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# On the Computational Complexity of Temporal Projection and Plan Validation*

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## Abstract

One kind of temporal reasoning is *temporal projection*—the computation of the consequences of a set of events. This problem is related to a number of other temporal reasoning tasks such as *story understanding*, *planning*, and *plan validation*. We show that one particular simple case of temporal projection on partially ordered events turns out to be harder than previously conjectured. However, given the restrictions of this problem, story understanding, planning, and plan validation appear to be easy. In fact, we show that plan validation, one of the intended applications of temporal projection, is tractable for an even larger class of plans.

## Introduction

The problem of *temporal projection* is to compute the consequences of a set of events. Dean and Boddy [1988] analyze this problem for sets of partially ordered events assuming a propositional STRIPS-like [Fikes and Nilsson, 1971] representation of events. They investigate the computational complexity of a number of restricted problems and conclude that even for severely restricted cases the problem is NP-hard, which motivate them to develop a tractable and sound but incomplete decision procedure for the temporal projection problem.

Among the restricted problems they analyze, there is one they conjecture to be solvable in polynomial time. As it turns out, however, even in this case temporal projection is NP-hard, as is shown in this paper. The result is somewhat surprising, because *planning*, *plan validation*, and *story understanding* seem to be easily solvable given the restriction of this temporal projection problem.

This observation casts some doubts on whether temporal projection is indeed the problem underlying plan validation and story understanding, as suggested by Dean and Boddy [1988]. It seems natural to assume that the *validation of plans* is not harder than planning. Our NP-hardness result for the simple temporal projection problem seems to suggest the contrary, though.

One of the most problematical points in the definition of the temporal projection problem by Dean and Boddy seems to be that event sequences are permitted to contain events that do not affect the world because their preconditions are not satisfied. If we define the plan validation problem in a way such that all possible event sequences have to contain only events that affect the world, plan validation is tractable for the class of plans containing only unconditional events, a point already suggested by Chapman [1987]. In fact, deciding a conjunction of temporal projection problems that is equivalent to the plan validation problem appears to be easier than deciding each conjunct in isolation.

## Temporal Projection

Given a description of the state of the world and a description of which events will occur, we are usually able to predict what the world will look like. This kind of reasoning is called *temporal projection*. It seems to be the easiest and most basic kind of temporal reasoning. Depending on the representation, however, there are subtle difficulties hidden in this reasoning task.

The formalization of the temporal projection problem for partially ordered events given below closely follows the presentation by Dean and Boddy [1988, Sect. 2].

**Definition 1** *A* **causal structure** *is given by a tuple* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$*, where*

- $\mathcal{P} = \{p_1, \ldots, p_n\}$ *is a set of propositional atoms, the* **conditions**,
- $\mathcal{E} = \{\epsilon_1, \ldots, \epsilon_m\}$ *is a set of* **event types**,
- $\mathcal{R} = \{r_1, \ldots, r_o\}$ *is a set of* **causal rules** *of the form* $r_i = \langle \epsilon_i, \varphi_i, \alpha_i, \delta_i \rangle$*, where*
  - $\epsilon_i \in \mathcal{E}$ *is the* **triggering event type**,

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- $\varphi_i \subseteq \mathcal{P}$ *is a set of* **preconditions**,
- $\alpha_i \subseteq \mathcal{P}$ *is the* **add list**,
- *and* $\delta_i \subseteq \mathcal{P}$ *is the* **delete list**.

In order to give an example, assume a toy scenario with a hall, a room $A$, and another room $B$. Room $A$ contains a public phone, and room $B$ contains an electric outlet. The robot Robby can be in the hall (denoted by the atom h), in room $A$ (a), or in room $B$ (b). Robby can have a phone card (p) or coins (c). Additionally, when Robby uses the phone, he can inform his master on the phone that everything is in order (i). Robby can be fully charged (f), almost empty (e), or, in unlucky circumstances, his batteries can be damaged (d). Summarizing, the set of conditions for our tiny causal structure is the following:

$$\mathcal{P} = \{\mathtt{a}, \mathtt{b}, \mathtt{h}, \mathtt{p}, \mathtt{c}, \mathtt{i}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}.$$

Robby can do the following. He can move from the hall to either room ($\epsilon_{h \rightarrow a}$, $\epsilon_{h \rightarrow b}$) and *vice versa* ($\epsilon_{a \rightarrow h}$, $\epsilon_{b \rightarrow h}$). Provided he is in room a and he has a phone card or coins, he can call his master ($\epsilon_{call}$). Additionally, if Robby is in room b, he can recharge himself ($\epsilon_{charge}$). However, if Robby is already fully charged, this results in damaging his batteries. Summarizing, we have the following set of event types:

$$\mathcal{E} = \{\epsilon_{h \rightarrow a}, \epsilon_{h \rightarrow b}, \epsilon_{a \rightarrow h}, \epsilon_{b \rightarrow h}, \epsilon_{call}, \epsilon_{charge}\},$$

and the following set of causal rules:

$$\begin{array}{rllll} \mathcal{R} = \{ & \langle \epsilon_{h \rightarrow a}, & \{\mathtt{h}\}, & \{\mathtt{a}\}, & \{\mathtt{h}\}\rangle, \\ & \langle \epsilon_{h \rightarrow b}, & \{\mathtt{h}\}, & \{\mathtt{b}\}, & \{\mathtt{h}\}\rangle, \\ & \langle \epsilon_{a \rightarrow h}, & \{\mathtt{a}\}, & \{\mathtt{h}\}, & \{\mathtt{a}\}\rangle, \\ & \langle \epsilon_{b \rightarrow h}, & \{\mathtt{b}\}, & \{\mathtt{h}\}, & \{\mathtt{b}\}\rangle, \\ & \langle \epsilon_{call}, & \{\mathtt{a}, \mathtt{p}\}, & \{\mathtt{i}\}, & \emptyset\rangle, \\ & \langle \epsilon_{call}, & \{\mathtt{a}, \mathtt{c}\}, & \{\mathtt{i}\}, & \{\mathtt{c}\}\rangle, \\ & \langle \epsilon_{charge}, & \{\mathtt{b}, \mathtt{e}\}, & \{\mathtt{f}\}, & \{\mathtt{e}\}\rangle, \\ & \langle \epsilon_{charge}, & \{\mathtt{b}, \mathtt{f}\}, & \{\mathtt{d}\}, & \{\mathtt{f}\}\rangle\}. \end{array}$$

In order to talk about sets of concrete events and temporal constraints over them, the notion of a *partially ordered event set* is introduced.[1]

**Definition 2** *Assuming a causal structure* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R}\rangle$, *a* **partially ordered event set** *(POE) over* $\Phi$ *is a pair* $\Delta_\Phi = \langle \mathcal{A}_\Phi, \prec\rangle$ *consisting of a set of* **actual events** $\mathcal{A}_\Phi = \{e_1, \ldots, e_p\}$ *such that* $type(e_i) \in \mathcal{E}$, *and a strict partial order*[2] $\prec$ *over* $\mathcal{A}_\Phi$.

Continuing our example, we assume a set of six actual events $\mathcal{A} = \{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}\}$, such that

$$\begin{array}{rcl} type(\mathtt{A}) & = & \epsilon_{h \rightarrow a} \\ type(\mathtt{B}) & = & \epsilon_{call} \\ type(\mathtt{C}) & = & \epsilon_{a \rightarrow h} \\ type(\mathtt{D}) & = & \epsilon_{h \rightarrow b} \\ type(\mathtt{E}) & = & \epsilon_{charge} \\ type(\mathtt{F}) & = & \epsilon_{b \rightarrow h}, \end{array}$$

and

$$\begin{array}{ccccc} \mathtt{A} & \prec & \mathtt{B} & \prec & \mathtt{C} \\ \mathtt{D} & \prec & \mathtt{E} & \prec & \mathtt{F}. \end{array}$$

POEs denote sets of possible sequences of events satisfying the partial order. A **partial event sequence** of length $m$ over such a POE $\langle \mathcal{A}, \prec\rangle$ is a sequence $\mathbf{f} = \langle f_1, \ldots, f_m\rangle$ such that (1) $\{f_1, \ldots, f_m\} \subseteq \mathcal{A}$, (2) $f_i \neq f_j$ if $i \neq j$, and (3) for each pair $f_i, f_j$ of events appearing in $\mathbf{f}$, if $f_i \prec f_j$ then $i < j$. For instance, $\langle \mathtt{A}, \mathtt{B}, \mathtt{C}\rangle$ is a partial event sequence of length three over the POE given above, while $\langle \mathtt{A}, \mathtt{C}, \mathtt{B}\rangle$ is not. If the event sequence is of length $|\mathcal{A}|$, it is called a **complete event sequence** over the POE. The sequences $\langle \mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}\rangle$ and $\langle \mathtt{A}, \mathtt{D}, \mathtt{B}, \mathtt{E}, \mathtt{C}, \mathtt{F}\rangle$ are complete event sequences, for instance. The set of all complete event sequences over a POE $\Delta$ is denoted by $CS(\Delta)$.

If $\mathbf{f} = \langle f_1, \ldots, f_k, \ldots, f_m\rangle$ is an event sequence, then $\langle f_1, \ldots, f_k\rangle$ is the initial sequence of $\mathbf{f}$ up to $f_k$, written $\mathbf{f}/f_k$. Similarly, $\mathbf{f}\backslash f_k$ denotes the initial sequence $\langle f_1, \ldots, f_{k-1}\rangle$ consisting of all events before $f_k$. Further, we write $\mathbf{f}; g$ to denote $\langle f_1, \ldots, f_m, g\rangle$.

Each event maps states (subsets of $\mathcal{P}$) to states. Let $S \subseteq \mathcal{P}$ denote a state and let $e$ be an event. Then we say that the causal rule $r$ is **applicable** in state $S$ iff $r = \langle type(e), \varphi, \alpha, \delta\rangle$ and $\varphi \subseteq S$. Given $e$ and $S$, $app(S, e)$ denotes the set of all **applicable rules** for $e$ in state $S$. An event $e$ is said to **affect** the world in a state $S$ iff $app(S, e) \neq \emptyset$. In order to simplify notation, we write $\varphi(r)$, $\alpha(r)$, $\delta(r)$ to denote the sets $\varphi$, $\alpha$, and $\delta$, respectively, appearing in the rule $r = \langle \epsilon, \varphi, \alpha, \delta\rangle$. If there is only one causal rule associated with the event type $type(e)$, we will also use the notation $\varphi(e)$, $\alpha(e)$, and $\delta(e)$. Based on this notation, we define what we mean by the *result* of a sequence of events relative to a state $S$.

**Definition 3** *The function "Res" from states and event sequences to states is defined recursively by:*[3]

$$\begin{array}{rcl} Res\Big(S, \langle\rangle\Big) & = & S \\ Res\Big(S, (\mathbf{f}; g)\Big) & = & Res(S, \mathbf{f}) - \\ & & \{\delta(r) |\, r \in app(Res(S, \mathbf{f}), g)\} \cup \\ & & \{\alpha(r) |\, r \in app(Res(S, \mathbf{f}), g)\}. \end{array}$$

It is easy to verify that the following equation holds for our example scenario:

$$Res(\{\mathtt{h}, \mathtt{e}, \mathtt{c}\}, \langle \mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}\rangle) = \{\mathtt{h}, \mathtt{f}, \mathtt{i}\}.$$

The definition of the function *Res* permits sequences of events where events occur that do not affect the world. For instance, it is possible to ask what the result of $\langle \mathtt{A}, \mathtt{D}, \mathtt{B}, \mathtt{E}, \mathtt{C}, \mathtt{F}\rangle$ in state $\{\mathtt{h}, \mathtt{e}, \mathtt{c}\}$ will be:

$$Res(\{\mathtt{h}, \mathtt{e}, \mathtt{c}\}, \langle \mathtt{A}, \mathtt{D}, \mathtt{B}, \mathtt{E}, \mathtt{C}, \mathtt{F}\rangle) = \{\mathtt{h}, \mathtt{e}, \mathtt{i}\}.$$

[1] This notion is similar to the notion of a *nonlinear plan*.

[2] A strict partial order is a transitive and irreflexive relation.

[3] Note that it can happen that two rules are applicable in a state, one adding and one deleting the same atom $p$. In this case, we follow [Dean and Boddy, 1988] and assume that $p$ holds after the event as reflected by the definition of *Res*.

Although perfectly well-defined, this result seems to be strange because the events D, E, and F occurred without having any effect on the state of the world.

Given a state $S$, we will often restrict our attention to event sequences such that all events affect the world. These sequences are called **admissible event sequences** relative to the state $S$. The set of all complete event sequences over $\Delta$ that are admissible relative to $S$ are denoted by $ACS(\Delta, S)$.

In the following, we will often talk about which consequences a POE will have on some initial state. For this purpose, the notion of an *event system* is introduced.

**Definition 4** *An* **event system** $\Theta$ *is a pair* $\langle \Delta_\Phi, \mathcal{I} \rangle$, *where* $\Delta_\Phi$ *is a POE over the causal structure* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$, *and* $\mathcal{I} \subseteq \mathcal{P}$ *is the* **initial state**.

In order to simplify notation, the functions $CS$ and $ACS$ are extended to event systems with the obvious meaning, i.e., $CS(\langle \Delta, S \rangle) = CS(\Delta)$ and $ACS(\langle \Delta, S \rangle) = ACS(\Delta, S)$. Further, if $CS(\Theta) = ACS(\Theta)$, $\Theta$ is called **coherent.**

The problem of temporal projection as formulated by Dean and Boddy [1988] is to determine whether some condition holds, *possibly* or *necessarily*, after a particular event of an event system.

**Definition 5** *Given an event system* $\Theta$, *an event* $e \in \mathcal{A}$, *and a condition* $p \in \mathcal{P}$:

$$p \in Poss(e, \Theta) \quad \text{iff} \quad \exists \mathbf{f} \in CS(\Theta)\colon p \in Res(\mathcal{I}, \mathbf{f}/e)$$
$$p \in Nec(e, \Theta) \quad \text{iff} \quad \forall \mathbf{f} \in CS(\Theta)\colon p \in Res(\mathcal{I}, \mathbf{f}/e).$$

Continuing our example, let us assume the initial state $\mathcal{I} = \{h, e, c\}$. Then the following can be easily verified:

$$\begin{array}{lll} \mathtt{i} \in Poss(\mathtt{B}, \Theta) & \qquad & \mathtt{i} \notin Nec(\mathtt{B}, \Theta) \\ \mathtt{d} \notin Poss(\mathtt{E}, \Theta) & \qquad & \mathtt{d} \notin Nec(\mathtt{E}, \Theta). \end{array}$$

In plain words, Robby is only possibly but not necessarily successful in calling his master. On the positive side, however, we know that Robby's batteries will not be damaged, regardless of in which order the events happen.

Given a set of conditions $S$ and a sequence $\mathbf{f}$, $Res(S, \mathbf{f})$ can easily be computed in polynomial time. Since the set $CS(\Theta)$ may contain exponentially many sequences, however, it is not obvious whether $p \in Poss(e, \Theta)$ and $p \in Nec(e, \Theta)$ can be decided in polynomial time.

## A "Simple" Temporal Projection Problem

In the general case, temporal projection is quite difficult. Dean and Boddy [1988] show that the decision problems $p \in Poss(e, \Theta)$ and $p \in Nec(e, \Theta)$ are NP-complete and co-NP-complete, respectively, even under some severe restrictions, such as restricting $\alpha$ or $\delta$ to be empty for all rules, or requiring that there is only one causal rule associated with each event type.

**Definition 6** *An event system is called* **unconditional** *iff for each* $\epsilon \in \mathcal{E}$, *there exists only one causal rule with the triggering event type* $\epsilon$. *An event system is called* **simple** *iff it is unconditional,* $\mathcal{I}$ *is a singleton, and for each causal rule* $r = \langle \epsilon, \varphi, \alpha, \delta \rangle$, *the sets* $\varphi$, $\alpha$, *and* $\delta$ *are singletons and* $\varphi = \delta$.

Dean and Boddy conjecture that temporal projection is a polynomial-time problem for simple event systems [Dean and Boddy, 1988, p. 379]. As it turns out, however, also this problem is computationally difficult.

**Theorem 1** *For simple event systems* $\Theta$, *deciding* $p \in Poss(e, \Theta)$ *is* NP-*complete and deciding* $p \in Nec(e, \Theta)$ *is co*-NP-*complete.*

**Proof Sketch.** First we show NP-completeness of $p \in Poss(e, \Theta)$.

Membership in NP is obvious. Guess an event sequence $\mathbf{f}$ and verify in polynomial time that $\mathbf{f} \in CS(\Theta)$ and $p \in Res(\mathcal{I}, \mathbf{f}/e)$.

In order to prove NP-hardness, we give a polynomial transformation from **path with forbidden pairs** (PWFP) to the temporal projection problem. The former problem is defined as follows:

> Given a directed graph $G = (V, A)$, two vertices $s, t \in V$, and a collection $C = \Big\{\{a_1, b_1\}, \ldots, \{a_n, b_n\}\Big\}$ of pairs of arcs from $A$, is there a directed path from $s$ to $t$ in $G$ that contains at most one arc from each pair in $C$?

This problem is NP-complete, even if the graph is acyclic and all pairs are disjoint [Garey and Johnson, 1979, p. 203].

We now construct an instance of the simple temporal projection problem from a given instance of the PWFP problem, assuming that the graph is acyclic and the forbidden pairs are all disjoint. Let $G = (V, A)$ be a DAG, where $V = \{v_1, \ldots, v_k\}$, and let $C$ be a collection of "forbidden pairs" of arcs from $A$. Further, let $s$ and $t$ be two vertices from $V$ and assume without loss of generality that there is no arc $(t, v_i) \in A$. Then define

$$\begin{array}{rcl} \mathcal{P} & = & \{v_1, \ldots, v_k\} \cup \{*\} \\ \mathcal{E} & = & \{\epsilon_{i,j} \,|\, (v_i, v_j) \in A\} \cup \{\epsilon_*\} \\ \mathcal{R} & = & \{\langle \epsilon_{i,j}, \{v_i\}, \{v_j\}, \{v_i\} \rangle \,|\, (v_i, v_j) \in A\} \cup \\ & & \{\langle \epsilon_*, \{*\}, \{*\}, \{*\} \rangle\} \\ \mathcal{A} & = & \{e_{i,j} \,|\, \epsilon_{i,j} \in \mathcal{E}\} \cup \{e_*\} \\ type(e_{i,j}) & = & \epsilon_{i,j} \text{ for all } e_{i,j} \in \mathcal{A} - \{e_*\} \\ type(e_*) & = & \epsilon_* \\ e & \prec & e_* \text{ for all } e \in \mathcal{A} - \{e_*\} \\ e_{k,l} & \prec & e_{i,j} \text{ iff } \{(v_i, v_j), (v_k, v_l)\} \in C \text{ and} \\ & & \qquad \text{there is a path from } v_j \text{ to } v_k \\ \mathcal{I} & = & \{s\}. \end{array}$$

Note that $\Theta$ can be constructed in polynomial time and that $\Theta$ is a simple event system. Further note that since the forbidden pairs are pairwise disjoint, there is no set of events $\{f_1, f_2, f_3\} \subseteq \mathcal{A} - \{e_*\}$ such that $f_1 \prec f_2 \prec f_3$.

It is now easy to verify that there is a path from $s$ to $t$ in $G$ that contains at most one arc from each pair in $C$ if, and only if, $t \in Poss(e_*, \Theta)$.

The co-NP-hardness result for the second problem follows by a slight modification of the above transformation. Membership in co-NP is again obvious.[4] ■

This result is somewhat surprising because one might suspect that story understanding and planning are easy under the restrictions imposed on the structure of event systems. In fact, a highly abstract form of story understanding is a polynomial-time problem under these restriction [Nebel and Bäckström, 1991; Bäckström and Nebel, 1992]. Also planning is an easy problem in this context. Planning can usually be transformed to the problem of finding a shortest path in a graph, which is a polynomial time problem. In the general case, the size of the graph is exponential in the size of the problem, but it turns out that the simple problem corresponds to a linearly sized graph. Hence, the problem can be solved in polynomial time. Similar tractability results have been obtained by Bylander [1991], Erol et al [1991] and Bäckström and Klein [1991] for more complicated planning problems. Some relations between these results and the complexity results for temporal projection are discussed in the full paper [Nebel and Bäckström, 1991].

One reason for analyzing the temporal projection problem is that it seems to constitute the heart of plan validation. If we now consider the restrictions placed on the simple temporal projection problem, we have already noted that planning itself—a problem one would expect to be harder than validation—is quite easy. One explanation for this apparent paradoxical situation could be that a planner could create the complicated structure we used in the proof of Theorem 1, but it never would do so. Hence, the theoretical complexity never shows up in reality. This explanation is unsatisfying, however. If this would be really the case, we should be able to characterize the structure of the nonlinear plans planning systems create and validate. The real reason is more subtle, as will be shown below.

## Temporal Projection and Plan Validation

Dean and Boddy [1988, p. 378] suggest that temporal projection is the basic underlying problem in plan validation:

> A nonlinear plan is represented as a set of actions $\{e_1, \ldots, e_n\}$ partially ordered by $\prec$. Each action has some set of *intended effects:* Intended$(e_i) \subseteq \mathcal{P}$. A nonlinear plan is said to be *valid* just in case Intended$(e_i) \subseteq$ Necessary$(e_i)$, for $1 \leq i \leq n$.

[4] Complete proofs can be found in the full paper [Nebel and Bäckström, 1991].

Although this definition sounds reasonable, there are some points which are arguable. We use a slightly different definition of plan validation in the following.

**Definition 7** *A POE $\Delta_\Phi$ over a causal structure $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$ is called a* **valid nonlinear plan** *with respect to an initial state $\mathcal{I} \subseteq \mathcal{P}$ and a goal state $\mathcal{G} \subseteq \mathcal{P}$ iff $\Delta_\Phi$ achieves its goal, i.e., $\mathcal{G} \subseteq Res(\mathcal{I}, \mathbf{f})$ for all $\mathbf{f} \in CS(\Delta_\Phi)$, and $\langle \Delta_\Phi, \mathcal{I} \rangle$ is coherent.*

Note that our definition coincides with Chapman's [1987, p. 340] definition of when a plan *solves* a problem. In contrast to Dean and Boddy's formulation, our definition does not refer to the *intended effects of particular events* but to the effects of the *overall plan* and to the state *before* particular events. Further note that plan validation can be reduced to deciding coherence of an event system in linear time. If $\Delta_\Phi$ is a POE and $\mathcal{G}$ is the goal state, $\Delta_\Phi^\mathcal{G}$ shall denote the POE $\Delta_\Phi$ extended by an event $e_*$ such that $e_*$ has to occur last and there is exactly one causal rule associated with $e_*$ such that $\varphi(e_*) = \mathcal{G}$.

**Proposition 2** *A POE $\Delta_\Phi$ is a valid nonlinear plan with respect to $\mathcal{I}$ and $\mathcal{G}$ iff $\langle \Delta_\Phi^\mathcal{G}, \mathcal{I} \rangle$ is a coherent event system.*

In what follows, we show that coherence, and, hence, the validity of nonlinear plans, can be decided in polynomial time, provided the event system is *unconditional.* Although the restriction may sound severe, it shows that plan validation is tractable for a considerably larger class of plans than temporal projection. In the full paper [Nebel and Bäckström, 1991] we argue that the restriction to unconditional actions is not very severe given the formalism used in this paper.

First of all, we note that coherence cannot be easily reduced to temporal projection as defined by Dean and Boddy since coherence refers to the state *before* an event occurs. For this reason, we define a variant of the temporal projection problem.

**Definition 8** *Given an event system $\Theta$, an event $e \in \mathcal{A}$, and a condition $p \in \mathcal{P}$:*

$$p \in Poss_b(e, \Theta) \quad \text{iff} \quad \exists \mathbf{f} \in CS(\Theta)\colon p \in Res(\mathcal{I}, \mathbf{f} \backslash e)$$
$$p \in Nec_b(e, \Theta) \quad \text{iff} \quad \forall \mathbf{f} \in CS(\Theta)\colon p \in Res(\mathcal{I}, \mathbf{f} \backslash e).$$

Deciding $p \in Nec_b(e, \Theta)$ instead of $p \in Nec(e, \Theta)$ does not simplify anything. All the NP-hardness proofs for *Nec* can be easily used to show NP-hardness for $Nec_b$. Nevertheless, using this variant of temporal projection we can decide coherence for unconditional event systems.

**Proposition 3** *An unconditional event system $\Theta$ is coherent iff*

$$\forall e \in \mathcal{A}\colon \varphi(e) \subseteq Nec_b(e, \Theta).$$

In order to simplify the following discussion, we will restrict ourselves to **consistent** unconditional event systems, which have to meet the restrictions that

$\alpha(e) \cap \delta(e) = \emptyset$, for all $e \in \mathcal{A}$. Note that any unconditional event system $\Theta$ can be transformed into an equivalent consistent unconditional event system $\Theta'$ in linear time by replacing $\delta(e)$ with $\delta(e) - \alpha(e)$ for all $e \in \mathcal{A}$.

As a first step to specifying a polynomial-time algorithm that decides coherence for unconditional event systems, we define a simple syntactic criterion, written $Maybe_b(e, \Theta)$, that approximates $Nec_b(e, \Theta)$.

**Definition 9** *Given a consistent unconditional event system $\Theta$, an atom $p \in \mathcal{P}$, and an event $e \in \mathcal{A}$, $Maybe_b(e, \Theta)$ is defined as follows:*

$$p \in Maybe_b(e, \Theta)$$
$$\textit{iff}$$
$$(1)\ p \in \mathcal{I} \vee \exists e' \in \mathcal{A}: [e' \prec e \wedge p \in \alpha(e')] \wedge$$
$$(2)\ \neg\exists e' \in \mathcal{A} - \{e\}: [e' \not\prec e \wedge e \not\prec e' \wedge p \in \delta(e')] \wedge$$
$$(3)\ \forall e' \in \mathcal{A}: [(e' \prec e \wedge p \in \delta(e')) \rightarrow \exists e'' \in \mathcal{A}: (e' \prec e'' \prec e \wedge p \in \alpha(e''))].$$

This definition resembles Chapman's [1987] *modal truth criterion*. The first condition states that $p$ has to be established before $e$. The second condition makes sure that there is no event unordered w.r.t. $e$ that could delete $p$, and the third condition enforces that for all events that could delete $p$ and that occur before $e$, some other event will reestablish $p$. It is obvious that this criterion can be checked in polynomial time.

$Maybe_b$ is neither sound nor complete w.r.t. $Nec_b$ in the general case because we do not know whether the events referred to in the definition actually affect the world. However, $Maybe_b$ coincides with $Nec_b$ in the important special case that the event system is coherent.

**Lemma 4** *Let $\Theta$ be an consistent unconditional event system. If $\Theta$ is coherent, then*

$$\forall e \in \mathcal{A}: Nec_b(e, \Theta) = Maybe_b(e, \Theta).$$

**Proof Sketch.** "$\subseteq$": Suppose that the first condition does not hold for some event $e$ and atom $p \in Nec_b(e, \Theta)$. Since $\Theta$ is coherent, we can construct an admissible complete event sequence $\mathbf{f} = \langle f_1, \ldots, e, \ldots \rangle$ such that $\mathbf{g} = \mathbf{f} \backslash e$ contains only events $g_i$ such that $g_i \prec e$. By induction over the length of $\mathbf{f} \backslash e$, we get $p \notin Res(\mathcal{I}, \mathbf{f} \backslash e)$, which is a contradiction.

Suppose that the second condition does not hold, i.e., there exists an event $e'$ unordered with respect to $e$ such that $p \in \delta(e')$. Then there exists a complete event sequence $\mathbf{f} = \langle f_1, \ldots, e', e, \ldots \rangle$. Since $\Theta$ is coherent, and thus $e'$ affects the world, it is obvious that $p \notin Res(\mathcal{I}, \mathbf{f}/e') = Res(\mathcal{I}, \mathbf{f} \backslash e)$, which is a contradiction.

Suppose the third condition is not satisfied, i.e., there exists $p \in Nec_b(e, \Theta)$ and an event $e' \prec e$ such that $p \in \delta(e')$, but there is no $e''$ such that $e' \prec e'' \prec e$ and $p \in \alpha(e'')$. Consider a complete event sequence $\mathbf{f}$ such that there are only events $e_i$ between $e'$ and $e$ that have to occur between them. Because $p \notin Res(\mathcal{I}, \mathbf{f}/e')$ and because by assumption $p \notin \alpha(e_i)$ for all events $e_i$ occurring between $e'$ and $e$, we can infer $p \notin Res(\mathcal{I}, \mathbf{f} \backslash e) \supseteq Nec_b(e, \Theta)$, which is again a contradiction.

"$\supseteq$": Assume $p \in Maybe_b(e, \Theta)$. Consider any complete event sequence $\mathbf{g} \in CS(\Theta)$. We want to show that $p \in Res(\mathcal{I}, \mathbf{g} \backslash e)$. By condition (1) of the definition of $Maybe_b$ and the fact that all complete event sequences are admissible, we know that there exists $g_i \in \mathcal{A}$ such that $|\mathbf{g} \backslash g_i| \leq |\mathbf{g} \backslash e|$ and $p \in Res(\mathcal{I}, \mathbf{g} \backslash g_i)$. Consider the latest such event, i.e., $g_i$ with a maximal $i$. Since all event sequences are finite, such an event must exist. If $g_i = e$, we are ready. Otherwise, because of conditions (2) and (3), $i$ cannot be maximal. ■

Now we can give a necessary and sufficient condition for coherence of consistent unconditional event systems.

**Theorem 5** *A consistent unconditional event system $\Theta$ is coherent iff*

$$\forall e \in \mathcal{A}: \varphi(e) \subseteq Maybe_b(e, \Theta).$$

**Proof Sketch.** "$\Rightarrow$": Follows immediately from Lemma 4.

"$\Leftarrow$": For the converse direction, we use induction on the number of conditions appearing in the preconditions of events over the entire event system: $k = \sum_{e \in \mathcal{A}} |\varphi(e)|$.

For the base step, $k = 0$, the claim holds trivially.

For the induction step assume an event system $\Theta$ with $k+1$ preconditions such that $\varphi(e) \subseteq Maybe_b(e, \Theta)$ for all $e \in \mathcal{A}$. Consider an event system $\Theta'$ that is identical to $\Theta$ except that for one event $f$ such that $\varphi(f) \neq \emptyset$ we set $\varphi'(f) = \emptyset$. Because $k \geq \sum_{e \in \mathcal{A}'} |\varphi'(e)|$, we can apply our induction hypothesis and conclude that $\Theta'$ is coherent. By Lemma 4, we have $\varphi(f) \subseteq Maybe_b(f, \Theta) = Maybe_b(f, \Theta') = Nec_b(f, \Theta')$. Hence, any sequence $\mathbf{g} \in CS(\Theta')$ that contains $f$ is an admissible sequence even if we would have $\varphi'(f) = \varphi(f)$. Since $CS(\Theta) = CS(\Theta')$, it follows that $\Theta$ is coherent. ■

Since plan validation can be reduced to coherence in linear time, it is a polynomial-time problem if the causal structure is unconditional.

**Theorem 6** *Plan validation for unconditional causal structures is a polynomial-time problem.*

**Proof Sketch.** Follows from Proposition 2, from Theorem 5, the fact that any unconditional event structures can be transformed into a consistent one in linear time, and the fact that $Maybe_b$ can be decided in polynomial time. ■

One interesting point to note about this result is that it appears to be easier to decide a big conjunction of the form

$$\bigwedge_{e \in \mathcal{A}} \varphi(e) \subseteq Nec_b(e, \Theta)$$

than to decide one of the conjuncts. In other words, the claim by Dean and Boddy [1988] that temporal projection (in some form) is the underlying problem of plan validation is conceptually correct. However, it turns out that solving the subproblems is harder than solving the original problem (assuming $\mathsf{NP} \neq \mathsf{P}$).

Intuitively, temporal projection is difficult because we cannot avoid to consider all elements of $CS(\Theta)$ as demonstrated in the proof of Theorem 1. Plan validation for unconditional causal structures is easy, on the other hand, since satisfaction of all preconditions can be reduced to a local syntactic property.

Although maybe surprising, the result is not new. Chapman [1987] used a similar technique to prove plan validation to be a polynomial-time problem for a slightly different formalism. It should be noted, however, that Chapman's [1987, p. 368] proof of the correctness and soundness of the *modal truth criterion* is correct only if we make the assumption that the plan is already coherent—a property we want to decide. In fact, it seems to be the case that Chapman missed to prove the second half of our Theorem 5.

## Discussion

Reconsidering the problem of temporal projection for sets of partially ordered events as defined by Dean and Boddy [1988], we noted that one special case conjectured to be tractable turned out to be NP-complete. Although this result does not undermine the arguments of Dean and Boddy [1988] that temporal projection is a quite difficult problem, it leads to a counter-intuitive conclusion, namely, that planning is easier than temporal projection in this special case.

Further, we showed that plan validation, if defined appropriately, is tractable for a more general problem, namely validation of unconditional nonlinear plans. This means that the problem of validating a plan as a whole is easier than validating all its actions separately. In other words, what might look like a *divide and conquer* strategy at a first glance is rather the opposite.

These two observations lead to the question of whether the formalization of temporal projection [Dean and Boddy, 1988] really captures one of the intended applications, namely, validation of nonlinear plans. In particular, one may ask whether the incomplete decision procedure for temporal projection developed by Dean and Boddy [1988] is based on the right assumptions. It turns out that the incomplete decision procedure fails on plans that could be validated in polynomial time using the techniques described above [Nebel and Bäckström, 1991; Bäckström and Nebel, 1992].

As a final remark, it should be noted that the criticisms expressed in this paper are possible only because Dean and Boddy [1988] made their ideas and claims very explicit and formal.


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