers*. The focus takes the hammer as RO (because it is mentioned first) and inserts it in the spatial array. In the next step, the focus moves to the left (the linguistic process of generating the reverse relation is not part of our model, but see Clark, 1969) and inserts the pliers into the next free position. Then, the model reads the next premise: *The screwdriver is to the left of the pliers*. The focus is still on the pliers and inserts the screwdriver in the next free cell to the left.



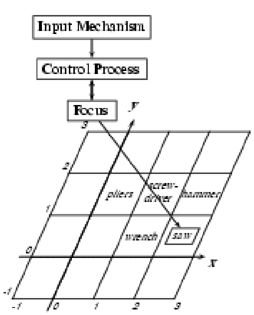
Then, the model reads the next premise: *The wrench is in front of the screwdriver*. The focus changes its direction and moves to the front cell (of its last cell) and inserts the wrench. The model reads the next premise: *The saw is in front of the pliers*. The focus moves back to the pliers. After that, it moves one step in front of the pliers and inserts the saw. Since in this case the model is determinate, there is no model variation phase. In the *inspection* phase the machine checks the putative conclusion: Which relation holds between the wrench and the saw? The focus now checks if the position that it is on contains an object mentioned in the conclusion. In this case it is so, as the last operation performed was the insertion of the saw.

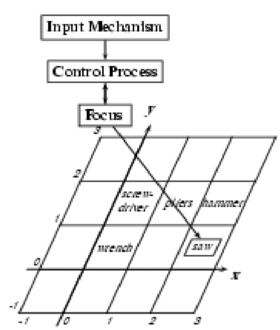


Then, the focus moves from the saw to the wrench and checks in which direction it moves and if it changes its direction. The output is: *The wrench is left of the saw*. This process is more complex for indeterminate modelsas the SRM model uses annotations to "remember" the indeterminate models. To give an example, assume that the following premises are given:

> The hammer is to the right of the pliers.
> The screwdriver is to the left of the hammer
> The wrench is in front of the screwdriver.
> The saw is in front of the hammer.

There are two possible models:





The SRM model works on the premises in the following way: it reads the first premise and constructs the model in analogy to the described procedure. For the second premise the machine proceeds as follows: it reads the second premise, moves to the reference object (the hammer, because it is in the model), from there it then moves to the left, and the focus scans (reads) the inserted screwdriver. Obviously, the model cannot insert an object there, because the cell is already occupied with the pliers. Therefore, the model makes an annotation at the hammer, saying that the screwdriver is to the left (the model "saves" the information that the screwdriver is left to the hammer), it then moves to the left of the pliers and inserts the screwdriver. This gives us the first model, and this model is constructed according to a principle we refer to as *first free fit (fff)*. It says that an object is inserted at the first *free* position. The alternative principle we call the *first fit (ff)* principle and this gives us the second possible model. The ff principle always inserts the object at the *next* position that fulfills the spatial relation from the premise. This sometimes means that another object that is already in the model must be moved. The fff-principle results in the first model and the ff-principle in the second model.

A Complexity Measure

We define three categories of relational problems: those that have only one model, the determinate problems, those that have only a small number of models, all of which can be checked by humans, and those at which the number of models exceeds the capacities of the human working memory. To capture the notion of practically feasible problems, we must limit our computational device to only run for a number of steps that is bounded by a function. Should this function be bounded in the length of the input, i.e. the number of premises, which would be a standard definition in the theory of computation? We do not think this is a cognitively plausible criterion. Instead we believe that the *number of relations* that have to be handled by the cognitive system is the limiting factor (Maybery et al., 1986). Based on this assumption, we now introduce a complexity measure, that can be used to classify the difficulty of different reasoning problems. The main concept is thus an abstract "unit" that stands for the number of operations in the array and the number of relations that must be handled. Although this concept is quite abstract, there are many experimental findings in support of the assumption that the capacities of the cognitive system are limited by both, in terms of visuo-spatial capacities (Luck & Vogel, 1997; Sperling, 1960) and in terms of relational complexity (Maybery et al., 1986).

The model maps the spatial working memory to a two-dimensional spatial array and a focus function. Since the processing consists mainly in model construction, -inspection, and -variation, and since these processes are done by the focus, the difficulty of tasks clearly depends on the movement of the focus. Assume that we only have a set of premises for which the focus moves along one direction and inserts successively an element into the array. This problem is of course a lot easier than a problem, where the focus has to change the direction several times and to insert several objects in-between other objects. As it is common in complexity theory we abstract from different costs for different operations of this machine and use *only one uniform complexity measure*. Nonetheless, we can show that the empirical differences in reasoning difficulty can be captured by this measure. The focus has, as has been shown, several functions. The first function: the focus can scan the model, i.e. this scanning process consists of a sequence of movements of the focus to the left, right, front, or behind (cf. with the scanning process, Schaeken et al., 1996, pp. 211). In our account, each movement of the focus costs one unit, each direction change costs an additional unit. The second function: the focus can insert or delete an object in the model. This operation also costs one unit. The third function: the focus can shift an object or a group (of objects). This operation also costs one unit. Finally, objects can be grouped together. Here the number of relations comes into play: the grouping operation costs one unit for each relation between neighbored objects in the grouping. This is according to Maybery, et al. (1986) a sensible cost measure from a cognitive point of view, and recent brain imaging studies also have shown that it correlates with neural activity in working memory related brain areas (e.g. Waltz, et. al., 1999). To give an example, we want to group these three objects:

screwdriver hammer pliers

This means that the screwdriver is left of the hammer, and the hammer is left to of the pliers. This arrangement is perfectly described by these relations, and we do not need the relation that the screwdriver is left of the pliers. The grouping process, in this case, can be compared to composing the two binary relations to a ternary relation. Or to be more general, for *(n+1)* objects to be grouped, we have *n* binary relations, and the grouping consists of a *n*–ary relation, and this grouping costs *n* units.

How does this complexity measure help to explain the different difficulties of reasoning problems? First, it follows that the premises

The screwdriver is to the left of the hammer.
The hammer is to the left of the pliers.

are the easiest problem. Many studies have shown that such tasks are very easy to solve because they are compatible with only one model, and the last term of the first premise is the first term in the second premise (Johnson-Laird & Byrne, 1991). The costs to build this model is *three* units (one for each of the three objects). The harder problem is:

The hammer is to the right of the screwdriver.
The hammer is to the left of the pliers.

First, the SRM model inserts the hammer, moves to the left to insert the screwdriver, then moves two steps right (because one step left of the inserted screwdriver is the hammer), and then inserts the pliers. The costs for building this model is five units (because of the direction change). The two examples already indicate that the model with the complexity measure can differentiate between problems with different term and premise orders (and thus also for the "figural effect" in spatial reasoning; Johnson-Laird & Bara, 1984; 1982; Knauff, et al. 1998). The approach explains the PMM, which is computationally the cheapest, and also accounts for the difference in difficulty between determinate and indeterminate problems, since in the model variation phase (see above) the machine generates according to the annotations the other models. This generation which consists of shifting and grouping operations incurs additional costs.

First Free Fit, First fit, and other Empirical Data

We here very briefly compare the performance of the SRM model to some experimental findings with human subjects. A more detailed evaluation will be provided in Ragni, Knauff, & Nebel (in prep.). First, the SRM model is able to reconstruct the effect of term order in spatial reasoning. Many studies have shown that problems are easier to be solved if the end term of the first premise is the first term in the second premise (for an overview see Manktelow, 1999). Our model can explain this by the different costs in terms of necessary units. Another well-known effect is that indeterminate problems are more difficult than determinate problems (Byrne & Johnson-Laird, 1989). In the previous section we have described that this difference can also be explained by the SRM model. Our main attention, however, is focused on the two insertion principles, first free fit (fff) and first fit (ff), and the explanation of preferred mental models. First, we have some initial evidence that in problems of our indeterminate example the model on the left-hand side is preferred over the right-hand side model. In Knauff & Ragni (in prep) 20 volunteers (all logically naïve undergraduate students) participated in a paper-and pencil test in which they had to draw "just one model" that was consistent with a set of premises. In nine of the twelve problems the participants generated a model that followed the same principle as the construction of the left-hand side model. The other possible model was generated very rarely. More empirical evidence comes from a study by Jahn, Knauff, & Johnson-Laird (in press). In this study, we directly tested the ff and the fff principle against each other. Twenty-four students of the University of Tübingen served as paid participants. The reasoning problems consisted of three premises that referred to horizontal one-dimensional layouts of four objects. The premises were consistent with four different arrangements. The problems were displayed on the computer screen, and the presentation was self-paced. Each trial began with the initial two premises. When participants pressed the space-key, the third premise replaced the initial premise. The third premise was presented together with the prompt "Is there a layout for which all premises are true?". After the participants had responded "yes" or "no" with one of the response keys, they used the initial letters of the four objects to write down a layout on an answer sheet. The results of this study clearly support the first free fit (fff) principle. Sixty-five percent of the generated models agreed with this principle, whereas the other 35 percent were distributed over the three other models. This means that only about 10 percent followed the ff principle. The details of this study can be found in Jahn, Knauff, & Johnson-Laird (in press). The most important result from this study is the following: if the first possible position in the model that fulfils the spatial relation of a premise is already occupied by another object, human subjects prefer to sacrifice adjacency (ff) in favor of outside insertion. In other words, they avoid relocating an object that is already in the model to make the first possible position free. Instead, they place the object in question at the end of the line, where the relation is also fulfilled. Our SRM model and the related complexity measure predict exactly this behavior. In other words: the fff principle is less expensive than the ff principle.

Discussion

The presented SRM model allows us to construct and manipulate mental models. This computational model implies a complexity measure based on abstract "units" that might have a cognitive counterpart: Nonetheless, our SRM model can predict problem difficulty and model preferences.

What are the limits of the SRM model? One could object that the spatial array structure, which is in fact a discretization of space, seems to be too restrictive. Nonetheless, this is not a real restriction because, for all psychologically relevant reasoning problems, the number of objects is bounded. Moreover, the shift operator can simulate a non-discrete space, even if a third object cannot be placed between two objects. This is a kind of "continuousness" that is sufficiently general in our context.

What are the differences to existing computational models for (spatial) reasoning? The models of Schlieder and Berendt (1998) also make use of a focus and explain model preferences. Both models, however, are restricted on intervals as elements and a quite technical set of relations. A fundamental difference is that our model is much more "natural" because it uses solid objects and the most common verbal relations from natural language (and reasoning research). Our computational model shares the most features with the UNICORE model developed by Bara et al. (2001). Both models are based on the same three considerations: a model must include a grid of positions that are assigned to tokens (our spatial array), those tokens must have a name (our objects), and some objects may be in relation. The main difference between Bara's model and the SRM model is that our model reproduces reasoning steps involved in spatial reasoning, the UNICORE model does not have this property. Another advantage of the SRM model is that we have introduced a complexity measure which explains the difficulty of reasoning problems.

Acknowledgments

This research was supported by grants from the DFG within the SFB/TR 8 Spatial Cognition to MK and BN. MK is also supported by a Heisenberg Award from the DFG.

References

Bara, B., Bucciarelli M., & Lombardo V. (2001*).* Model theory of deduction: a unified computational approach. *Cognitive Science*, 25, 839-901.

Byrne, R. M. J., & Johnson-Laird, P. N. (1989). Spatial reasoning. *Journal of Memory and Language, 28,* 564-575.

Clark, H.H. (1969) Linguistic processes in deductive reasoning. *Psychological Review,* 76, 387-404.

Huttenlocher, J. (1968). Constructing spatial images: A strategy in reasoning. Psychological Review, 75, 550-560.

Jahn, G., Knauff, M., & Johnson-Laird, P. (in press). Preferred Mental Models in Spatial Reasoning. In *Proceedings of the Twenty Seventh Annual Conference of the Cognitive Science Society,* Mahwah, NJ: Erlbaum.

Johnson-Laird, P. N. (2001). Mental models and deduction. *Trends in Cognitive Sciences*, *5*, 434-442.

Johnson-Laird, P. N., & Byrne, R. M. J. (1991). *Deduction*. Hove, UK: Erlbaum.

Knauff, M. (1999). The cognitive adequacy of Allen's interval calculus for qualitative spatial representation and reasoning. *Spatial Cognition and Computation* 1, 261-290

Knauff, M., & Johnson-Laird, P. N. (2002). Visual imagery can impede reasoning. *Memory & Cognition*, 30, 363-371.

Knauff, M., Rauh, R., & Schlieder, C. (1995). Preferred mental models in qualitative spatial reasoning: A cognitive assessment of Allen's calculus. In *Proceedings of the Seventeenth Annual Conference of the Cognitive Science Society* (pp. 200-205). Mahwah, NJ: Erlbaum.

Knauff, M., Rauh, R., Schlieder, C., & Strube, G. (1998). Continuity effect and figural bias in spatial relational inference. In *Proceedings of the Twentieth Annual Conference of the Cognitive Science Society* (pp. 573-578). Mahwah, NJ: Erlbaum.

Knauff, M., Strube, G., Jola, C., Rauh, R., & Schlieder, C. (2004). The psychological validity of qualitative spatial reasoning in one dimension. *Spatial Cognition and Computation* 4, 167-188.

Luck, S. J. & Vogel, E. K. (1997). The capacity of visual working memory for features and conjunctions. *Nature* 390, 279–281.

Manktelow, K.I. (1999). *Reasoning and Thinking*. Hove (UK): Psychology Press.

Maybery, M.T., Bain, J.D., & Halford, G.S. (1986): Information processing demands of transitive inference. *Journal of Experimental Psychology: Learning, Memory, and Cognition*. 12, 600-613.

Miller, G. A. & Johnson-Laird, P.N. (1976).: Language and Perception. Cambridge: Cambridge University press.

Rauh, R., Hagen, C., Knauff, M., Kuß, T., Schlieder, C. and Strube, G (2005). Preferred and alternative mental models in spatial reasoning. *Spatial Cognition and Computation* (in press).

Schlieder, C., & Berendt, B. (1998). Mental model construction in spatial reasoning: A comparison of two computational theories. In U. Schmid, J. F. Krems & F. Wysotzki (Eds.), *Mind modelling: A cognitive science approach to reasoning* (pp. 133-162). Lengerich: Pabst.

Sperling, G. (1960). The information available in brief visual presentations. *Psychological. Monographs*. 74 (Whole number 498).

Vandierendonck, A., Dierckx, V., & Vooght G. D. (2004). Mental model construction in linear reasoning: Evidence for the construction of initial annotated models. *The Quarterly Journal of Experimental Psychology. 57A, 1369-1391.*

Waltz, J. A., Knowlton, B. J., Holyoak, K. J., Boone, K. B., Mishkin, F. S., Menezes Santos, M. de, Thomas, C. R., & Miller, B. L. (1999). A system for relational reasoning in human prefrontal cortex. *Psychological Science*, 10, 119-125