Computational Properties of Qualitative Spatial Reasoning: First Results*

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Abstract. While the computational properties of qualitative temporal reasoning have been analyzed quite thoroughly, the computational properties of qualitative spatial reasoning are not very well investigated. In fact, almost no completeness results are known for qualitative spatial calculi and no computational complexity analysis has been carried out yet. In this paper, we will focus on the so-called RCC approach and use Bennett's encoding of spatial reasoning in intuitionistic logic in order to show that consistency checking for the topological base relations can be done efficiently. Further, we show that path-consistency is sufficient for deciding global consistency. As a side-effect we prove a particular fragment of propositional intuitionistic logic to be tractable.

1 Introduction

If precise, quantitative information is either not available or desirable, qualitative representation and reasoning can be worthwhile. If, for instance, the layout of documents that are to be analyzed should be described, a qualitative description seems much more favorable than a rigid, quantitative description.

While the computational properties of qualitative temporal reasoning [1] have been analyzed quite thoroughly [20, 19, 12], for qualitative spatial reasoning the situation appears to be completely different. First of all, it is not clear what the *representational primitives* should be. Contrary to the one-dimensional temporal case, for spatial reasoning there are a number of different interesting aspects along which one can abstract in order to derive qualitative descriptions (see e.g. [5]). Secondly, for almost all proposals there are no formal results concerning completeness or computational complexity of the proposed calculi.

In this paper, we will focus on the so-called RCC approach [15, 17]. This approach is based on modeling qualitative spatial binary relations between *regions* using first-order predicate logic. While a description of the theory in terms of first-order logic is certainly worthwhile for formalizing the approach, it does not lead to efficient computation by itself. In fact, even computing a *composition*

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table for the basic relations turns out to be very time-consuming if one relies on the first-order formalization [16]. Bennett [2] used an encoding of the topological RCC relations in *intuitionistic logic* in order to overcome this problem and demonstrated that a propositional intuitionistic theorem prover could compute a composition table much more efficiently than a first-order theorem prover such as OTTER.

Interestingly, Bennett's [2] encoding of qualitative spatial relations using intuitionistic logic uses a quite restricted fragment of this logic. Indeed, as noted by Bennett [2], this can lead to a possible reduction of the search space that makes proofs in the restricted fragment much more efficient than in general intuitionistic logic.

As we will show below, the intuitionistic fragment needed to encode qualitative spatial relations is a polynomial-time fragment, implying that the composition table could be generated quite efficiently. Further, the fragment is in NC, i.e., efficiently solvable on parallel machines.[2] This is even more surprising when considering the fact that general propositional intuitionistic logic is PSPACE-complete. Based on the tractability proof for this fragment of intuitionistic logic, it is also possible to show that a *path-consistent* [10, 11] network of topological base relations is globally consistent.

The rest of the paper is structured as follows. In Section 2, the topological relations defined in the RCC theory and Bennett's encoding in a fragment of intuitionistic logic are sketched. In Section 3, we show that reasoning in this fragment is tractable. Based on this proof, in Section 4 we show that path consistency of a network of topological base relations is sufficient for deciding global consistency. Finally, in Section 5 we discuss the results and give a list of interesting open problems.

2 Spatial Reasoning and Intuitionistic Inference

Figure 1 gives 2-dimensional examples for the 8 pairwise disjoint and exhausting relations that form the basis of the set of binary topological relations definable in the RCC framework [17].[3] These relations will be denoted by the capital letters R, T, S.[4] In order to describe the configuration of a set of **regions** (denoted by x, y, z), one can use atomic formulae of the form $R(x, y)$, which we will call **spatial formulae** (denoted by $\mathbf{r}, \mathbf{s}, \mathbf{t}$). Finite sets of such spatial formulae are denoted by Θ.

One computational problem in this context is to check whether such a set Θ of spatial formulae is **consistent**, i.e., whether it is possible to assign regions to

[2] See, for example, [8]. NC is the class of problems that can be solved in polylogarithmic time on machines with polynomially many processors.

[3] These relations are identical to those considered by Egenhofer [4].

[4] In the general case, one may also want to consider *unions* of the base relations in order to express indefinite information. In the present paper, however, we will only consider base relations (but see Section 5).

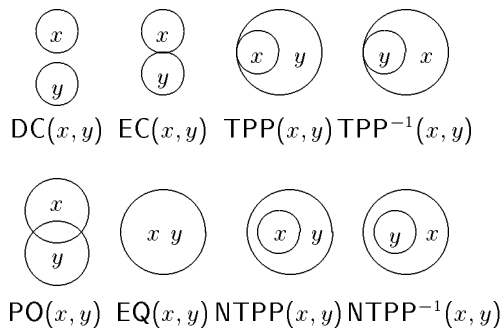

Fig. 1. Basic topological relations in the RCC theory

all variables such that the specified relations hold. For instance, assuming

$$\Theta = \{\mathsf{TPP}(y, x), \mathsf{TPP}(z, y)\}, \tag{1}$$

it is obvious that there exists an instantiation of all variables to regions such that the stated relations hold. This is even true if the formula $\mathsf{NTPP}(z, x)$ is added to Θ. However, if we add $\mathsf{PO}(x, z)$ to Θ, the set becomes inconsistent.

Although the problem of deciding consistency for a set of spatial formulae is important if one wants to reason with topological relations, no complexity results are known. The proposed procedures for this problem are either not known to be complete (those based on constraint propagation [4, 6]) or they appear to inefficient (those based on logical formalizations [17, 2]).

In order to analyze the complexity of the consistency problem for topological base relations, we will use Bennett's [2] representation of topological relations in *propositional intuitionistic logic*. Using the interpretation of propositional atoms as *open sets in a topological space* [18], Bennett [2] describes an approach that uses reasoning in propositional intuitionistic logic in order to decide the consistency of a set of spatial formulae. In this approach, each of the topological base relations is associated with a set of *model constraints* $\mathcal{M}$ and a set of *entailment constraints* $\mathcal{E}$, which are formulated using intuitionistic logic.

Table (1) specifies the constraints for all base relations. We use the symbols x, y, z to denote propositional atoms (corresponding to regions), and $\wedge$, $\vee$, $\sim$, and $\Rightarrow$ are used to denote the intuitionistic connectors conjunction, disjunction, negation, and arrow, respectively.

The model and entailment constraints associated with a spatial formula $\mathbf{r}$ will be denoted by $\mathcal{M}(\mathbf{r})$ or $\mathcal{E}(\mathbf{r})$. The constraints associated with a set of spatial formulae Θ is simply the union over all constraints associated with each element, and they are written as $\mathcal{M}(\Theta)$ or $\mathcal{E}(\Theta)$, respectively. Propositional intuitionistic formulae are denoted by φ and ψ, and for intuitionistic entailment the symbol $\vdash_I$ is used.

Relation	*Model Constraints*	*Entailment Constraints*
$\mathsf{DC}(x,y)$	$\sim x \vee \sim y$	$\sim x, \sim y$
$\mathsf{EC}(x,y)$	$\sim(x \wedge y)$	$\sim x, \sim y, \sim x \vee \sim y$
$\mathsf{PO}(x,y)$		$\sim x, \sim y, \sim(x \wedge y), x \Rightarrow y, y \Rightarrow x$
$\mathsf{TPP}(x,y)$	$x \Rightarrow y$	$\sim x, \sim y, \sim x \vee y, y \Rightarrow x$
$\mathsf{TPP}^{-1}(x,y)$	$y \Rightarrow x$	$\sim x, \sim y, \sim y \vee x, x \Rightarrow y$
$\mathsf{NTPP}(x,y)$	$\sim x \vee y$	$\sim x, \sim y, y \Rightarrow x$
$\mathsf{NTPP}^{-1}(x,y)$	$\sim y \vee x$	$\sim x, \sim y, x \Rightarrow y$
$\mathsf{EQ}(x,y)$	$x \Rightarrow y, y \Rightarrow x$	$\sim x, \sim y$

Table 1. Model and entailment constraints for the 8 base relations

The main intuition behind the model and entailment constraints is that the model constraint must hold if the respective spatial formula is asserted, while the entailment constraints should not be forced to be true, i.e., they should not be entailed by the model constraints. Otherwise, the set of spatial formulae is inconsistent. This intuition is formalized in Bennett's [2] theorem that reduces inconsistency of spatial formulae to entailment in intuitionistic logic, as spelled out below.

Theorem 1 (Bennett 1994). *A finite set Θ of spatial formulae is consistent iff for all $\varphi \in \mathcal{E}(\Theta)$: $\mathcal{M}(\Theta) \not\vdash_I \varphi$.*

As noted by Bennett [2], the move to propositional intuitionistic logic did not only result in decidability, but because of the restricted form of the formulae, reasoning appeared to be significantly easier than reasoning with general intuitionistic logic, which is known to be PSPACE-complete [7]. Below we will see that reasoning in the fragment of propositional intuitionistic logic used in Bennett's Theorem is indeed a polynomial-time problem, which implies that the consistency of a set of spatial formulae can be checked efficiently.

3 Reasoning Efficiently in Binary-Clause Intuitionistic Logic

The formulae appearing in the entailment problems in Bennett's Theorem are all of a particular simple form. They are all composed out of one or two propositional atoms, perhaps one binary connector and perhaps negation, but without nesting of negation or arrow in the scope of another negation or arrow. This description covers the formulae

$$\sim x, \sim x \vee \sim y, \sim(x \wedge y), \sim x \vee y, x \Rightarrow y, \tag{2}$$

which appear all in Table 1, as well as

$$x, x \vee y, x \wedge y, \sim x \wedge y, \sim x \wedge \sim y. \tag{3}$$

We will call the fragment of propositional intuitionistic logic that contains only formulae of the form given in (2) and (3) the **binary-clause fragment**, symbolically $\mathcal{I}_b$.

It appears to be plausible that it is possible to find a specialized intuitionistic proof calculus that permits efficient deduction in $\mathcal{I}_b$. In fact, Bennett [2] mentions that "the nondeterministic and extremely computationally expensive rule for eliminating implications from the left side of a sequent can be replaced by other rules," leading to a reduction in the search space. However, this modification of the calculus did obviously not lead to a completely deterministic proof system.

In order to show that intuitionistic entailment for $\mathcal{I}_b$ is tractable, we will use a (classical) possible-worlds semantics for intuitionistic logic [9] and rely on a tableaux-based proof method (see [13]) to generate counter-examples invalidating an intuitionistic formula.

A **possible-world model** for propositional intuitionistic logic is a triple

$$\mathcal{P} = \langle W, \preceq, \pi \rangle, \tag{4}$$

where $(W, \preceq)$ is a partially ordered set of **worlds** and π is a function from W to sets of propositional atoms such that

$$\text{for all } w, w' \text{ with } w \preceq w'\text{: } \pi(w) \subseteq \pi(w'). \tag{5}$$

We now define the **forcing** relation, symbolically $\Vdash$, between worlds and formulae by induction over the structure of the formulae for full propositional intuitionistic logic:

$$w \Vdash x \quad \text{iff} \quad x \in \pi(w) \tag{6}$$
$$w \Vdash (\varphi \Rightarrow \psi) \quad \text{iff} \quad \text{for all } v \succeq w\text{: } v \Vdash \varphi \text{ implies } v \Vdash \psi \tag{7}$$
$$w \Vdash {\sim}\varphi \quad \text{iff} \quad \text{for all } v \succeq w\text{: } v \not\Vdash \varphi \tag{8}$$
$$w \Vdash (\varphi \wedge \psi) \quad \text{iff} \quad w \Vdash \varphi \text{ and } w \Vdash \psi \tag{9}$$
$$w \Vdash (\varphi \vee \psi) \quad \text{iff} \quad w \Vdash \varphi \text{ or } w \Vdash \psi \tag{10}$$

We say that a formula is **forced in a model** $\mathcal{P}$ if it is forced by every world in $\mathcal{P}$. We say that a formula is **intuitionistically valid** if it is forced in all models. As shown by Kripke [9], this notion coincides with the property that the formula has an intuitionistic proof.

The formula ψ is called an **intuitionistic consequence** of $\varphi_1, \ldots, \varphi_n$, symbolically $\varphi_1, \ldots, \varphi_n \models_I \psi$, if for any model and any world w, $w \Vdash \varphi_1, \ldots, \varphi_n$ implies $w \Vdash \psi$. Obviously, this notion, which coincides with intuitionistic entailment, can be reduced to intuitionistic validity:

$$\varphi_1, \ldots, \varphi_n \models_I \psi \quad \text{iff} \quad \varphi_1 \wedge \ldots \wedge \varphi_n \Rightarrow \psi \text{ is valid.} \tag{11}$$

One way to show that a formula is valid (or invalid) is to systematically construct models that contain a world *not* forcing the formula. If one of these attempts succeeds, we know that the formula is not valid, otherwise it must be valid. This is precisely what tableaux-based proof methods do. However, instead of presenting now tableaux rules for intuitionistic logic [13], in our tractability proof we will rely on the reader's intuition that we have exhausted all possible ways of constructing counter-examples.

The key observations for the tractability proof of entailment in $\mathcal{I}_b$ is that entailment can be reduced to **2SAT**, i.e., to the problem of deciding whether a classical propositional formula in CNF containing only binary clauses[5] – also called **Krom** formula – is satisfiable. Such a formula can be constructed because:

1. A counter-example to the validity of a formula corresponding to an entailment problem in $\mathcal{I}_b$ needs not more than four worlds.
2. The forcing relation for propositional atoms can be expressed by a classical propositional Krom formula.
3. Finally, the constraint (5) in the definition of an intuitionistic possible-world model is expressible as a Krom formula as well.

Lemma 2. *Intuitionistic entailment in $\mathcal{I}_b$ can be log-space reduced to 2SAT.*

Proof. First of all, it should be obvious that we can ignore the third, fourth, and fifth formula form in (3). On the left hand side, they are equivalent to sets of unit clauses. On the right hand side, the entailment of a conjunction can be reduced to the entailment of each conjunct.

As a first step, we construct the generic structure of a model that provides a counter-example to the validity of the formula

$$\varphi_1 \wedge \ldots \wedge \varphi_n \Rightarrow \psi, \tag{12}$$

where $\varphi_i, \psi \in \mathcal{I}_b$. This formula is not forced in a world u iff there exists a world v with $u \preceq v$ such that the left hand side of the arrow is forced and the right hand side is not forced, i.e.,

$$\exists v \succeq u\colon\ v \not\Vdash \psi,\ v \Vdash \varphi_1 \wedge \ldots \wedge \varphi_n. \tag{13}$$

For the purpose of constructing a counter-example model, we can here and in the following safely assume that existentially quantified worlds are different from all other already introduced worlds.[6]

As a second step, we consider all the forms ψ can have and continue constructing the counter-example model:

1. $\psi = \sim x$. This formula is not forced by v iff
$$\exists w \succeq v\colon\ w \Vdash x. \tag{14}$$
2. $\psi = \sim x \vee \sim y$. This formula is not forced by v iff
$$\exists w', w'' \succeq v\colon\ w' \Vdash x,\ w'' \Vdash y. \tag{15}$$
For the purpose of constructing a counter-example, we can assume without loss of generality that w' and w'' are incomparable with respect to $\preceq$.

[5] We make no assumption about whether the two literals are different!

[6] In fact, here it would also be safe to assume that $u = v$.

3. $\psi = \sim(x \wedge y)$. This formula is not forced by v iff

$$\exists w \succeq v\colon\ w \Vdash x,\ w \Vdash y. \tag{16}$$

4. $\psi = \sim x \vee y$. This formula is not forced by v iff

$$\exists w \succeq v\colon\ v \not\Vdash y,\ w \Vdash x. \tag{17}$$

5. $\psi = x \Rightarrow y$. This formula is not forced iff

$$\exists w \succeq v\colon\ w \Vdash x,\ w \not\Vdash y. \tag{18}$$

6. $\psi = x$. This formula is not forced by v iff

$$v \not\Vdash x. \tag{19}$$

7. $\psi = x \vee y$. This formula is not forced by v iff

$$v \not\Vdash x,\ v \not\Vdash y. \tag{20}$$

This means that in all cases we have at most four possible worlds and the forcing of propositional atoms is deterministic. For the purpose of specifying a reduction, we will assume that we always have w' and w'' as different worlds.

As the third step, we add the forcing conditions implied by $\varphi_1 \wedge \ldots \wedge \varphi_n$. All of the conjuncts of this formula must be forced by v:

1. $\varphi_i = \sim x$. This formula is forced by v iff x is not forced in all worlds larger or equal to v:

$$\forall w \succeq v\colon\ w \not\Vdash x. \tag{21}$$

2. $\varphi_i = \sim x \vee \sim y$. This formula is forced by v iff $\sim x$ or $\sim y$ is forced by v, i.e., if x is not forced by all worlds larger or equal to v or if y is not forced by these worlds:

$$\forall w \succeq v\colon\ w \not\Vdash x \quad \text{or} \quad \forall w \succeq v\colon\ w \not\Vdash y. \tag{22}$$

   Note that in our counter-example model this can be expressed by saying that either x or y is not forced by all maximal worlds (of which we have maximally two).

3. $\varphi_i = \sim(x \wedge y)$. This formula is forced by v iff in all worlds w larger or equal to v x or y is not forced:

$$\forall w \succeq v\colon\ (w \not\Vdash x \ \text{or}\ w \not\Vdash y). \tag{23}$$

4. $\varphi_i = \sim x \vee y$. This formula is forced by v iff y is forced by v or for all worlds larger or equal x is not forced:

$$v \Vdash y \quad \text{or} \quad \forall w \succeq v\colon\ w \not\Vdash x. \tag{24}$$

   Note that in our counter-example model this could be equivalently expressed by requiring that y is forced by v or x is not forced in all maximal worlds.

5. $\varphi_i = x \Rightarrow y$. This formula is forced iff for all worlds larger and equal to v, y is forced if x is forced

$$\forall w \succeq v\colon\ (w \Vdash x \text{ implies } w \Vdash y). \tag{25}$$

6. $\varphi_i = x$. This formula is forced by v iff x forced by v (and therefore in all larger worlds):

$$v \Vdash x. \tag{26}$$

7. $\varphi_i = x \vee y$. This formula is forced by v iff x or y are forced by v:

$$v \Vdash x \quad \text{or} \quad v \Vdash y. \tag{27}$$

We now consider whether it is possible to construct a model that has the structure and forcing relations as specified for ψ satisfying at the same time the forcing requirements for all the φ_i's. These requirements can be obviously met if all forcing and non-forcing requirements for all propositional atoms can be satisfied.

In order to solve this problem, we translate the forcing requirements to classical propositional logic. If x must be forced by w, we use the formula consisting of the positive literal x_w to describe this. Similarly, the requirement that $w \nVdash x$ is translated to $\neg x_w$.

First of all, we notice that all forcing requirements generated by ψ result in unit clauses. Secondly, we note that requirement (5) on intuitionistic possible-world models can be expressed by binary clauses. For each atom x we add $\neg x_v \vee x_w$ for all pairs of worlds worlds with $v \preceq w$. Thirdly, we note that the conditions for the φ_i's can be expressed as binary clauses in the worlds, and in case of $\varphi_i = {\sim}x \vee {\sim}y$ or $\varphi_i = {\sim}x \vee y$ as binary clauses between atoms in different worlds. For $\varphi_i = {\sim}x \vee {\sim}y$ and the case that we have two maximal worlds w' and w'', the encoding is $(\neg x_{w'} \wedge \neg x_{w''}) \vee (\neg y_{w'} \wedge \neg y_{w''})$ which can be transformed to Krom. For $\varphi_i = {\sim}x \vee y$, the condition would be $y_v \vee (\neg x_{w'} \wedge \neg x_{w''})$, which can be transformed to Krom as well.

Summarizing, the forcing conditions on atoms in a potential counter-example model can be expressed as a Krom formula. In other words, the question whether a counter-example can be constructed reduces to the question of whether the classical propositional Krom formula is satisfiable.

Since the reduction produces a classical Krom formula that is polynomially bounded in size by the number of propositional atoms in the intuitionistic formula, and hence by the size of the intuitionistic formula, and since the reduction is "local" in the sense that one binary source clause produces a goal formula of constant size, the entire reduction can be carried out in logarithmic space. ■

Because we can reduce entailment in $\mathcal{I}_b$ to 2SAT in logarithmic space, i.e., in polynomial time as well, and 2SAT can be decided in polynomial time, it follows that deciding entailment in $\mathcal{I}_b$ is a polynomial-time problem. In fact, because the reduction can be carried out in log-space and 2SAT is in NC, a stronger result follows.

Theorem 3. *Deciding entailment for the binary-clause fragment of intuitionistic logic is in NC.*

Since the reduction in Bennett's Theorem can be carried out in log-space, the positive complexity result for $\mathcal{I}_b$ is inherited by the spatial reasoning task we have considered.

Theorem 4. *Deciding consistency of a set of spatial formulae is in NC.*

This result can be easily generalized. The only prerequisite for the theorem to hold is that the relations are representable by model and entailment constraints using $\mathcal{I}_b$.

4 Path-Consistency and Global Consistency

Most of the work concerning reasoning with qualitative spatial information seems to focus on how to compute *composition tables* [4, 6, 16, 2]. The implicit assumption seems to be that once we have such a table, constraint propagation can do the rest. While it is true that constraint propagation in the form of the path-consistency algorithm [10] leads to sound conclusions, it is not clear whether the method is complete for topological relations, even if we consider only constraints that are base relations – as we do in this paper.[7] Incompleteness of the path-consistency algorithm would not matter much if the reasoning problem was NP-hard, because in this case the path-consistency algorithm would provide us with a polynomial-time approximation. As we have seen, however, determining consistency for sets of base relations is computationally tractable.

In other words, the path-consistency algorithm should better be complete in our case. Otherwise it would not be a method one should consider for implementing spatial reasoning systems.

As it turns out, however, the path-consistency method is indeed complete. The key observation for proving completeness is that any inconsistency of a set of (classical) binary clauses corresponds to a cycle passing x and $\neg x$ in the graph formed by the implications corresponding to the clauses. Further, this cycle corresponds to a path through the constraint network. By renaming nodes in the cycle, one can produce a cycle which by assumption should be inconsistent, but the corresponding path in the constraint network must be consistent because of path-consistency.

Theorem 5. *The path-consistency algorithm is refutation complete for constraint networks containing only base relations.*

[7] While one might conjecture that a path-consistent network that contains only base relations (from some relation algebra) is globally consistent, this is unfortunately not true. Robin Hirsh gave me a counter-example (which he attributed to Roger Maddux) invalidating this conjecture.

Proof Sketch. A constraint network containing only base relations corresponds to a set Θ of spatial formulae constraining the relations between all regions to base relations. The path-consistency algorithm ensures that for all paths starting at x and returning to x an instantiation of the variables can be found such that all the constraints (relations) between two regions are satisfied, where for multiple occurrences of the same variable y on the path different instantiations are allowed [10, 11]. If this is not possible, the path-consistency algorithm signals an inconsistency.

We will assume that the path-consistency algorithm has not signalled an inconsistency on Θ, although Θ is inconsistent. The latter implies that one of the entailments in Bennett's Theorem hold, i.e., the corresponding Krom formula constructed in Lemma 2 is unsatisfiable.

A Krom formula is unsatisfiable iff a particular graph corresponding to the formula has a cycle that contains the propositional atom x and its negation [14, Theorem 9.1]. This cycle corresponds to path through the constraint network starting and ending at a node x. Now, multiple occurrences of literals corresponding to one node in the constraint-network can be renamed such that the Krom-graph still contains a cycle, but the path through the constraint network has all multiple occurring nodes renamed to different "copies" of this node. Because we assumed path-consistency, this path with different copies must be consistent, which is a contradiction. Hence, a path-consistent constraint network that contains only base relations must be globally consistent. ■

As in the previous section, this result can be straightforwardly generalized to all relations that can be represented by $\mathcal{I}_b$-constraints.

5 Discussion and Outlook

Representation of qualitative spatial knowledge and reasoning with it has become a very lively research topic [5]. However, computational complexity issues of the reasoning problems have been ignored. In most cases, constraint propagation techniques have been proposed (e.g., [4, 6]), but it remained unclear what the coverage of these techniques is.

In this paper, we made first steps to analyze the computational complexity of the underlying reasoning problems and to validate the use of constraint propagation techniques. Using Bennett's Theorem [2] that relates reasoning about topological relations in the RCC framework with intuitionistic logic, we showed that reasoning with the base relations is a polynomial-time problem. As a side-effect, a particular fragment of propositional intuitionistic logic was shown to be tractable. Based on this proof, we showed that the path-consistency algorithm is in fact refutation-complete for constraint networks that contain only base relations.

While these results answer some questions concerning the computational properties of qualitative spatial reasoning, they also raise a number of other questions:

- How far can we generalize Theorem 4 and Theorem 5 to other than the base relations? We already pointed out some obvious generalizations, but these have to be made more concrete and there may be other possible ways to go.
- Is it possible to strengthen Theorem 5 in the direction such that path-consistency implies minimality of the constraint network? While for networks of base relations, this is trivially true, it becomes an issue if disjunctive relations are permitted.
- Is reasoning with arbitrary disjunctive topological relations NP-hard?
- What are the computational properties of calculi extending the topological relation system [17, 6]? Bennett's [3] encoding of most of the RCC relations in modal logic may be a good starting point for applying the techniques used in our paper in order to get an answer for the RCC relations.

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