IJCAI-2011 Workshop 27

Benchmarks and Applications of Spatial Reasoning

Workshop Proceedings

Barcelona, Spain
July 17, 2011

Workshop Co-Chairs

Jochen Renz (Australian National University - Canberra, AU)
Anthony G. Cohn (University of Leeds, UK)
Stefan Wölfl (University of Freiburg, DE)

Program Committee

Alia Abdelmoty (Cardiff University, UK)
Marco Aiello (University of Groningen, NL)
Mehul Bhatt (University of Bremen, DE)
Jean-Francois Condotta (University of Artois, FR)
Matt Duckham (University of Melbourne, AU)
Antony Galton (University of Exeter, UK)
Jason Jingshi Li (Ecole Polytechnique Federale de Lausanne, CH)
Reinhard Moratz (University of Maine, Orono (ME), US)
Bernhard Nebel (University of Freiburg, DE)
Marco Ragni (University of Freiburg, DE)
Christoph Schlieder (University of Bamberg, DE)
Steven Schockaert (Ghent University, BE)
Nico van de Weghe (Ghent University, BE)
Jan-Oliver Wallgrün (University of Bremen, DE)

Contents

A new perspective on reasoning with qualitative spatial knowledge

Jae Hee Lee and **Diedrich Wolter**
SFB/TR 8 Spatial Cognition
Universität Bremen
P.O. Box 330440 28334 Bremen, Germany

Abstract

In this paper we call for considering a paradigm shift in the reasoning methods that underly qualitative spatial representations. As alternatives to conventional methods we propose exploiting methods from linear programming and real algebraic geometry. We argue that using mathematical theories of the spatial domain at hand might be the key to effective reasoning methods, and thus to practical applications.

1 Introduction

Qualitative spatial knowledge is ubiquitous in natural language. Thus, it is essential in human-computer interaction, which is an integral part of our everyday life where interaction with digital equipments is omnipresent. In the field of artificial intelligence, reasoning with qualitative spatial knowledge has been researched under the umbrella term *Qualitative Spatial Reasoning (QSR)* [Cohn and Renz, 2008]. QSR pursues a relation-algebraic approach that provides universal means to deal with any type of qualitative spatial knowledge (e.g., topology, direction, distance). It has been assumed that the relation-algebraic approach will allow for an efficient, effective, universal reasoning method. Despite its promising properties, however, the relation-algebraic approach suffers from its incompleteness for many representations of qualitative spatial knowledge. Furthermore, it is not capable of generating a model for given constraints, which is a desirable feature for many real-world applications.

In this paper we call for considering a paradigm shift in the reasoning methods that underly qualitative spatial representations. As alternatives to conventional methods we propose exploiting methods from linear programming and real algebraic geometry. We argue that using mathematical theories of the spatial domain at hand might be the key to effective reasoning methods, and thus to practical applications.

2 The Relation-Algebraic Approach and Its Limitations

The building blocks of QSR are a spatial domain $\mathcal{D}$, a finite set $\mathcal{R} = \{R_1, R_2, \ldots, R_n\}$ of binary relations on $\mathcal{D}$ which partitions $\mathcal{D}^2$, and a map $\circ : \mathcal{R} \times \mathcal{R} \to 2^{\mathcal{D}}, R_1 \circ R_2 = \{y \in \mathcal{D} \mid xR_1y \text{ and } yR_2z\}$, which is called the *composition*. A prominent, simple example is the one-dimensional space (e.g., a queue) equipped with the relations *before*, *behind*, *equal* and the usual notion of composition, e.g., if Alice is *behind* Bob and Bob is *behind* Charlie than Alice is *behind* Charlie (i.e., *behind* $\circ$ *behind* $=$ *behind*).

For a given domain $\mathcal{D}$ (e.g., a queue), a partition $\mathcal{R}$ (e.g., *before*, *behind*, *equal*), a composition $\circ$, a set of variables (e.g., Alice, Bob, Charlie), and a set of spatial constraints (e.g., Alice is *behind* Bob, Bob is *behind* Charlie, Charlie is *behind* Alice), a common *reasoning task* is figuring out whether there is an instantiation of the variables over the domain $\mathcal{D}$, such that the given spatial constraints are consistent (the example is not consistent, as there is no instantiation for Alice, Bob and Charlie that satisfies the constraints). For this reasoning problem QSR employs the path-consistency method, which is used for solving constraint satisfaction problems over *finite* domains. Since the domain $\mathcal{D}$ of interest in QSR is usually *infinite* as opposed to the domain of a finite CSP, partition $\mathcal{R}$ and composition $\circ$ have to meet certain requirements, such that the path-consistency method is applicable to the constraints (See [Renz and Nebel, 2007] and [Renz and Ligozat, 2005] for more details). A triple $(\mathcal{D}, \mathcal{R}, \circ)$ that meets those requirements forms a non-associative algebra; it forms a relation algebra, if it is additionally closed under composition [Ligozat, 2005].

We will call the reasoning approach that utilizes the path-consistency method the *relation-algebraic* approach. The main deficiency of the relation-algebraic approach is that there is no guarantee for its completeness, i.e., the algorithm can fail to identify all inconsistent scenarios. Accordingly, research has been concentrated on finding out whether the consistency of constraints defined by a triple $(\mathcal{D}, \mathcal{R}, \circ)$ can be decided with the path-consistency method. The recent result showed that spatial representations for directional information *cannot* be decided by the path-consistency method in general [Wolter and Lee, 2010]. Thus, we have to question the idea of keeping the relation-algebraic approach as a universal means, and should be open to search for alternative methods for a sound and complete reasoning.

The relation-algebraic approach is also not capable of providing models for the given input constraints. However, in real application domains (e.g., computer-aided design, geographic information systems) not only deciding the consistency of constraints, but also determining the positions of spatial objects satisfying those constraints is desired.

In the next two sections we introduce a selection of qualitative spatial representations for directional information, and methods for reasoning with those representations, which overcome the deficiencies of the relation-algebraic approach.

3 Representations for Qualitative Spatial Knowledge

If a set of spatial objects are represented by a finite number of points in the Euclidian space—which is generally the case in many applications—then the qualitative spatial relations between those objects can be described by a system of polynomial equations or inequalities. For example, we can model people in a queue as points in $\mathbb{R}$ and represent "Alice is *behind* Bob, Bob is *behind* Charlie, Charlie is *behind* Alice" with the system $x_A - x_B > 0 \wedge x_B - x_C > 0 \wedge x_C - x_A > 0$, where $x_A, x_B, x_C \in \mathbb{R}$.

If we leave the one-dimensional Euclidian space $\mathbb{R}$ and move to the two-dimensional Euclidian space $\mathbb{R}^2$, new constraints emerge which were not existent in the one-dimensional case. An important new constraint in the two-dimensional case is based on the relative positions of three points, i.e., whether the points are oriented in clockwise (CW) order, counterclockwise (CCW) order, or collinear. Formally, such a constraint can be expressed as a polynomial inequality or equation based on three points $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$ and $p_3 = (x_3, y_3)$ in the following way

$$x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 < 0 \qquad (\textit{CW})$$

$$x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 > 0 \qquad (\textit{CCW})$$

$$x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 = 0, \qquad (\textit{collin.})$$

where the polynomials on the lefthand side are obtained from

$$\det \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix}, \qquad (1)$$

where $\det$ stands for determinant. The importance and ubiquity of this relationship of three points in a plane will be evident in the next subsections, where we introduce a selection of qualitative spatial representations for directional information. In each of the subsections we will show how a relation of each spatial representation can be translated to a polynomial constraint, which is based on the relative position of three points presented above.

3.1 The $\mathcal{LR}$ calculus

The domain of the $\mathcal{LR}$ calculus [Scivos and Nebel, 2005] is the set of all points in the Euclidian plane. A $\mathcal{LR}$ relation describes for three points $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$, $p_3 = (x_3, y_3)$ the relative position of p_3 with respect to p_1, where the orientation of p_1 is determined by p_2. There are altogether nine $\mathcal{LR}$ relations; seven relations for points, which are depicted in Figure 1 are: **l**eft, **r**ight, **f**ront, **s**tart, **i**nbetween, **e**nd, **b**ack. In Figure 1 the Euclidian plane is partitioned by points p_1 and p_2, $p_1 \neq p_2$ into seven regions: two half-planes (**l**, **r**), two half-lines (**f**, **b**), two points (**s**, **e**), and a line segment (**i**). These regions determine the relation of the third point

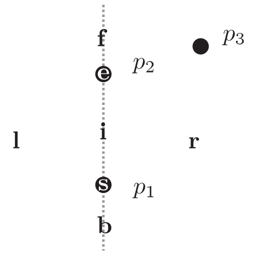


Figure 1: Illustration of $\mathcal{LR}$ relation $p_1\ p_2\ \mathbf{r}\ p_3$

$$\begin{aligned}
p_1\,p_2\,\mathbf{l}\,p_3 &\Leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 > 0 \\
p_1\,p_2\,\mathbf{r}\,p_3 &\Leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 < 0 \\
p_1\,p_2\,\mathbf{b}\,p_3 &\Leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 = 0 \\
&\qquad \wedge\ p_1\,p_2\,\mathbf{r}\,p_4\ \wedge\ p_4\,p_1\,\mathbf{l}\,p_3 \\
p_1\,p_2\,\mathbf{s}\,p_3 &\Leftrightarrow x_3 = x_1\ \wedge\ y_3 = y_1\ \wedge\ x_3 \neq x_2\ \wedge\ y_3 \neq y_2 \\
p_1\,p_2\,\mathbf{i}\,p_3 &\Leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 = 0 \\
&\qquad \wedge\ p_1\,p_2\,\mathbf{r}\,p_4\ \wedge\ p_4\,p_1\,\mathbf{r}\,p_3\ \wedge\ p_4\,p_2\,\mathbf{l}\,p_3 \\
p_1\,p_2\,\mathbf{e}\,p_3 &\Leftrightarrow x_3 = x_2\ \wedge\ y_3 = y_2\ \wedge\ x_3 \neq x_1\ \wedge\ y_3 \neq y_1 \\
p_1\,p_2\,\mathbf{f}\,p_3 &\Leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 = 0 \\
&\qquad \wedge\ p_1\,p_2\,\mathbf{r}\,p_4\ \wedge\ p_4\,p_2\,\mathbf{r}\,p_3 \\
p_1\,p_2\,\mathbf{d}\,p_3 &\Leftrightarrow x_1 = x_2\ \wedge\ y_1 = y_2\ \wedge\ x_1 \neq x_3\ \wedge\ y_1 \neq y_3 \\
p_1\,p_2\,\mathbf{t}\,p_3 &\Leftrightarrow x_1 = x_2 = x_3\ \wedge\ y_1 = y_2 = y_3,
\end{aligned}$$

Table 1: A correspondence table for the $\mathcal{LR}$ calculus.

to p_1 and p_2. The remaining two relations are: **d**ouble $:= \{(p_1, p_2, p_3) \mid p_1, p_2, p_3 \in \mathbb{R}^2, p_1 = p_2, p_1 \neq p_3\}$, **t**riple $:= \{(p_1, p_2, p_3) \mid p_1, p_2, p_3 \in \mathbb{R}^2, p_1 = p_2 = p_3\}$. By describing the relations using polynomial constraints, we obtain the correspondences in Table 1, where we introduce a new point p_4 when required. We note that an inequation "$\neq$" can be written as a disjunction of "$>$" and "$<$".

3.2 The $\mathcal{OPRA}_m$ calculus

The domain of the $\mathcal{OPRA}_m$ calculus [Moratz, 2006] is the set of all oriented points. An oriented point p is a quadruple (x, y, v, w), $x, y, v, w \in \mathbb{R}$, where (x, y) is the location of p, and (v, w) defines the orientation of p by means of an orientation vector $\vec{o}_p := (v, w) - (x, y)$. Two orientated points p_1 and p_2 are equal, if their positions and orientations are equal. With m lines passing through p, we can partition the whole plane (without the point itself) equally into $2m$ open sectors and $2m$ half-lines, where exactly one distinguished half-line has the same orientation as $\vec{o}_p$. Starting with the distinguished half-line, and going through the sectors and half-lines alternately in the counterclockwise order, we can assign numbers 0 to $4m - 1$ to the open sectors and half-lines (see Figure 2). An $\mathcal{OPRA}_m$ relation is a binary relation which describes for points p_1 and p_2 their positions to each other with respect to the aforementioned partitioning. This is represented by the notation $p_1\ {}_m\angle_i^j\ p_2$, where m is as defined

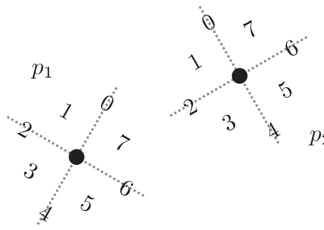


Figure 2: Illustration of $\mathcal{OPRA}_2$ relation $p_1 \,{}_2\angle_7^2\, p_2$

before, i is number of the sector (or half-line) of p_1, in which p_2 is located, and j is the number of the sector (or half-line) of p_2, in which p_1 is located. We write $p_1 \,{}_m\angle_=\, p_2$ if they share the same position.[1] Then for $p_1 = (x_1, y_1, v_1, w_1)$, $p_2 = (x_2, y_2, v_2, w_2)$, and the rotation map

$$\begin{pmatrix} r_x(v,w,k) \\ r_y(v,w,k) \end{pmatrix} := \begin{pmatrix} \cos(k \cdot \frac{\pi}{m}) & -\sin(k \cdot \frac{\pi}{m}) \\ \sin(k \cdot \frac{\pi}{m}) & \cos(k \cdot \frac{\pi}{m}) \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \qquad (2)$$

we can define for $i = 0, 2, \ldots, m-4, m-2$:

$$\begin{aligned} p_1 \,{}_m\angle_i^*\, p_2 \;:\Leftrightarrow\; &\det \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & r_x(v_1,w_1,\frac{i}{2}) & r_y(v_1,w_1,\frac{i}{2}) \\ 1 & x_2 & y_2 \end{pmatrix} = 0 \\ \wedge\; &\det \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & r_x(v_1,w_1,\frac{i}{2}+1) & r_y(v_1,w_1,\frac{i}{2}+1) \\ 1 & x_2 & y_2 \end{pmatrix} < 0, \end{aligned}$$

which describe that p_2 is in half-line i of p_1, and for $i = 1, 3, \ldots, m-3, m-1$:

$$\begin{aligned} p_1 \,{}_m\angle_i^*\, p_2 \;:\Leftrightarrow\; &\det \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & r_x(v_1,w_1,\frac{i-1}{2}) & r_y(v_1,w_1,\frac{i-1}{2}) \\ 1 & x_2 & y_2 \end{pmatrix} > 0 \\ \wedge\; &\det \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & r_x(v_1,w_1,\frac{i+1}{2}) & r_y(v_1,w_1,\frac{i+1}{2}) \\ 1 & x_2 & y_2 \end{pmatrix} < 0, \end{aligned}$$

which describe that p_2 is in sector i of p_1. Then

$$p_1 \,{}_m\angle_i^j\, p_2 \quad\Leftrightarrow\quad p_1 \,{}_m\angle_i^*\, p_2 \quad\wedge\quad p_2 \,{}_m\angle_j^*\, p_1$$

and

$$p_1 \,{}_m\angle_=\, p_2 \quad\Leftrightarrow\quad (x_1, y_1) = (x_2, y_2),$$

and we obtain the desired polynomial constraints.

The polynomial constraints from $\mathcal{OPRA}_m$ relations consist of quadratic polynomials with real algebraic numbers[2] as their coefficients. Dealing with real algebraic numbers requires more computing effort than with rational numbers. As the real algebraic numbers are resulted from $\cos(k\frac{\pi}{m})$ and $\sin(k\frac{\pi}{m})$ from (2) which are responsible for the positions of the half-lines, we can avoid real algebraic numbers by slightly modifying the definition for the positions of the half-lines so as to have only rational numbers as the coefficients.

3.3 The $\mathcal{STAR}_m$ calculus

The $\mathcal{STAR}_m$ calculus [Renz and Mitra, 2004] is similar to the $\mathcal{OPRA}_m$ calculus except it has a fixed reference direction. Consequently, for all oriented points $p = (x, y, v, w)$ the values for (v, w) are fixed to $v = x$, $w = y + 1$ to allow $\vec{o}_p = (v, w) - (x, y) = (0, 1)$ as the orientation for all points. This restriction on the expressibility of the representation has a computational advantage that the resulting polynomial constraints require less variables and they are linear and not quadratic. Hence, they can be solved more efficiently, for example, by the simplex method in subsection 4.1.

So far, we have seen the correspondences between qualitative spatial constraints and polynomial constraints from several spatial representations. Once we have these correspondences, deciding the consistency or finding a model of a set of constraints amounts to solving a system (i.e., a conjunction) of corresponding polynomial equations or inequalities. The approaches to this very problem is discussed in the following section.

4 Alternative Methods for Reasoning with Qualitative Spatial Knowledge

This section introduces methods for solving constraints coming from qualitative spatial relations. As seen in the preceding section, directional constraints can be translated to a system of polynomial equations and inequalities. If the polynomials in the system have degree at most 1 (i.e., the systems is linear), than the simplex method from linear programming can be applied. Otherwise, the Gröner base method from algebraic geometry, or the cylindrical algebraic decomposition method from real algebraic geometry can be applied to polynomial systems with arbitrary degrees.

4.1 The Simplex Method

Many mathematical optimization problems can be formulated as a *Linear Programming* [Dantzig and Thapa, 1997] problem, i.e., finding a maximum (or minimum) of a linear function subject to a set of constraints which is given by a system of linear inequalities. The *simplex method* is one of the techniques in linear programming that is widely used. The simplex method is divided in two phases. In Phase I, it searches for a feasible solution of the given linear system. If a solution is found, then the solution is used in Phase II to find an optimal solution. As our objective is solving a linear system and not optimization, only the algorithm for Phase I is relevant.

The simplex method is a sound and complete method, and has single exponential time complexity.

4.2 The Gröbner Base Method

Several methods have been developed to solve systems of multivariate polynomial equations over the complex field. Gröbner bases introduced by Buchberger [Buchberger, 1985] offer a computational approach that allows us to rewrite a set of polynomial equations, not altering their common zero set. In spirit, the approach of computing Gröbner bases is related Wu's method [Wu, 1978; 1986] as both methods determine elimination polynomials to rewrite polynomials by means of polynomial division. The rewriting process cancels variables and thus leads to equations that are easier to handle. Both elimination techniques are common foundations of algebraic approaches to geometric theorem proving. When computing the Gröbner basis a normalization step is usually carried out to

[1] The original paper [Moratz, 2006] introduces also the so-called *same relations* that further differentiate $p_1 \,{}_m\angle_=\, p_2$ by the orientations of p_1 and p_2.

[2] A real algebraic number is a real number that is a root of a polynomial with integer coefficients (e.g., $\sqrt{2}$ as a root of x^2).

obtain the basis in normal form, called the *reduced Gröbner basis*. This form exhibits a remarkable feature: when the initial set of polynomials does not have a common solution, then the reduced Gröbner basis is equal to $\{1\}$. This property suggests that Gröbner basis enable a straight-forward approach to test the zero set for emptiness, but recall that polynomial equations can also involve complex roots. Henceforth, in cases where the reduced Gröbner basis does not equal $\{1\}$, a common solution is known to exist, but one still needs to check whether the common solution is real-valued. The approach of first computing the Gröbner basis and then further examining existence of real-valued solutions can handle problems arising when analyzing constraint calculi [Wolter, to appear], e.g., automatically computing the composition operation. However, this approach does not provide us with a complete decision procedure and it appears to be very difficult to turn it into a provenly complete one.

4.3 Cylindrical Algebraic Decomposition

The *Cylindrical Algebraic Decomposition* (CAD) [Collins, 1975; Arnon *et al.*, 1984] overcomes the deficiencies of the two previously introduced methods; compared to the simplex method, CAD can handle any polynomial systems and is not limited to linear systems, and where as the Gröber base method is not complete, CAD provides a complete algorithm.

Given a finite set of polynomials $f_1, \ldots, f_m$ in r variables with coefficients from $\mathbb{Q}$, the CAD algorithm computes a finite subset S of $\mathbb{R}^r$, such that

$$\begin{aligned} &\{(\operatorname{sgn}(f_1(s)), \ldots, \operatorname{sgn}(f_n(s))) \mid s \in S\} \\ &\qquad = \{(\operatorname{sgn}(f_1(x)), \ldots, \operatorname{sgn}(f_r(x))) \mid x \in \mathbb{R}^r\}, \end{aligned} \tag{3}$$

where sgn is a real-valued function that returns the sign (i.e., -1, 0, or 1) of its argument. Thus, solving a system of polynomial equations and inequalities having $f_1, \ldots, f_m$ on the left-hand side of the system can be accomplished by evaluating $f_1, \ldots, f_m$ over the elements of S and checking their signs. Due to condition (3) this decision procedure is sound and complete. It also terminates as S is finite.

To generate the set of sample points S the CAD algorithm decomposes $\mathbb{R}^r$, the domain of variables $x_1, \ldots, x_r$, into finitely many subsets $C_1, \ldots, C_K$ of $\mathbb{R}^r$, such that each cell C_i is *sign-invariant* with respect to $f_1, \ldots, f_m$, meaning that the signs of $f_1, \ldots, f_m$ are constant when evaluated over C_i. Set S is then obtained by calculating a sample point in each of the cells $C_1, \ldots, C_K$.

The complexity of CAD is doubly exponential in the number r of the variables.

CAD is designed for general polynomial systems. As a consequence, it is not optimized for particular polynomial systems translated from qualitative spatial relations. For instance, the fact that most polynomial constraints coming from directional relations have their origins in the determinant expression in (1) is not deployed. This lack of integration results in the low performance of the CAD algorithm when dealing with qualitative spatial constraints. We observe in the evaluation of the computer algebra system Mathematica[3] in Figure 5 that CAD is not able to deal with more than 5 objects efficiently.

[3]http://www.wolfram.com/mathematica

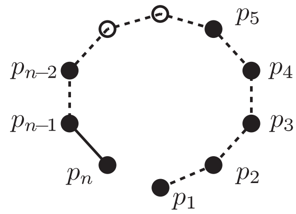


Figure 3: The benchmark problem LR-ALL-LEFT(n) consists of a set of $\mathcal{LR}$ constraints $\{p_i\ p_j\ \mathbf{l}\ p_k \mid 1 \leq i < j < k \leq n\}$ over n varibles, which are consistent by construction.

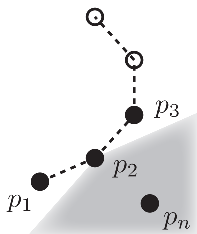


Figure 4: The benchmark problem LR-INDIAN-TENT(n) is a generalization of the Indian Tent Problem for four points (see [Wallgrün *et al.*, 2007]). The problem consists of the same set as LR-ALL-LEFT(n) except two constraints $p_1\ p_2\ \mathbf{l}\ \ p_n$ and $p_2\ p_3\ \mathbf{l}\ p_n$ are substituted with $p_1\ p_2\ \mathbf{r}\ p_n$ and $p_2\ p_3\ \mathbf{r}\ p_n$. These new two constraints contradict $p_1\ p_3\ \mathbf{l}\ p_n$, because they force p_n to be placed in the shaded region. Hence, LR-INDIAN-TENT(n) is inconsistent for all $n \geq 4$.

Accordingly, future research has to concentrate on the theoretical analysis of the interaction between the CAD algorithm and qualitative spatial constraints, and also on the tight integration thereof to achieve better performance.

5 Conclusions

In this paper we have discussed several approaches that propose themselves as alternatives to the conventional relation-algebraic method. From the three presented approaches the simplex method and CAD provide sound and complete algorithms, which are also constructive and are therefore able to generate models for consistent constraints. The simplex method, which runs faster than CAD, is well suited for qualitative spatial constraints that can be translated to a system of linear equations and inequalities (e.g., constraints from the $\mathcal{STAR}_m$ calculus). On the other hand, CAD is versatile, and can deal with any system of polynomial constraints. However, CAD suffers from its poor performance in solving qualitative spatial constraints, since it is a general solver and is therefore not tailored to these specific constraints. We see this deficiency of CAD as an open research question. To overcome this issue, a thorough analysis of the input polynomials is needed in the future. Analyzing the determinant expression (1) and adapting the result to the CAD algorithm might be a key to the improvement of this approach.

In summary, there is a need to adopt the mentioned new

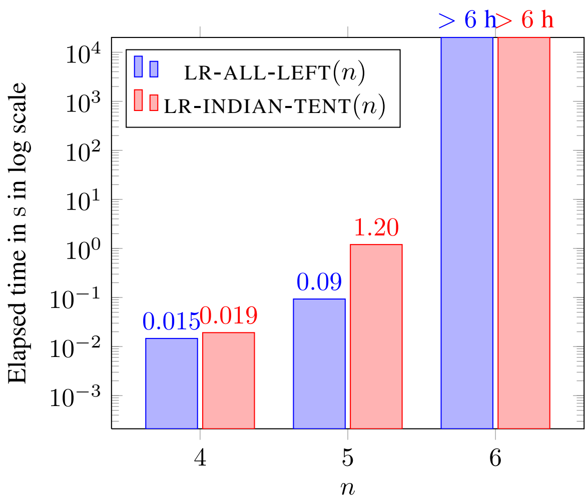


Figure 5: Evaluation of Mathematica™ ver. 8.0.1.0 with benchmark problems LR-ALL-LEFT(n) (see Figure 3) and LR-INDIAN-TENT(n) (see Figure 4) using the function `FindInstance`. Although Mathematica finds consistent instances for LR-ALL-LEFT(4) and LR-ALL-LEFT(5), and inconsistencies of LR-INDIAN-TENT(4) and LR-INDIAN-TENT(5) in less than few seconds, it was not able to decide consistency of LR-ALL-LEFT(6) and inconsistency of LR-INDIAN-TENT(6) within 6 hours. The evaluation was done on an OS X machine with Intel Core 2 Duo 2.66 GHz processor and 4 GB memory.

approaches for reasoning with qualitative spatial information. The future research in qualitative spatial reasoning should therefore consider—besides investigating qualitative spatial representations with regard to their relation-algebraic properties—analyzing and optimizing the introduced new approaches by exploiting the structure of polynomials from qualitative spatial constraints.

Acknowledgements

This work was supported by the DFG Transregional Collaborative Research Center SFB/TR 8 Spatial Cognition. Funding by the German Research Foundation (DFG) is gratefully acknowledged.

References

[Arnon *et al.*, 1984] Dennis S. Arnon, George E. Collins, and Scott McCallum. Cylindrical algebraic decomposition i: the basic algorithm. *SIAM J. Comput.*, 13:865–877, November 1984.

[Buchberger, 1985] Benno Buchberger. Göbner-bases: An algorithmic method in polynomial ideal theory. In N.K. Bose, editor, *Multidimensional Systems Theory - Progress, Directions and Open Problems in Multidimensional Systems*, chapter 6, pages 184–232. Reidel Publishing Company, Dordrecht—Boston—Lancaster, 1985.

[Cohn and Renz, 2008] Anthony G. Cohn and Jochen Renz. Chapter 13 qualitative spatial representation and reasoning. In Vladimir Lifschitz Frank van Harmelen and Bruce Porter, editors, *Handbook of Knowledge Representation*, volume 3 of *Foundations of Artificial Intelligence*, pages 551 – 596. Elsevier, 2008.

[Collins, 1975] George Collins. Quantifier elimination for real closed fields by cylindrical algebraic decompostion. In H. Brakhage, editor, *Automata Theory and Formal Languages 2nd GI Conference Kaiserslautern, May 20–23, 1975*, volume 33 of *Lecture Notes in Computer Science*, pages 134–183. Springer Berlin / Heidelberg, 1975.

[Dantzig and Thapa, 1997] George B. Dantzig and Mukund N. Thapa. *Linear programming 1: introduction*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1997.

[Ligozat, 2005] Gérard Ligozat. Categorical methods in qualitative reasoning: The case for weak representations. In Anthony Cohn and David Mark, editors, *Spatial Information Theory*, volume 3693 of *Lecture Notes in Computer Science*, pages 265–282. Springer Berlin / Heidelberg, 2005.

[Moratz, 2006] Reinhard Moratz. Representing relative direction as a binary relation of oriented points. In *Proceeding of the 2006 conference on ECAI 2006: 17th European Conference on Artificial Intelligence August 29 – September 1, 2006, Riva del Garda, Italy*, pages 407–411, Amsterdam, The Netherlands, The Netherlands, 2006. IOS Press.

[Renz and Ligozat, 2005] Jochen Renz and Gérard Ligozat. Weak composition for qualitative spatial and temporal reasoning. In Peter van Beek, editor, *Principles and Practice of Constraint Programming - CP 2005*, volume 3709 of *Lecture Notes in Computer Science*, pages 534–548. Springer Berlin / Heidelberg, 2005.

[Renz and Mitra, 2004] Jochen Renz and Debasis Mitra. Qualitative direction calculi with arbitrary granularity. In Chengqi Zhang, Hans W. Guesgen, and Wai-Kiang Yeap, editors, *PRICAI 2004: Trends in Artificial Intelligence*, volume 3157 of *Lecture Notes in Computer Science*, pages 65–74. Springer Berlin / Heidelberg, 2004.

[Renz and Nebel, 2007] Jochen Renz and Bernhard Nebel. Qualitative spatial reasoning using constraint calculi. In Marco Aiello, Ian Pratt-Hartmann, and Johan Benthem, editors, *Handbook of Spatial Logics*, pages 161–215. Springer Netherlands, 2007.

[Scivos and Nebel, 2005] Alexander Scivos and Bernhard Nebel. The finest of its class: The natural point-based ternary calculus for qualitative spatial reasoning. In Christian Freksa, Markus Knauff, Bernd Krieg-Brückner, Bernhard Nebel, and Thomas Barkowsky, editors, *Spatial Cognition IV. Reasoning, Action, and Interaction*, volume 3343 of *Lecture Notes in Computer Science*, pages 283–303. Springer Berlin / Heidelberg, 2005.

[Wallgrün *et al.*, 2007] Jan Wallgrün, Lutz Frommberger, Diedrich Wolter, Frank Dylla, and Christian Freksa. Qualitative spatial representation and reasoning in the sparq-toolbox. In Thomas Barkowsky, Markus Knauff, Gérard

Ligozat, and Daniel Montello, editors, *Spatial Cognition V Reasoning, Action, Interaction*, volume 4387 of *Lecture Notes in Computer Science*, pages 39–58. Springer Berlin / Heidelberg, 2007.

[Wolter and Lee, 2010] D. Wolter and J.H. Lee. Qualitative reasoning with directional relations. *Artificial Intelligence*, 174(18):1498 – 1507, 2010.

[Wolter, to appear] Diedrich Wolter. Analyzing qualitative spatio-temporal calculi using algebraic geometry. *Spatial Cognition and Computation*, to appear.

[Wu, 1978] Wen-Tsun Wu. On the decision problem and the mechanization of theorem proving in elementary geometry. *Scientia Sinica*, 21:157–179, 1978.

[Wu, 1986] Wen-Tsun Wu. Basic principles of mechanical theorem proving in elementary geometries. *Journal of Automated Reasoning*, 2:221–252, 1986.

Solving Qualitative Constraints Involving Landmarks *

Weiming Liu[1], **Shengsheng Wang**[2], **Sanjiang Li**[1]
[1]Centre for Quantum Computation and Intelligent Systems,
Faculty of Engineering and Information Technology, University of Technology Sydney, Australia
[2]College of Computer Science and Technology, Jilin University, Changchun, China

Abstract

Consistency checking plays a central role in qualitative spatial and temporal reasoning. Given a set of variables V, and a set of constraints Γ taken from a qualitative calculus (e.g. the Interval Algebra (IA) or RCC-8), the aim is to decide if Γ is consistent. The consistency problem has been investigated extensively in the literature. Practical applications e.g. urban planning often impose, in addition to those between undetermined entities (variables), constraints between determined entities (constants or landmarks) and variables. This paper introduces this as a new class of qualitative constraints satisfaction problems, and investigates its consistency in several well-known qualitative calculi, e.g. IA, RCC-5, and RCC-8. We show that the usual local consistency checking algorithm works for IA but fails in RCC-5 and RCC-8. We further show that, if the landmarks are represented as polygons, then the new consistency problem of RCC-5 is tractable but that of RCC-8 is NP-complete.

1 Introduction

Qualitative constraints are widely used in temporal and spatial reasoning. This is partially because they are close to the way humans represent and reason about commonsense knowledge. Moreover, qualitative constraints are easy to specify and provide a flexible way to deal with incomplete knowledge.

Usually, these constraints are taken from a qualitative calculus, which is a set $\mathcal{M}$ of relations defined on an infinite universe U of entities [6]. Well-known qualitative calculi include the Interval Algebra [1], RCC-5 and RCC-8 [9], and the cardinal direction calculus (for point-like objects) [7].

A central problem of reasoning with a qualitative calculus is the *consistency problem*. For a qualitative calculus $\mathcal{M}$ on U, an instance of the consistency problem over $\mathcal{M}$ is a network Γ of constraints like $x\alpha y$, where x, y are variables taken from a finite set V, and α is a relation in $\mathcal{M}$. Consistency checking has applications in many areas, e.g. temporal or spatial query preprocessing, planning, natural language understanding, etc. Moreover, several other reasoning problems e.g. the minimal label problem and the entailment problem can be reduced in polynomially time to the consistency problem.

The consistency problem has been studied extensively for many different qualitative calculi (cf. [2]). These works almost unanimously assume that the qualitative constraints involve only *unknown* entities. In other words, the precise (geometric) information of *every* object is totally unknown. In practical applications, however, we often meet constraints that involve both known and unknown entities, i.e. constants and variables.

For example, consider a class scheduling problem in a primary school. In addition to constraints between unknown intervals (e.g. a Math class is *followed by* a Music class), we may also impose constraints involving determined intervals (e.g. a P.E. class should be *during* afternoon).

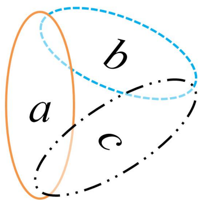
Constraints involving known entities are especially common in spatial reasoning tasks such as urban planning. For example, to find a best location for a landfill, we need to formulate constraints between the unknown landfill and significant landmarks, e.g. lake, university, hospital etc.

In this paper, we explicitly introduce *landmarks* (defined as known entities) into the definition of the consistency problem, and call the consistency problem involving landmarks the *hybrid* consistency problem. In comparison, we call the usual consistency problem (involving no landmarks) the *pure* consistency problem.

In general, solving constraint networks involving landmarks is different from solving constraint networks involving no landmarks. For example, consider the simple RCC-5 algebra. It is a well-known result that a path-consistent constraint network Γ is consistent when Γ involves no landmarks. But the following example shows that this fails to hold when landmarks are involved. Suppose a, b, c are the three regions shown below. Let x be a spatial variable, which is required to be a subset of a, b, c. This network is path-consistent, but inconsistent since the three landmarks have no common points.

The aim of this paper is to investigate how landmarks affect the consistency of constraint networks in several very important qualitative calculi. The rest of this paper proceeds as follows. Section 2 introduces basic notions in qualitative

*This work was partly supported by an ARC Future Fellowship (FT0990811).



constraint solving and examples of qualitative calculi. The new consistency problem, as well as several basic results, is also presented here. Assuming that all landmarks are represented as polygons, Section 3 then provides a polynomial decision procedure for the consistency of hybrid basic RCC-5 networks. Besides, if the network is consistent, a solution is constructed in polynomial time; Section 4 shows that consistency problem for hybrid basic RCC-8 networks is NP-hard. The last section then concludes the paper.

2 Qualitative Calculi and The Consistency Problem

Most qualitative approaches to spatial and temporal knowledge representation and reasoning are based on qualitative calculi. Suppose U is a universe of spatial or temporal entities. Write $\mathbf{Rel}(U)$ for the algebra of binary relations on U. A qualitative calculus on U is a sub-Boolean algebra of $\mathbf{Rel}(U)$ generated by a set $\mathcal{B}$ of jointly exhaustive and pairwise disjoint (JEPD) relations on U. Relations in $\mathcal{B}$ are called basic relations of the qualitative calculus.

We next recall the well-known Interval Algebra (IA) [1] and the two RCC algebras.

Example 2.1 (Interval Algebra)**.** Let U be the set of closed intervals on the real line. Thirteen binary relations between two intervals $x = [x^-, x^+]$ and $y = [y^-, y^+]$ are defined by comparing the order relations between the endpoints of x and y. These are the basic relations of IA.

Example 2.2 (RCC-5 and RCC-8 Algebras[1])**.** Let U be the set of bounded regions in the real plane, where a region is a nonempty regular set. The RCC-8 algebra is generated by the eight topological relations

$$\mathbf{DC}, \mathbf{EC}, \mathbf{PO}, \mathbf{EQ}, \mathbf{TPP}, \mathbf{NTPP}, \mathbf{TPP}^\sim, \mathbf{NTPP}^\sim, \quad (1)$$

where $\mathbf{DC}, \mathbf{EC}, \mathbf{PO}, \mathbf{TPP}$ and $\mathbf{NTPP}$ are defined in Table 1, $\mathbf{EQ}$ is the identity relation, and $\mathbf{TPP}^\sim$ and $\mathbf{NTPP}^\sim$ are the converses of $\mathbf{TPP}$ and $\mathbf{NTPP}$, respectively, see Fig. 1 for illustraion. The RCC-5 algebra is the sub-algebra of RCC-8 generated by the five part-whole relations

$$\mathbf{DR}, \mathbf{PO}, \mathbf{EQ}, \mathbf{PP}, \mathbf{PP}^\sim, \quad (2)$$

where $\mathbf{DR} = \mathbf{DC} \cup \mathbf{EC}$, $\mathbf{PP} = \mathbf{TPP} \cup \mathbf{NTPP}$, and $\mathbf{PP}^\sim = \mathbf{TPP}^\sim \cup \mathbf{NTPP}^\sim$.

A qualitative calculus provides a useful constraint language. Suppose $\mathcal{M}$ is a qualitative calculus defined on domain U. Relations in $\mathcal{M}$ can be used to express constraints

[1]We note that the RCC algebras have interpretations in arbitrary topological spaces. In this paper, we only consider the most important interpretation in the real plane.

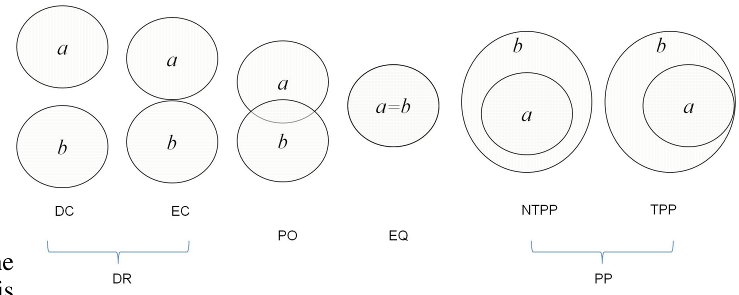


Figure 1: Illustrations of the basic relations in RCC-8.

Table 1: A topological interpretation of basic RCC-8 relations in the plane, where a, b are two bounded plane regions, and a°, b° are the interiors of a, b, respectively.

Relation	Meaning
DC	$a \cap b = \varnothing$
EC	$a \cap b \neq \varnothing, a^\circ \cap b^\circ = \varnothing$
PO	$a \not\subseteq b, b \not\subseteq a, a^\circ \cap b^\circ \neq \varnothing$
TPP	$a \subset b, a \not\subset b^\circ$
NTPP	$a \subset b^\circ$

about variables which takes values in U. A constraint has the form

$$x\alpha y, \text{ or } x\alpha c, \text{ or } c\alpha x,$$

where α is a relation in $\mathcal{M}$, c is a constant in U (called *landmark* in this paper), x, y are variables taking values in U. Such a constraint is *basic* if α is a basic relation in $\mathcal{M}$.

Given a finite set Γ of constraints, write $V(\Gamma)$ ($L(\Gamma)$, resp.) for the set of variables (constants, resp.) appearing in Γ. A solution of Γ is an assignment of values in U to variables in $V(\Gamma)$ such that all constraints in Γ are satisfied. If Γ has a solution, we say Γ is *consistent* or *satisfiable*. Two sets of constraint Γ and Γ' are *equivalent* if they have the same set of solutions.

A set Γ of constraints is said to be a *complete constraint network* if there is a unique constraint between each pair of variables/constants appearing in Γ.

Definition 2.1. Let $\mathcal{M}$ be a qualitative calculus on U. The hybrid consistency problem of $\mathcal{M}$ is, given a constraint network Γ in $\mathcal{M}$, decide the consistency of Γ in $\mathcal{M}$, i.e. decide if there is an assignment of elements in U to variables in Γ that satisfies all the constraints in Γ. The pure consistency problem of $\mathcal{M}$ is the sub-consistency problem that considers constraint networks that involve no landmarks.

The hybrid consistency problem of $\mathcal{M}$ can be approximated by a variant of the path-consistency algorithm. We say a complete constraint network Γ is *path-consistent* if for any three objects l_i, l_j, l_k in $V(\Gamma) \cup L(\Gamma)$, we have

$$\alpha_{ij} = \alpha_{ji}^\sim \ \& \ \alpha_{ij} \subseteq \alpha_{ik} \circ_w \alpha_{kj}, \quad (3)$$

where $\circ_w$ is the weak composition [4; 6] in $\mathcal{M}$ and $\alpha \circ_w \beta$ is defined to be the smallest relation in $\mathcal{M}$ which contains the usual composition of α and β. It is clear that each complete network can be transformed in polynomial time into an equivalent complete network that is path-consistent. Because the consistency problem is in general NP-hard, we do not expect that a local consistency algorithm can solve the general consistency problem. However, it has been proved that the path-consistency algorithm suffices to decide the pure consistency problem for large fragments of some well-known qualitative calculi, e.g. IA, RCC-5, and RCC-8 (cf. [2]). This shows that, at least for these calculi, the pure consistency problem can be solved by path-consistency algorithm and the backtracking method.

The remainder of this paper will investigate the hybrid consistency problem for the above calculi. In the following discussion, we assume Γ is a complete basic network that involves at least one landmark.

For IA, endpoints of the intervals in different solutions of a complete basic constraint network respect the same ordering. This suggests that any partial solution of a consistent network can be extended to a complete solution.

Proposition 2.1. *Suppose Γ is a basic network of IA constraints that involves landmarks and variables. Then Γ is consistent iff it is path-consistent.*

This result shows that, for IA, the hybrid consistency problem can be solved in the same way as the pure consistency problem. Similar conclusion also holds for some other calculi, e.g. the Point Algebra, the Rectangle Algebra, and the Cardinal Direction Calculus (for point-like objects) [7]. This property, however, does not hold in general. Take the RCC-5 as example. If a basic network Γ involves no landmark, then we know Γ is consistent if it is path-consistent. If Γ involves landmarks, we have seen in the introduction a path-consistent but inconsistent basic RCC-5 network.

In the next two sections, we investigate how landmarks affect the consistency of RCC-5 and RCC-8 topological constraints. We stress that, in this paper, we *only consider* the standard (and the most important) interpretation of the RCC language in the real plane, as given in Example 2.2. When restricting landmarks to polygons, we first show that the consistency of a hybrid basic RCC-5 network can still be decided in polynomial time (Section 4), but the that of RCC-8 networks is NP-hard.

3 The Hybrid Consistency Problem of RCC-5

We begin with a short review of the realization algorithm for pure consistency problem of RCC-5 [5; 3]. Suppose Γ involves only spatial variables $v_1, v_2, \cdots, v_n$. We define a finite set X_i of *control points* for each v_i as follows:

- Add a point P_i to X_i;
- For any $j > i$, add a new point P_{ij} to both X_i and X_j if $(v_i \mathbf{PO} v_j) \in \Gamma$;
- For any j, put all points in X_i into X_j if $(v_i \mathbf{PP} v_j) \in \Gamma$.

Take $\varepsilon > 0$ such that the distance between any two different points in $\bigcup_{i=1}^{n} X_i$ is greater than 2ε. Let $B(P, \varepsilon)$ be the closed disk with radius ε centred at P. By the choice of ε, different disks are disjoint. Let $a_i = \bigcup\{B(P, \varepsilon) : P \in X_i\}$. It is easy to check that the assignment is a solution of Γ, if Γ is consistent.

Assume Γ is a basic RCC-5 network involving landmarks $L = \{l_1, \cdots, l_m\}$ in the real plane and variables $V = \{v_1, \cdots, v_n\}$. Write ∂L for the union of the boundaries of the landmarks. An equivalence relation $\sim_L$ can be defined on the plane as follows: For $P, Q \notin \partial L$,

$$P \sim_L Q \quad \text{iff} \quad (\forall 1 \leq j \leq m)[P \in l_j \leftrightarrow Q \in l_j] \tag{4}$$

A *block* is defined as an equivalent class under $\sim_L$. Because $\sim_L$ is defined only for points that are not on the boundaries of the landmarks, it is easy to see that each block is an open set. It is also clear that the complement of the union of all landmarks (which are bounded) is the unique unbounded block. We write $\mathbb{B}$ for the set of all blocks.

For each landmark l_i, we write $I(l_i)$ for the set of blocks that l_i contains, and write $E(l_i)$ for the set of rest blocks, i.e. the blocks that are disjoint from l_i. That is,

$$I(l_i) = \{b \in \mathbb{B} : b \subseteq l_i\}, \tag{5}$$

$$E(l_i) = \{b \in \mathbb{B} : b \cap l_i = \varnothing\}. \tag{6}$$

It is easy to see that the interior (exterior, resp.) of l_i is exactly the regularized union (i.e. the interior of its closure) of all blocks in $I(l_i)$ ($E(l_i)$,resp.). Moreover, each block is in either $I(l_i)$ or $E(l_i)$, but not both, i.e., $I(l_i) \cup E(l_i) = \mathbb{B}$ and $I(l_i) \cap E(l_i) = \varnothing$.

These constructions can be extended from landmarks to variables as

$$I(v_i) = \bigcup\{I(l_j) : l_j \mathbf{PP} v_i\}, \tag{7}$$

$$E(v_i) = \bigcup\{I(l_j) : l_j \mathbf{DR} v_i\} \cup \bigcup\{E(l_j) : v_i \mathbf{PP} l_j\}. \tag{8}$$

Intuitively, $I(v_i)$ is the set of blocks that v_i must contain, and $E(v_i)$ is the set of blocks that should be excluded from v_i.

The following proposition claims that no block can appear in both $I(v_i)$ and $E(v_i)$.

Proposition 3.1. *Suppose Γ is a basic RCC-5 constraint network that involves at least one landmark. If Γ is path-consistent, then $I(v_i) \cap E(v_i) = \varnothing$.*

We have the following theorem.

Theorem 3.1. *Suppose Γ is a basic RCC-5 constraint network that involves at least one landmark. If Γ is consistent, then we have*

- *For any $v_i \in V$,*
$$E(v_i) \subsetneq \mathbb{B}. \tag{9}$$
- *For any $v_i \in V$ and $w \in L \cup V$ such that $(v_i \mathbf{PO} w) \in \Gamma$,*
$$E(v_i) \cup E(w) \subsetneq \mathbb{B}, \tag{10}$$
$$E(v_i) \cup I(w) \subsetneq \mathbb{B}, \tag{11}$$
$$I(v_i) \cup E(w) \subsetneq \mathbb{B}. \tag{12}$$
- *For any $v_i \in V$ and $l_j \in L$ such that $(v_i \mathbf{PP} l_j) \in \Gamma$,*
$$I(v_i) \subsetneq I(l_j). \tag{13}$$
- *For any $v_i \in V$ and $l_j \in L$ such that $(l_j \mathbf{PP} v_i) \in \Gamma$,*
$$E(v_i) \subsetneq E(l_j). \tag{14}$$
- *For any $v_i, v_j \in V$ such that $(v_i \mathbf{PP} v_j) \in \Gamma$,*
$$I(v_i) \cup E(v_j) \subsetneq \mathbb{B}. \tag{15}$$

These conditions are also sufficient to determine the consistency of a path-consistent basic RCC-5 network. We show this by devising a realization algorithm. The construction is similar to that for the pure consistency problem. For each v_i, we define a finite set X_i of control points as follows, where for clarity, we write

$$P(v_i) = \mathbb{B} - I(v_i) - E(v_i). \tag{16}$$

- For each block b in $P(v_i)$, select a fresh point in b and add the point into X_i.
- For any $j > i$ with $(v_i\mathbf{PO}v_j) \in \Gamma$, select a fresh point in some block b in $P(v_i) \cap P(v_j)$ (if it is not empty), and add the point into X_i and X_j.
- For any j, put all points in X_j into X_i if $(v_j\mathbf{PP}v_i) \in \Gamma$.

We note that the points selected from a block b for different v_i, or in different steps, should be pairwise different. Recall that each point in $\bigcup_{i=1}^n X_i$ is not at the boundary of any block. We choose $\varepsilon > 0$ such that $B(P, \varepsilon)$ does not intersect either the boundary of a block or another disk $B(Q, \varepsilon)$. Furthermore, we can assume that ε is small enough such that the union of all the disks $B(P, \varepsilon)$ does not cover any block in $\mathbb{B}$.

Let

$$\hat{a}_i = \bigcup\{B(P, \varepsilon) : P \in X_i\} \cup \bigcup\{l_j : l_j\mathbf{PP}v_i\}. \tag{17}$$

We claim that $\{\hat{a}_1, \cdots, \hat{a}_t\}$ is a solution of Γ. To prove this, we need the following lemma.

Lemma 3.1. *Let Γ be a path-consistent basic RCC-5 constraint network that involves at least one landmark. Suppose $\mathbb{B}$ is the block set of Γ. Then, for each $b \in \mathbb{B}$, we have*

- *$b \in I(v_i)$ iff $b \subseteq \hat{a}_i$.*
- *If $b \in E(v_i)$ iff $b \cap \hat{a}_i = \varnothing$.*
- *If $b \in P(v_i)$ iff $b \nsubseteq \hat{a}_i$ and $b \cap \hat{a}_i \neq \varnothing$.*

Remark 3.1. Since $\{I(v_i), E(v_i), P(v_i)\}$ is a partition of the blocks in B, it is easy to see the conditions in Lemma 3.1 are also sufficient. That is, for example, $b \in I(v_i)$ iff $b \subseteq \hat{a}_i$.

We next prove that $\{\hat{a}_1, \cdots, \hat{a}_t\}$ is a solution of Γ.

Theorem 3.2. *Suppose Γ is a complete basic RCC-5 network involving landmarks L and variables V. Assume Γ is path-consistent and satisfies the conditions in Theorem 3.1. Then Γ is consistent and $\{\hat{a}_1, \cdots, \hat{a}_t\}$, as constructed in (17), is a solution of Γ.*

It is worth noting that the complexity of deciding the consistency of a hybrid basic RCC-5 network includes two parts, viz. the complexity of computing the blocks, and that of checking the conditions in Theorem 3.1. The latter part alone can be completed in $O(|\mathbb{B}|n(n+m))$ time, where $|\mathbb{B}|$ is the number of the blocks. In the worst situation, the number of blocks may be up to 2^m. This suggests that the decision method described above is in general inefficient. The following theorem, however, asserts that this method is still polynomial in the size of the input instance, provided that the landmarks are all represented as polygons.

Theorem 3.3. *Suppose Γ is a basic RCC-5 constraint network, and $V(\Gamma) = \{v_1, \cdots, v_n\}$ and $L(\Gamma) = \{l_1, \cdots, l_m\}$ are the set of variables and, respectively, the set of landmarks appearing in Γ. Assume each landmark l_i is represented by a (complex) polygon with less than k vertices. Then the consistency of Γ can be decided in $O((m+n)^6k^6)$ time.*

4 The Hybrid Consistency Problem of RCC-8

Suppose Γ is a complete basic RCC-8 network that involves no landmarks. Then Γ is consistent if it is path-consistent [8; 10]. Moreover, a solution can be constructed for each path-consistent basic network in cubic time [5; 3]. This section shows that, however, when considering polygons, it is NP-hard to determine if a complete basic RCC-8 network involving landmarks has a solution. We achieve this by devising a polynomial reduction from 3-SAT.

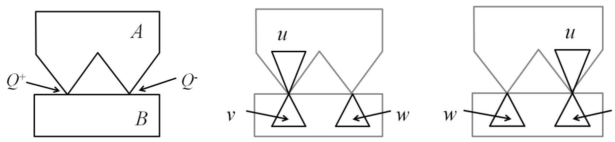
In this section, for clarity, we use upper case letters A, B, C (with indices) to denote landmarks, and use lower case letters u, v, w (with indices) to denote spatial variables.

The NP-hardness stems from the fact that two externally connected polygons, say A, B, may have more than one tangential points. Assume v is a spatial variable that is required to be a tangentially proper part of A but externally connected to B. Then it is undetermined at which tangential point(s) v and B should meet.

Precisely, consider the configuration shown in Fig. 2 (a), where A and B are two externally connected landmarks, meeting at two tangential points, say Q^+ and Q^-. Assume $\{u, v, w\}$ are variables that are subject to the following constraints

$$u\mathbf{TPP}A, u\mathbf{EC}B,$$
$$v\mathbf{TPP}B, v\mathbf{EC}A, w\mathbf{TPP}B, w\mathbf{EC}A,$$
$$u\mathbf{EC}v, u\mathbf{DC}w, v\mathbf{DC}w.$$

It is easy to see that u and B are required to meet at either Q^+



(a) (b) (c)

Figure 2: Two landmarks A, B that are externally connected at two tangential points Q^+ and Q^-.

or Q^-, but not both (cf Fig. 2(b,c)). The correspondence between these two configurations and the two truth values (true or false) of a propositional variable is exploited in the following reduction.

Let $\phi = \bigwedge_{k=1}^m \varphi_k$ be a 3-SAT instance over propositional variables set $\{p_1, \cdots, p_n\}$. Each clause φ_k has the form $p_r^* \vee p_s^* \vee p_t^*$, where literal p_i^* is either p_i or $\neg p_i$ for $i = r, s, t$. We next construct a set of polygons L and a complete basic RCC-8 network Γ_ϕ, such that ϕ is satisfiable iff Γ_ϕ is satisfiable.

First, define $A, B_1, B_2, \cdots, B_n$ as in Fig. 3. For each $1 \leq i \leq n$, A is externally connected to B_i. Let Q_i^+ and Q_i^- be the two tangential points.

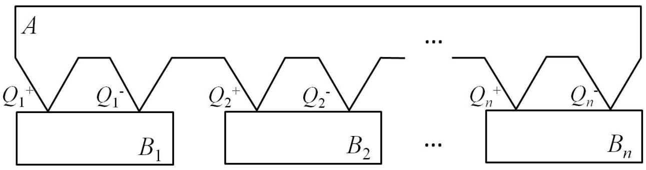


Figure 3: Illustration of landmarks $A, B_1, \cdots, B_n$.

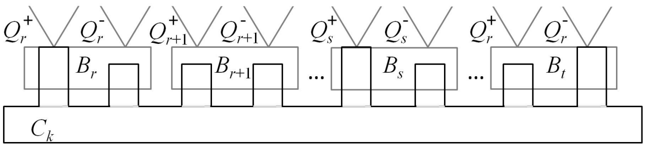


Figure 4: Illustration of landmark C_k.

The variable set of Γ is $V = \{u, v_1, \cdots, v_n, w_1, \cdots, w_n\}$. We impose the following constraints to the variables in V.

$$u\mathbf{TPP}A, \quad u\mathbf{EC}B_i, \tag{18}$$

$$v_i\mathbf{EC}A, \quad v_i\mathbf{TPP}B_i, \quad v_i\mathbf{DC}B_j\ (j \neq i), \tag{19}$$

$$w_i\mathbf{EC}A, \quad w_i\mathbf{TPP}B_i, \quad w_i\mathbf{DC}B_j\ (j \neq i), \tag{20}$$

$$u\mathbf{EC}v_i, \quad u\mathbf{DC}w_i, \tag{21}$$

$$v_i\mathbf{DC}w_j, \quad v_i\mathbf{DC}v_j\ (j \neq i), \quad w_i\mathbf{DC}w_j\ (j \neq i). \tag{22}$$

From the discussion above, we know u is required to meet with each B_i at either Q_i^- or Q_i^+, but not both.

For each clause φ_k, we introduce an additional landmark C_k, which externally connects A at three tangential points, and partially overlaps B_i. The three tangential points of C_k and A are determined by the literals in φ_k. Precisely, suppose $\varphi_k = p_r^* \vee p_s^* \vee p_t^*$, then the first tangential point of A and C_k is constructed to be Q_r^+ if $p_r^* = p_r$, or Q_r^- if $p_r^* = \neg p_r$. The second and the third tangential points are selected from $\{Q_s^+, Q_s^-\}$ and $\{Q_t^+, Q_t^-\}$ similarly. Take clause $p_r \vee \neg p_s \vee p_t$ for example, the tangential points between landmarks C_k and A should be Q_r^+, Q_s^-, and Q_t^+, as shown in Fig. 4.

The constraints between C_k and variables in V are specified as

$$u\mathbf{EC}C_k, \quad v_i\mathbf{PO}C_k, \quad w_i\mathbf{PO}C_k. \tag{23}$$

Since C_k and A have three tangential points, the constraints $u\mathbf{TPP}A$ and $u\mathbf{EC}C_k$ imply that u should occupy at least one of the three tangential points. This corresponds to the fact that if φ_k is true under some assignment, then at least one of its three literals is assigned true.

Lemma 4.1. *Suppose $\phi = \bigwedge_{k=1}^m \varphi_k$ is a 3-SAT instance over propositional variables set $\{p_1, p_2, \cdots, p_n\}$. Let Γ_ϕ be the basic RCC-8 network composed with constraints in (18)-(23), involving landmarks $\{A, B_1, \cdots, B_n, C_1, \cdots, C_m\}$ and spatial variables $\{u, v_1, \cdots, v_n, w_1, \cdots, w_n\}$. Then ϕ is satisfiable iff Γ_ϕ is satisfiable.*

The following corollary follows directly.

Corollary 4.1. *Deciding the consistency of a complete basic RCC-8 network involving landmarks is NP-hard.*

Is this consistency problem still in NP? As long as the landmarks are polygons, the answer is yes!

Theorem 4.1. *Suppose all landmarks in a hybrid basic RCC-8 network are represented by (complex) polygons. Then deciding the consistency of a complete basic RCC-8 network involving at least one landmark is an NP-complete problem.*

5 Conclusion and Further Discussions

In this paper, we introduced a new paradigm of consistency checking problem for qualitative calculi, which supports definitions of constraints between a constant (landmark) and a variable. Constraints like these are very popular in practical applications such as urban planning and schedule planning. Therefore, this hybrid consistency problem is more practical. Our examinations showed that for some well-behaved qualitative calculi such as PA and IA, the new hybrid consistency problem can be solved in the same way; while for some calculi e.g. RCC-5 and RCC-8, the usual composition-based reasoning approach fails to solve the hybrid consistency problem. We provided necessary and sufficient conditions for deciding if a hybrid basic RCC-5 network is consistent. Under the assumption that each landmark is represented as a polygon, these conditions can be checked in polynomial time. As for the RCC-8, however, we show that it is NP-complete to determine the consistency of a basic network that involves polygonal landmarks.

The hybrid consistency problem is equivalent to determining if a partial solution can be extended to a complete solution. This is usually harder than the pure consistency problem. More close connections between the pure and hybrid consistency problems are still unknown. For example, suppose the consistency problem is in NP (decidable, resp.). Is the hybrid consistency problem always in NP (decidable, resp.)?

References

[1] J. F. Allen. Maintaining knowledge about temporal intervals. *Commun. ACM*, 26(11):832–843, 1983.

[2] A. G. Cohn and J. Renz. Qualitative spatial reasoning. In *Handbook of Knowledge Representation*. Elsevier, 2007.

[3] S. Li and H. Wang. RCC8 binary constraint network can be consistently extended. *Artif. Intell.*, 170(1):1–18, 2006.

[4] S. Li and M. Ying. Region connection calculus: Its models and composition table. *Artif. Intell.*, 145(1-2):121–146, 2003.

[5] S. Li. On topological consistency and realization. *Constraints*, 11(1):31–51, 2006.

[6] G. Ligozat and J. Renz. What is a qualitative calculus? A general framework. In *PRICAI*, pages 53–64. Springer, 2004.

[7] G. Ligozat. Reasoning about cardinal directions. *J. Vis. Lang. Comput.*, 9(1):23–44, 1998.

[8] B. Nebel. Computational properties of qualitative spatial reasoning: First results. In *KI*, pages 233–244. Springer, 1995.

[9] D. A. Randell, Z. Cui, and A. G. Cohn. A spatial logic based on regions and connection. In *KR*, pages 165–176, 1992.

[10] J. Renz and B. Nebel. On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the region connection calculus. *Artif. Intell.*, 108(1-2):69–123, 1999.

Benchmarking Qualitative Spatial Calculi for Video Activity Analysis

Muralikrishna Sridhar, Anthony G Cohn and David C Hogg

University Of Leeds, UK,
{krishna,agc,dch}@comp.leeds.ac.uk

Abstract. This paper presents a general way of addressing problems in video activity understanding using graph based relational learning. Video activities are described using relational spatio-temporal graphs, that represent qualitative spatio-temporal relations between interacting objects. A wide range of spatio-temporal relations are introduced, as being well suited for describing video activities. Then, a formulation is proposed, in which standard problems in video activity understanding such as event detection, are naturally mapped to problems in graph based relational learning. Experiments on video understanding tasks, for a video dataset consisting of common outdoor verbs, validate the significance of the proposed approach.

1 Introduction

One of the goals of AI is to enable machines to observe human activities and understand them. Many activities can be understood by an analysis of the interactions between objects in space and time. The authors in [13][14] introduce a representation of interactions between objects, using perceptually salient discretizations of space-time, in the form of qualitative spatio-temporal relationships. Then, they apply relational learning to learn event classes from this representation. This approach to understanding video activities using a qualitative spatio-temporal representation and relational learning is an alternative to much research on video activity analysis, which has largely focussed on a low-level pixel based representations e.g. [17].

This paper expands the scope of this research in the following two ways. Firstly, building on previous work [14], that has restricted itself to just simple topological relations, this work draws from a body of research in qualitative spatial relations [11][2], and proposes that these relations provide a natural way of representing video activities. This aspect is described in section 2. Secondly, this paper presents a general way of translating standard problems in video activity analysis [9] to problems in relational graph learning [4][1], by extending the application of a novel formulation proposed in [14]. This aspect is described in section 3. Sections 4 describes experimental analysis on real data. Section 5 concludes this chapter with pointers to future research.

[1] While this paper concentrates on graph based relational learning for reasons given below, we believe that this analysis can be carried over to logic based relational learning [7] [10].

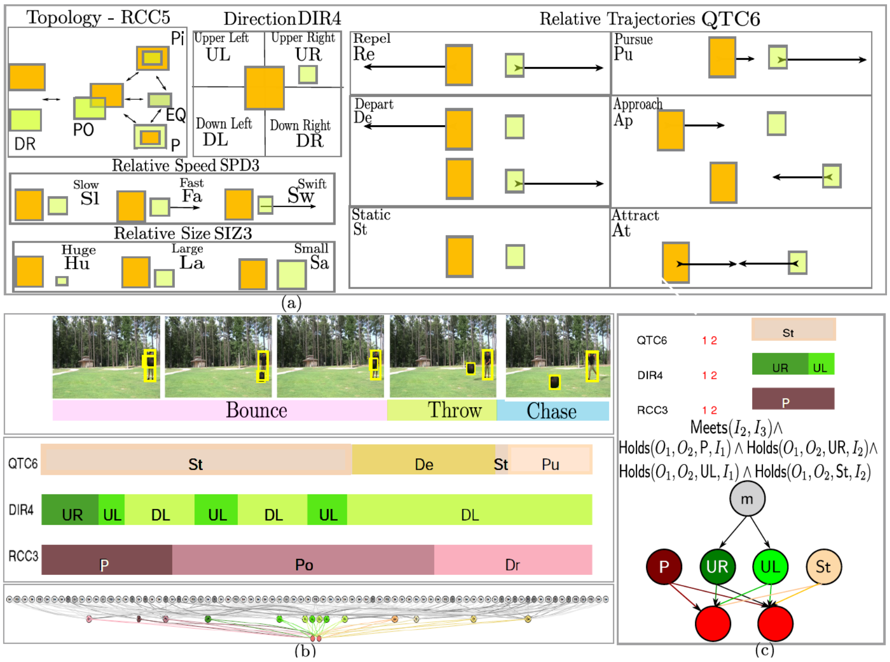


Fig. 1: (a) Five qualitative spatial relationships:(i)topology; (ii) direction; (iii) relative speed; (iv) relative size; (v) qualitative trajectories. (b) Three simple events: (i) bounce - characterized by a periodic change between the directional relationships UL and DL; (ii) throw - by the change from PO to DR and St to De; (iii) chase by a change from De to Pu. At the bottom is the corresponding spatio-temporal graph. (c) The same spatial relations in (b) for a short segment, its logical representation and equivalent relational interaction graph.

2 Graph Based Representation of Activities

We propose that qualitative spatial relations provide a natural way of representing interactions between objects participating in video activities. Qualitative relations form interesting features as they are the result of a particular way of discretizing quantitative measurements into qualitatively interesting concepts, such that these concepts signify *perceptually salient relationships* [3]. The problem of abstracting qualitative relations from noisy video data, is facilitated by the use of a Hidden Markov Model based framework described in [15].

Five types of relations are illustrated in Fig. 1 (a). Their suitability for describing interactions is illustrated in Fig. 1 (b). At the top of Fig. 1 (b) is a sequence of images representing the interaction between a person and a ball, namely bounce, throw and chase. Below that is shown, three "parallel sequences of episodes". An episode [13] corresponds to an interval, during which a spatial relationship holds maximally, and can be described by logic (e.g. Holds(O_1, O_2, UR, I_2) as shown in Fig. 1(c) for a shorter sub-interval of the interval shown in Fig. 1(b)). Each sequence of episodes in Fig. 1(b) and (c) correspond to one of the three different types of qualitative relations, namely topology (RCC5), relative directions (DIR4) and relative trajectories (QTC6).

An alternative to the above "sequence of episodes" based representation is to relate the intervals corresponding to each pair of episodes, using Allen's temporal relationships [1], e.g. $\mathsf{Meets}(I_2, I_3)$, as shown in Fig. 1(c). This leads to a fully relational representation capturing many, if not all, qualitatively interesting temporal dependencies.

An alternative relational representation to logical predicates is to use interaction graphs [14], as shown in Fig. 1(c). They are three layered graphs, in which the layer 1 nodes are mapped to the interacting objects. Layer 2 nodes of the interaction graph represent the episodes between the respective pairs of tracks pointed to at layer 1 and are labelled with their respective maximal spatial relation as shown in Fig. 1(c). The layer 3 nodes of the activity graph are labelled with Allens temporal relations (e.g. m : meets, in Fig. 1(c)) between intervals corresponding to certain pairs [12] of layer 2 nodes.

Interaction graphs are a computationally efficient alternative to logical predicates, as they avoid repetition of object and episode variables and also provide a well defined and computationally efficient comparison of interactions, by means of suitable similarity measure. This measure is defined using a kernel on a feature space obtained by expressing a interaction graph in terms of a bag of sub-interaction subgraphs [14].

An *activity graph* is an interaction graph that captures the spatio-temporal relationships between all pairs of co-temporally observed objects that are involved in activities for an extended duration. Note that the activity graph may also represent the spatio-temporal graph for activities in several unrelated videos for the same domain, and not necessarily one single video.

3 Graph Based Relational Learning of Activities

The authors in [14] proposed a novel relational graph based learning formulation for video activity understanding, in the context of a specific unsupervised learning task. In the following, we use this formulation to describe a general way of translating standard problems in video activity analysis to standard problems in relational graph learning. We show how it can be more generally applied, in order to address many of the standard video activity understanding tasks.

One of the key underlying hypotheses in research on video activity understanding [9] is that activities are composed of events of different types. Based on this hypothesis, tasks such as learning event class models, event classification, clustering and detection are defined. In this work, we characterize events by a set of co-temporal tracklets (a tracklet is a one-piece segment of a track). Events having similar spatio-temporal relationships between their constituent tracklets tend to belong to the same event class. The set of all event classes is called $\mathcal{C}$. A set of events E is a "cover" of a set of tracks $\mathcal{T}$ iff the union of all tracklets in E is isomorphic to $\mathcal{T}$. In general there may be coincidental interactions between objects that that would not naturally be regarded as part of any event in an event class [2]. This notion of an event cover can be regarded as an global *explanation* of the activities in a video in terms of instances of event classes.

A set of tracks $\mathcal{T}$ can be abstractly represented using an activity graph $\mathcal{A}$, as described above. An event corresponds to a subgraph of $\mathcal{A}$, such that this subgraph is also

[2] We ignore this complexity here but see [12], [14]. The final paper would contain details of how coincidences can be incorporated into the learning algorithms. Here, there is no space to give further details here.

an interaction graph. An event cover in this formulation, thus becomes a set of interaction graphs, whose union is $\mathcal{A}$. These interaction graphs are called event graphs[3]. Similar event graphs tend to belong to the same event class. An *event model* is defined for the set of event classes $\mathcal{C}$, according to which, each class is a probability distribution over a finite set of interaction graphs. Finally, *observation noise* is modelled by allowing multiple possible activity graphs $\mathcal{A}$, for the same set of observed tracks $\mathcal{T}$. This formalism has been used to model the joint probability distribution of the above variables $\{\mathcal{C}, \mathcal{G}, \mathcal{A}, \mathcal{T}\}$ as:

$$P(\mathcal{C}, \mathcal{G}, \mathcal{A}, \mathcal{T}) \approx P(\mathcal{C})P(\mathcal{G}|\mathcal{C})P(\mathcal{A}|\mathcal{G}, \mathcal{C})P(\mathcal{T}|\mathcal{A})$$

We now apply this formulation to address the above video understanding tasks in terms of relational graph learning. The task of *learning an event model* translates to learning an event model for event classes $\mathcal{C}$ given $\mathcal{A}$ and a corresponding $\mathcal{G}$. A MAP formulation of this problem is

$$\hat{\mathcal{C}} = \arg\max_{\mathcal{C}} P(\mathcal{C})P(\mathcal{G}|\mathcal{C})$$

In this work, we learn a generative event model in the form of a simple mixture of Gaussians, in both supervised and unsupervised settings. In the unsupervised setting, we used a Bayesian Information Criterion to automatically determine the number of classes. More generally, techniques related to graph classification [5] [6] [8] and clustering [16], may be applied.

The *video event detection task* corresponds to the case, when given an event model $\mathcal{C}$, the goal is to detect the events, or more generally, learn a labelled cover $\mathcal{G}$, where the labels correspond to one of the event classes in $\mathcal{C}$, that is:

$$\hat{\mathcal{G}} = \arg\max_{\mathcal{G}} P(\mathcal{G}|\mathcal{C})P(\mathcal{A}|\mathcal{G}, \mathcal{C})$$

In this work, we form the cover $\mathcal{G}$, by simply searching for subgraphs in the activity graph that are most likely given the event model $\mathcal{C}$. That is, we find those graphs $g \in \mathcal{G}$ for which the likelihood $P(g|\mathcal{C})$, is above a threshold. We also simply assume a uniform distribution $P(\mathcal{A}|\mathcal{G}, \mathcal{C})$ for all possible event graph covers $\mathcal{G}$.

In a more general unsupervised video understanding setting, the goal is to learn the unknowns: $\mathcal{G}$, $\mathcal{C}$ and $\mathcal{A}$, given only the observed tracks $\mathcal{T}$, that is:

$$(\hat{\mathcal{C}}, \hat{\mathcal{G}}, \hat{A}) = \arg\max_{\mathcal{C}, \mathcal{G}, \mathcal{A}} P(\mathcal{C})P(\mathcal{G}|\mathcal{C})P(\mathcal{A}|\mathcal{G}, \mathcal{C})P(\mathcal{T}|\mathcal{A})$$

A Markov Chain Monte Carlo (MCMC) procedure is used in [14] to find the MAP solution. MCMC is used to efficiently search the space of possible activity graphs, possible covers of the activity graph and possible event models, in order to find the MAP solution.

4 Experiments

A real video dataset consisting of activities representing simple verbs such as throw (a ball), catch etc is used to evaluate the proposed approach. The dataset consists of 36 videos. Each video lasts for approximately 150–200 frames and contains one or more

[3] In practical situations, with co-temporal events, there will be co-incidental interaction graphs, which are a part of $\mathcal{A}$, but not a part of any event graph. We leave further details of this to the full paper.

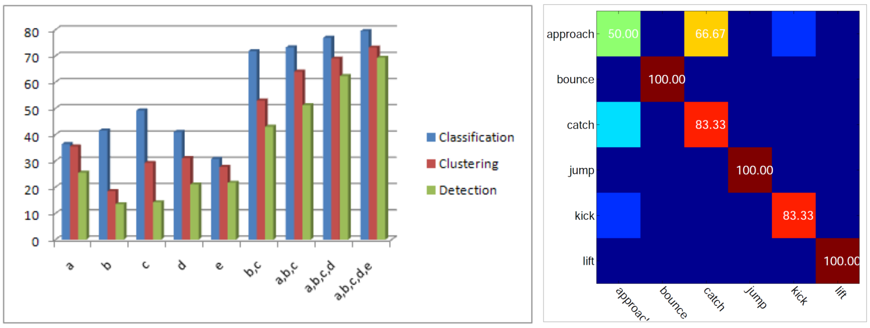


Fig. 2: Left: Accuracies for three tasks - classification, clustering and detection - for possible combinations of spatial relationships are shown. In order to make the results visually legible, only the top ranking combination for a fixed number of combinations are shown. The letters are given by letters: a - RCC5, b - QTC6, c - DIR4, d - SPD3, e - SIZ3 (See Fig. 1 (a) for further explanation of these acronyms). Right: Confusion matrix for the classification task.

of the following 6 verbs: approach, bounce, catch, jump, kick and lift. A ground truth, in terms of labelled intervals corresponding to each of the constituent verbs, in each of these videos is available. We process the dataset by detecting objects of interest using a multi-class object detector and then track the detected blobs.

This dataset is used to evaluate how possible combinations of these features perform for three of the learning tasks - event classification, event clustering and event detection - that arise out of the proposed formulation described above. In order to evaluate the performance of event recognition, a leave-one out cross validation scheme is adopted. For the classification task, an event model in terms of the interaction graphs, is learned from the training videos, in a supervised way using the available class labels. The interaction graph for the video corresponding to the test segment is classified using the learned event model. The classified label for the test segment evaluated against the ground truth label for this segment, in order to compute the average accuracy across different folds. In order to evaluate clustering, the segments for all the available videos are clustered and the accuracy of clustering is evaluated using Rand Index. Finally, the detection task is evaluated by a leave one out procedure, which uses 35 videos for training the event model. The event model is used to detect the events in the remaining video. An event is regarded as being detected if the detected interval overlaps the ground-truth interval by more than 50%.

The results for the classification, clustering and detection tasks are shown in Fig. 2 (left), for different combinations of spatial relationships. These results show that for all three learning tasks, the combination of all five types of qualitative spatial relations results in maximum accuracies. The results for the classification task for each of the six verbs is shown with the help of a confusion matrix in Fig. 2 (right). It can be seen that apart from the verb “approach”, which gets confused with “catch”, the rest of the verbs are classified with reasonably high accuracies.

5 Summary and Future Work

This paper firstly demonstrates the role of different types of qualitative spatio-temporal relations in bridging the gap between low level video input and high level activity understanding has been demonstrated. One direction for future research is to investigate the role of other qualitative relations and their role in representing activities. Another interesting direction is to model human actions by considering relationships between body parts. These body parts could be obtained using part-based models.

Another contribution is that this paper presents a general way of addressing problems in video activity understanding using graph based relational learning. In the future, it would be interesting to extend this formalism to other tasks in activity understanding such as anomaly detection, scene description and gap filling.

References

1. Allen, J.: Maintaining knowledge about temporal intervals. Commun. ACM 26(11), 832–843 (1983)
2. Cohn, A.G., Hazarika, S.M.: Qualitative spatial representation and reasoning: An overview. Fundamenta Informaticae (2001)
3. Cohn, A.G., Magee, D., Galata, A., Hogg, D.C., Hazarika, S.: Towards an architecture for cognitive vision using qualitative spatio-temporal representations and abduction. pp. 232–248 (2003)
4. Cook, D.J., Holder, L.B.: Mining Graph Data. Wiley-Interscience (2007)
5. Deshpande, M., Kuramochi, M., Wale, N., Karypis, G.: Frequent substructure-based approaches for classifying chemical compounds. IEEE Transactions on Knowledge and Data Engineering (TKDE) 17(8), 1036–1050 (2005)
6. Gärtner, T., Flach, P.A., Wrobel, S.: On graph kernels: Hardness results and efficient alternatives. In: Proceedings of the Conference On Learning Theory (COLT). pp. 129–143 (2003)
7. Getoor, L., Taskar, B.: Introduction to Statistical Relational Learning (2007)
8. Kudo, T., Maeda, E., Matsumoto, Y.: An application of boosting to graph classification. In: Proceedings of Neural Information Processing Systems (NIPS) (2004)
9. Lavee, G., Rivlin, E., Rudzsky, M.: Understanding video events: a survey of methods for automatic interpretation of semantic occurrences in video. IEEE Transactions on Systems, Man, and Cybernetics pp. 489–504 (2009)
10. Raedt, L.D., Kersting, K.: Probabilistic inductive logic programming (2008)
11. Randell, D.A., Cui, Z., Cohn, A.G.: A spatial logic based on regions and connection. In: Proceedings of the Conference on Knowledge Representation and Reasoning (KR) (1992)
12. Sridhar, M.: Unsupervised Learning of Event and Object Classes from Video. University Of Leeds, http://www.comp.leeds.ac.uk/krishna/thesis.pdf
13. Sridhar, M., Cohn, A.G., Hogg, D.C.: Learning functional object-categories from a relational spatio-temporal representation. In: Proceedings of the European Conference on Artifical Intelligence (ECAI) (2008)
14. Sridhar, M., Cohn, A.G., Hogg, D.C.: Unsupervised learning of event classes from video. In: Proceedings of the AAAI Conference on Artificial Intelligence (AAAI) (2010)
15. Sridhar, M., Cohn, A.G., Hogg, D.C.: From video to RCC8: exploiting a distance based semantics to stabilise the interpretation of mereotopological relations. Proc. COSIT, In Press (2011)
16. Tsuda, K., Kurihara, K.: Graph mining with variational dirichlet process mixture models. In: Proceedings of SIAM International Conference on Data Mining (2008)
17. Wang, X., Ma, X., Grimson, E.: Unsupervised activity perception in crowded and complicated scenes using hierarchical Bayesian models. IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI) (2009)

Physical Puzzles—Challenging Spatio-Temporal Configuration Problems

Arne Kreutzmann and **Diedrich Wolter**
SFB/TR 8 Spatial Cognition, University Bremen

Abstract

This paper serves to promote studying spatio-temporal configurations problems in physical domains, called physical puzzled for short. The kind of physical puzzles we consider involve simple objects that are subject to the laws of mechanics, restricted to what is commonly considered to be accessible by common sense knowledge. Our problem specification involves several unknowns, creating uncertainty which inhibits analytic construction of solutions. Instead, tests need to be carried out in order to evaluate solution candidates. The objective is to find a solution whilst minimizing the number of tests required.

1 Introduction

Qualitative representations aim to capture human common-sense understanding and to enable efficient symbolic reasoning processes. Qualitative representations abstract from an overly detailed domain by only distinguishing between an essential set of meaningful concepts. Qualitative approaches are widely acknowledged for their ability to abstract from uncertainty, for example an uncertain measurement of a location can become a certain notion of region membership. Naturally, different tasks may call for different qualitative concepts to describe the state of affairs. This task-dependency lead to the development of a wide range of qualitative representations of space and time—see [Cohn and Renz, 2007] for an overview.

When benchmarking qualitative representation and reasoning it appears natural to consider adequacy of representation as well as effectiveness and efficiency of reasoning. Since qualitative representations are meant to provide us with a formal model for common-sense reasoning, we argue for studying problems which are easy to solve for humans but hard for computers. To this end, we examine spatial configuration problems in the physical domain, i.e., problems in which objects need to be arranged in a certain way in order to achieve a specific goal (like making a ball hit a goal). As claimed by Bredeweg and Struss, "reasoning about, and solving problems in, the physical world is one of the most fundamental capabilities of human intelligence and a fundamental subject for AI" [Bredeweg and Struss, 2003]. Problem solving in a physical context can thus be considered a well-suited benchmark domain for AI. The physical domain is also key to qualitative reasoning. As Williams and de Kleer put it, "[...] the heart of the qualitative reasoning enterprise is to develop computational theories of the core skills underlying engineers, scientists, and just plain folks's ability to hypothesize, test, predict, create, optimize, diagnose and debug physical mechanisms" [Williams and de Kleer, 1991]. Solving puzzles has some tradition is AI research. Recently, Cabalar and Santos accounted puzzles as an well-suited test bed for their ability to present challenging problems in small packages [Cabalar and Santos, 2011]. We argue for studying physical puzzles that involve dynamics, in particular we consider the problem of configuring an environment by arranging objects to make a ball bounce into a pre-defined goal region. A similar kind of bouncing ball problem also served as example to motivate the poverty conjecture in qualitative reasoning [Forbus *et al.*, 1991]. In the light of today's state of the art in qualitative spatial reasoning, Cohn and Renz take a more differentiated point of view [Cohn and Renz, 2007]. Therefore, we regard physical puzzles to be the domain of choice for evaluating advances in qualitative reasoning.

2 The Physical Puzzle Domain

Our proposal has been inspired by computer games that, among other difficulties, confront a player with tricky physical problems that involve spatio-temporal reasoning as well as reasoning about action and change. Two games exemplify the genre of physical puzzles we propose to study: the game Deflector published by Vortex Software in 1987 (see Figure 1 for a screenshot of the Commodore 64 version) requires the player to arrange a set of rotatable mirrors in such a way as to make a laser beam hit balloons. Hitting all balloons (and thereby making them burst) clears a level. This kind of puzzle is a purely spatial one. Obstacles placed in the level make it hard to foresee which mirror setup is required to point the laser to a specific point in space. Whilst this problem can be solved purely using computational geometry, the state space is too large to be enumerated by humans. Human players need to employ some means of heuristics and reasoning in order to construct solutions. The second game, called Crazy Machines developed by FAKT Software is similar but involves a complex physical domain (see Figure 2 for a screenshot). The objective is to arrange objects in such a way that they exhibit certain functionality (for instance, making a set of balloons

Figure 1: Screenshot of the computer game Deflektor in which rotatable mirrors have to be arranged such that all balloons (grey) are destroyed by the laser beam (yellow).

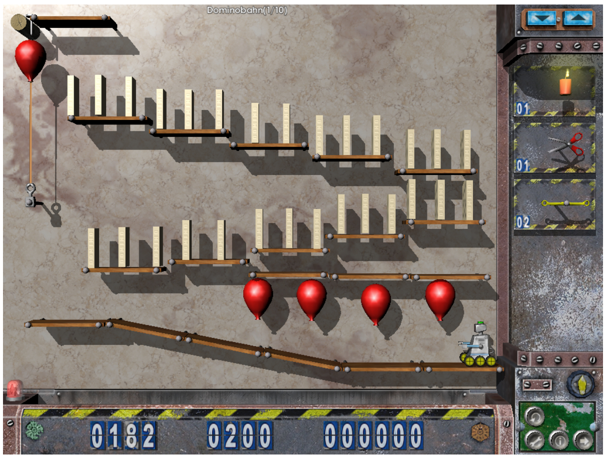


Figure 2: An exemplary physical puzzles from the computer game Crazy Machines. The objects to the right (a burning candle, scissors, and two levers) have to placed in such a way that all balloons burst. The robot in this puzzle is capable of carrying one object and it will only move straight on, eventually falling off the platform.

burst). This game is not only more complex as it involves a variety of physical laws (gravity, magnetism, etc.), but it also involves many unknowns. No physical constants like friction coefficients, mass, or density are known.

When details of the underlying physical model are unknown or cannot be handled computationally[1], a solution cannot be determined by a single computation. Instead it becomes necessary to first construct trial solutions and study how they perform. In order to position a trampoline such that it gets hit by a ball falling down (see Fig. 3 for illustration), it is necessary to first observe where the ball hits the ground. Then the trampoline can then be placed accordingly. Aside from such simple variations of where exactly to place the trampoline between objects O_1 and O_2, more complex relationships need to be assessed too. Again considering Fig. 3, it is by no

[1] for example, if inverse kinematics cannot be handled analytically

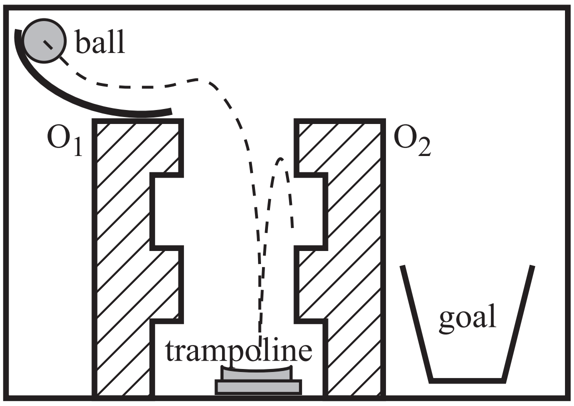


Figure 3: A physical puzzle in which trampolines need to placed to make a ball reach a goal area

means easy to see whether placing a trampoline on the ground between O_1 and O_2 would help to make the ball bounce over the obstacle O_2 in order to reach the goal area. In conclusion, unknowns introduce uncertainty on two levels, on the numerical level of fine-tuning a solution and on the qualitative level.

2.1 Reasoning in Physical Puzzles

Reasoning can help in different ways to solve a physical puzzle. First of all, qualitative assessment of a trial can help to guide the search for the right choice of parameters. To this end, a representation of some basic physical knowledge is required. From such background knowledge one can infer whether a parameter like the position of a trampoline needs to be shifted to the left or to the right. The same approach can also help to recognize that fine-tuning parameters will not lead to a solution. For example, if the trampoline does not make the ball jump high enough (top vertex of the parabola-like trajectory is not above O_2), it is pointless to fine-tune how the trampoline bounces back the ball.

More importantly, reasoning on the qualitative level can also help to identify solution candidates. To this end any solution to the physical puzzle also needs to be a solution of the qualitative abstraction of the puzzle. This allows for a generate-and-test approach based on qualitative reasoning. First, solution candidates are generated on the qualitative level and then it is studied by trials whether it the solution candidate is realizable in the concrete physical context given. A similar approach has recently been described by [Westphal *et al.*, 2011] in context of spatial planning.

To foster reasoning we treat the physical world as a black box and aim to minimize the number of trials required. We note that the necessity of performing trials is not an artificial burden but it is also common in engineering problems. Even when all physical effects involved are known, one might not be able to create reliable computer models. It is up to the researcher developing a reasoner (or up to the engineer, respectively) to minimize the amount of experiments necessary.

Even simple physical puzzles involve a dense and complex structured search space that does not become tractable until

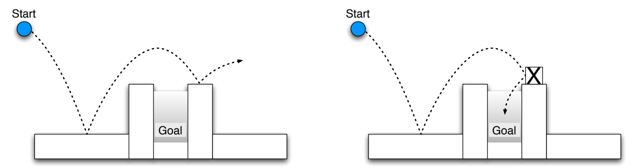


Figure 4: Specialized (extended) Bouncing Ball Puzzles BBP^0 (left) and $eBBP^0$ (right)

reasoning is applied. The level of difficulty can easily be fine-tuned by changing the number of static obstacles and by limiting the set of objects that can be placed. We conclude that physical puzzles are an excellent problem to study the utility of approaches to qualitative representation and reasoning.

3 Problem Specification

The physics considered in this proposal are the physic of rigid objects including gravity. For the sake of simplicity we only consider the task of throwing a ball into a basket. In order to solve a puzzle, one can change how the ball is thrown from a fixed start position (by choosing the initial velocity vector) and one may alter a given scene by placing objects from a given set of objects.

Definition 1. *A* trajectory *is a continuous function* $T : \mathbb{R}_0^+ \to \mathbb{R}^n$. *Let* $\mathfrak{T}^n$ *be the set of all trajectories in* $\mathbb{R}^n$.

A trajectory does not have to be continuously differentiable. In the kind of problems we consider we can regard all trajectories to converge to a fixed end position in finite time, thus trajectories can be represented as finite polygonal curves.

Definition 2. *A* simulation *is a function* $S^n : \mathfrak{C} \times \mathfrak{F} \times \mathfrak{U} \to \mathfrak{T}^n$ *mapping a set of problem parameters to a trajectory. We call* $\mathfrak{C}$ *the set of configuration parameters,* $\mathfrak{F}$ *and* $\mathfrak{U}$ *are sets of fixed problem parameters of which* $\mathfrak{U}$ *is called unknowns.*

Definition 3. *A* physical puzzle *is the tuple* $< S^n, \mathfrak{C}, F, G >$, *where* S^n *is a simulation,* $F \in \mathfrak{F}$, $U \in \mathfrak{U}$, *and* $G \subset \mathbb{R}^n$ *a set of goal positions. A configuration* $C \in \mathfrak{C}$ *is called a* solution *for a particular* U *iff* $\lim_{t\to\infty} S^n(P, F, U)(t) \in G$.

In the following the dimension n will be omitted and defaults to 2. When benchmarking a solution strategy to solve a puzzle, the number of tries is counted, i.e., calls to the S function until finding a solution. For reasons of comparability evaluations should be performed on a number of puzzles variants and involve statistics. We now list a set of distinct types of physical puzzle problems order by increasing difficulty.

The Simple Bouncing Ball Puzzle (sBBP) In this puzzle only two free parameters are available to control how the ball is thrown from a fixed start position: the angle and the size of the initial velocity vector. We single out two special variants:

sBBP^0 the goal area is enclosed be obstacles but can be entered freely by a ball from above (see Figure 4).

sBBP^{-1} this is the normal BBP but without gravity

The Bouncing Ball Puzzle (BBP) In this problem a number of objects can be placed to alter the path of the ball. Some special cases of these puzzles can be identified:

BBP^0 analogous to sBBP^0, but the ball has a fixed initial velocity. It can only be guided into the goal area by placing objects (see Figure 4).

BBP^{-1} BBP without gravity

4 Conclusion

In this proposal we argue for solving configuration problems in the physical domain to benchmark qualitative representation and reasoning techniques. Solving problems in physics of rigid objects is largely of spatio-temporal nature, but it involves unknowns, resulting in a steep scaling behavior with respect to problem complexity. We have chosen the physical world as our domain as it covers spatial, temporal, and general qualitative reasoning. Furthermore, it gives rise to the ultimate benchmark: to defeat human problem solvers in computer games. Using a physical simulator that takes quantitative input parameters and produces a quantitative output we allow different qualitative representations to be applied and successful puzzle solvers can also be expected to be relevant for serious applications.

Acknowledgements

This work is supported by the Deutsche Forschungsgemeinschaft (DFG) in context of the transregional collaborative research center *Spatial Cognition*. Financial support is gratefully acknowledged.

References

[Bredeweg and Struss, 2003] B Bredeweg and P Struss. Current topics in qualitative reasoning. *AI Magazine*, jan 2003.

[Cabalar and Santos, 2011] Pedro Cabalar and Paulo E. Santos. Formalising the fisherman's folly puzzle. *Artificial Intelligence*, 175(1):346–377, 2011.

[Cohn and Renz, 2007] Anthony G. Cohn and Jochen Renz. Qualitative spatial representation and reasoning. In F. van Harmelen, V. Lifschitz, and B. Porter, editors, *Handbook of Knowledge Representation*, pages 551–596. Elsevier, 2007.

[Forbus *et al.*, 1991] Kenneth D. Forbus, P. Nielsen, and B. Faltings. Qualitative spatial reasoning: The clock project. *Artificial Intelligence*, 51:417–471, 1991.

[Westphal *et al.*, 2011] Matthias Westphal, Christian Dornhege, Stefan Wölfl, Marc Gissler, and Bernhard Nebel. Guiding the generation of manipulation plans by qualitative spatial reasoning. *Spatial Cognition & Computation: An Interdisciplinary Journal*, 11(1):75–102, 2011.

[Williams and de Kleer, 1991] Brian C. Williams and Johan de Kleer. Qualitative Reasoning About Physical Systems - a Return to Roots. *Artificial Intelligence*, 51:1–9, 1991.

Streets to the $\mathcal{OPRA}$— Finding your destination with imprecise knowledge

Dominik Lücke
University of Bremen
SFB/TR8
Bremen, Germany

Till Mossakowski
DFKI GmbH Bremen
Safe and Secure Systems
Bremen, Germany

Reinhard Moratz
National Center for
Geographic Information and Analysis
Department of Spatial Information
Science and Engineering
Orono, Maine, USA

Abstract

Qualitative spatial calculi offer a method to describe spatial configurations in a framework based on a finite set of relations that abstracts from the underlying mathematical theory. But an open issue is whether they can be employed in applications. Further their cognitive adequacy is questionable or not investigated at all. In this paper we investigate the applicability of $\mathcal{OPRA}$ to navigation in street networks that are described via local observations. Further we scrutinize whether a description of directions that is deemed cognitively adequate and can be described in $\mathcal{OPRA}$ can perform that task. We are using an environment that we developed ourselves for these experiments, the used algorithms and the program itself are explained in detail.

1 Introduction

Since the emergence of Allen's interval algebra [Allen, 1983] qualitative spatial and temporal reasoning has become an interesting field in artificial intelligence research. A lot of the tools used and later refined for reasoning tasks has already been introduced by Allen, i.e. composition based reasoning. A multitude of qualitative spatial and temporal calculi have been defined dealing with different aspects of space and time. In the field of spatial calculi, we can spot two big classes of calculi, this is the ones dealing with topological aspects of space like $\mathcal{RCC}$ [Randell *et al.*, 1992] and others dealing with directions either with a local or global reference frame. $\mathcal{OPRA}$ is a calculus dealing with directions having a local reference frame. It is based on oriented points, i.e. points in the plane that have a position and an orientation. A feature of the $\mathcal{OPRA}$ calculus is its adjustable granularity, in fact for each $m \in \mathbb{N}$ with $m \geq 1$ a version of the $\mathcal{OPRA}$ calculus exists. The reference frame for $\mathcal{OPRA}_2$ is shown in Figure 1. The position of the basic entity of the $\mathcal{OPRA}$ calculus, the *oriented point*, is shown as the black dot in the middle and its direction as the arrow. $\mathcal{OPRA}_2$ means that the plane is divided into sectors by two intersecting lines with all angles between adjacent lines being the same. The lines and their intersection point divide the plane into one sector that is a point (the intersection point itself) four sectors on the lines and four

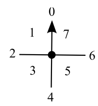


Figure 1: $\mathcal{OPRA}_2$ reference frame

planar sectors. If we call the point in Figure 1 A and another point B, we can determine in which sector B with respect to A lies. With rising granularity the relations of the $\mathcal{OPRA}$ calculus grow finer and finer and their number rises making reasoning very time consuming.

Although there are many qualitative spatial calculi and even more publication about them, only initial steps have been made towards applicability of qualitative spatial calculi to problems that arise in the real world. Moreover, for many calculi it is known that *algebraic closure* only approximates consistency, but it is not know if this approximation is "good enough" for tasks at hand.

We investigate the applicability of the $\mathcal{OPRA}$ calculus (with reasonable granularity) to navigation problems in a street network. For this task, we only rely on knowledge that a person can observe at the *decision points*, i.e. the crossings, of a street network in a qualitative way. In Figure 2 such a crossing is shown. The person driving in the car knows where

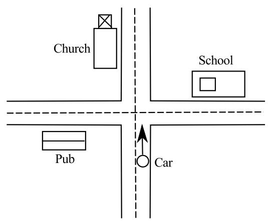


Figure 2: A crossing

she comes from and can observe that the street with the pub is to the left, the one with the church is straight ahead and the one with the school is to the right. But she cannot observe where the airport at the other end of the city is with respect to this. Further knowledge can be deduced from the observed one, but that knowledge is only as good as is the reasoning for the calculus at hand. The have to ask the question, if this knowledge is good enough. What helps us in this case is the fact that we are navigating in a grid that is pre-defined by the given street network. But there are still open questions, is the "straight ahead" or "left" defined by $\mathcal{OPRA}$ the "straight ahead" or "left" as perceived by humans.

As an overall scenario consider that a swarm of robots is exploring an unknown street network (or interior of a building). The robots can make observations at any crossing with respect to a qualitative calculus (in our case $\mathcal{OPRA}$) and they know what a street looks like, i.e. the connections between crossings. The robots can exchange and integrate the data they obtained, but they cannot triangulate their positions. When their work is done, a network of local observations is obtained, but nothing is known so far about non-local constraints. In that this means that all these non-local constraints are only restricted by the universal relations so far. So the issue is the non-existence of non-local knowledge in our network. It is desirable to refine those universal relations in a way that all relations that cannot hold with respect to the algebraic properties of the calculus at hand are thrown out. The standard approach in qualitative spatial reasoning is applying algebraic closure on the network. This approach is basically just an approximation, but this approximation might be good enough.

Research on "wayfinding choremes" by A. Klippel et al. [Klippel and Montello, 2007; Klippel *et al.*, 2005] claims a *cognitively adequate* representation of directions on decision points, i.e. crossings in our street networks. Basically there are 7 choremes that describe turning situations at crossings as depicted in Figure 3. These choremes are ignorant of the

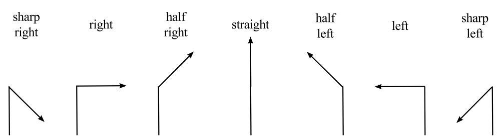


Figure 3: The seven wayfinding choremes

situation of "going back", which is formalized in $\mathcal{OPRA}$. Furthermore, for our navigation task the situation of running into a dead end can always appear and we need the possibility of turning around and leaving that dead end. The derivation of these choremes in based upon a sectorization of a circle as shown in Figure 4. With these sectors we would have

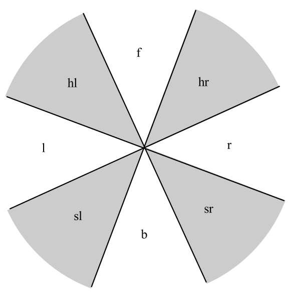


Figure 4: Sectors of a circle for wayfinding choremes

the choice of directions from l, r, f, b in Figure 2, sharp or half turns do not occur there. This sectorization clearly has a "back" sector and is quite close to the definition of the $\mathcal{OPRA}$ relations. The main difference is the lack of relations on a line. The size of the sectors in Figure 4 is only approximately described by Klippel. We are going to simulate these sectorization by $\mathcal{OPRA}$ relations of adequate granularity. Where the choice of granularity is a tradeoff between the minimum size of sectors and reasoning efficiency. We will use these Klippel's sectors encoded in $\mathcal{OPRA}$ to navigate our street network and examine its impact on the reasoning qualities.

We apply our techniques for techniques for deriving observations in $\mathcal{OPRA}$ and in the representation of Klippel's sectors in $\mathcal{OPRA}$ to test data to gain knowledge their fitness for navigation tasks in street networks. Since we believe that the best test data for street networks are the real ones, we use descriptions of street networks compiled out of maps from OpenStreetMap[1].

2 The $\mathcal{OPRA}$ calculus

The basic entity of the $\mathcal{OPRA}$ calculus are *oriented points*, these are points that have a position given by coordinates and an orientation. This orientation can be given as an angle with respect to an axis. A configuration of oriented points is shown in Figure 5.

Definition 1 (Oriented Point)**.** An *oriented point* is a tuple $\langle p, \varphi \rangle$, where p is a coordinate in $\mathbb{R}^2$ and φ an angle to an axis.

We also can describe an oriented point as a tuple of points $\langle p_0, p_i \rangle$ being located at p_0 and pointing to p_1. hence the direction is given by the vector from p_0 to p_1. From this description, we can compute the angle φ to the axis easily. By disregarding the lengths of the vectors, we arrive at Definition 1. The $\mathcal{OPRA}$ calculus defines relations between such

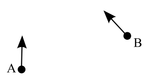


Figure 5: Oriented points

pairs of oriented points. These relations are of adjustable granularity, where this granularity is denoted by the index m of $\mathcal{OPRA}_m$. For the introduction of relations the plane around each oriented point is sectioned by m lines with one of them having the same orientation φ as the oriented point. The angles between all lines have to be equal. The sectors are numbered from 0 to $4m - 1$ counterclockwise. The label 0 is assigned to the direction with the same orientation as the oriented point itself. Such a sectioning is shown in Figure 6 this is in fact Figure 5 with the sectioning introduced. In fact, we introduce a set of angles

$$\bigcup_{0 \le i < 2m} \left\{ \left[i\frac{\pi}{m} \right], \left] i\frac{\pi}{m}, (i+1)\frac{\pi}{m} \right[\right\}$$

[1] http://www.openstreetmap.org/

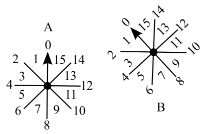


Figure 6: Oriented points with sectors

to partition the plane into the described sections. To introduce $\mathcal{OPRA}$ relations between two oriented points o and q, we need to distinguish between the two cases, if $\mathrm{pr}_1(o) = \mathrm{pr}_1(q)$ or not, where pr_1 is the projection to the first component of a tuple. I.e. we need to distinguish if both points have the same position in the plane.

A good auxiliary construction to introduce $\mathcal{OPRA}_m$ relations are *half relations*.

Definition 2. For two oriented points o and q we call $o \rhd q$ the *half relation* from o to q.

If we want to annotate a sector i or granularity m to a half relation, we shall write $o_m \rhd_i q$. A half relation determines the number i of the sector around o where q lies in if $\mathrm{pr}_1(o) \neq \mathrm{pr}_1(q)$ and the sector around o into q points into, if $\mathrm{pr}_1(o) = \mathrm{pr}_1(q)$. E.g. in Figure 6 the oriented points B lies in sector 13 of A and we obtain the half relation $A_4 \rhd_{13} B$. And for A with respect to B we get $B_4 \rhd_3 A$.

First we consider the case of $\mathrm{pr}_1(o) \neq \mathrm{pr}_1(q)$. We then get the $\mathcal{OPRA}_m$ relation from o to q as the product of $o_m \rhd_i q$ and $o_m \rhd_j q$, we will write this as $o_m\angle_i^j$. And for $\mathrm{pr}_1(o) = \mathrm{pr}_1(q)$, we get the $\mathcal{OPRA}_m$ relations as the product of $o\ s\ q$ and $o_m \rhd_j q$ written as $o_m\angle_s^j q$, where s is a special symbol describing the coincidence of the position of points.

The composition and converse tables for $\mathcal{OPRA}$ need to be calculated for any granularity of this calculus, fortunately there is a quite efficient algorithm for this task [Mossakowski and Moratz, to appear].

3 Factorizing the $\mathcal{OPRA}$ to cognitive adequacy

Investigations of Alexander Klippel et al. [Klippel *et al.*, 2005] investigated sector models as shown in Figure 7 for

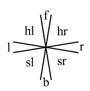


Figure 7: Klippel's relations

navigation tasks and claim their cognitive adequacy. They are using eight sectors

f	front
hl	half left
l	left
sl	sharp left
b	back
sr	sharp right
r	right
hr	half right

for their model. Nothing is said about the treatment of the borders of the sectors, i.e. about which sector the separating line belongs to, if it belongs to any. This question needs to be solved for simulating such a sectioning by a qualitative spatial calculus.

We are encoding Klippel's approach into $\mathcal{OPRA}_8$ (see Figure 8) and $\mathcal{OPRA}_{16}$ to be able to define f, b, l and r

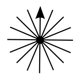


Figure 8: $\mathcal{OPRA}_8$

sectors that are suitably small and to get constraint network sizes that still can be handled by algebraic reasoners. The reasoner `GQR` [Gantner *et al.*, 2008] already needs 14GB of memory to start up with the $\mathcal{OPRA}_{16}$ composition table, without precaching the composition table for all general relations. For having suitably small sectors, we unite the $\mathcal{OPRA}_m$ ($m \in 2^n$ and $n > 2$) sectors via a mapping d as following.

$$\begin{array}{rcl} f & \mapsto & \{0, 1, 2, 4m-1, 4m-2\} \\ l & \mapsto & \{m-2, m-1, m, m+1, m+2\} \\ b & \mapsto & \{2m-2, 2m-1, 2m, 2m+1, 2m+2\} \\ r & \mapsto & \{3m-2, 3m-1, 3m, 3m+1, 3m+2\} \end{array}$$

The Klippel sectors hl, sl, sr and hr are formed by the remaining $\mathcal{OPRA}$ sectors. For $n = 2$ the sectors would overlap with this approach. We decided to add the border lines of f, b l and r to the respective relations, since this still yields sectors for these relations for $m \mapsto \infty$ for $\mathcal{OPRA}_m$. With this we would recover $\mathcal{OPRA}_2$ from Klippel's approach for $m \mapsto \infty$. To apply this sectioning to $\mathcal{OPRA}_m$, for all sets $d_1, d_2 \in d(K)$ apply $d_1 \times d_2$ where K are Klippel's sectors, and add the sets $\{s\} \times d_1$ we call these sets D. From these sets of sectors we can easily define predicates $p_1 \ldots p_8$ that are $true$ if and only if a certain $\mathcal{OPRA}_m$ relation belongs to such a set lifted to $\mathcal{OPRA}_m$.

Example 3. We want to encode Klippel's sectioning into the sectioning of $\mathcal{OPRA}_8$, which has the half relations $0 \ldots 31$. With the above definitions we obtain the mapping

$$\begin{array}{rcl} f & \mapsto & \{30, 31, 0, 1, 2\} \\ hl & \mapsto & \{3, 4, 5\} \\ l & \mapsto & \{6, 7, 8, 9, 10\} \\ sl & \mapsto & \{11, 12, 13\} \\ b & \mapsto & \{14, 15, 16, 17, 18\} \\ sr & \mapsto & \{19, 20, 21\} \\ r & \mapsto & \{22, 23, 24, 25, 26\} \\ hr & \mapsto & \{27, 28, 29\} \end{array}$$

This mapping of Klippel's sectors to the sectors of $\mathcal{OPRA}_8$ is shown in Figure 9. Please note that the $\mathcal{OPRA}_8$ emulations of f, l, b and r are still quite big sectors with 22.5°. Another drawback is that all sectors are the same size.

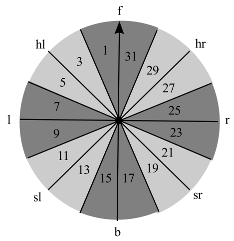


Figure 9: Mapping Klippel to $\mathcal{OPRA}$

Example 4. We can get smaller sectors by encoding Klippel's sectors into the sectors for $\mathcal{OPRA}_{16}$ as

$$\begin{array}{rcl} f & \mapsto & \{62, 63, 0, 1, 2\} \\ hl & \mapsto & \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\} \\ l & \mapsto & \{14, 15, 16, 17, 18\} \\ sl & \mapsto & \{19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\} \\ b & \mapsto & \{30, 31, 32, 33, 34\} \\ sr & \mapsto & \{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45\} \\ r & \mapsto & \{46, 47, 48, 49, 50\} \\ hr & \mapsto & \{51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61\} \end{array}$$

The sectors for f, l,b and r now have a size of $10.25°$ and the remaining sectors are bigger than them, what is closer to Klippel's intention. The issue with working with $\mathcal{OPRA}_{16}$ is already the sheer size of the composition table with 4160^2 entries and the long descriptions of constraint networks in 4160 base relations.

In the end we have a trade-off between staying close to Klippel's intentions, which can be done by a high arity $\mathcal{OPRA}$ calculus and the possibility to perform reasoning over constraint networks. But for our task of navigation the reasoning results to not have to be perfect, they just need to be good enough. Hence, we hope that on constraint networks of reasonable size $\mathcal{OPRA}_8$ and $\mathcal{OPRA}_{16}$ do the job. It would also be nice to have high arity $\mathcal{OPRA}$ calculi for having the possibility of being able to compare the impact of the size of f, l b and r in more detail.

4 From observations to a constraint network

As stated it is our aim to investigate navigation based on local observations using the $\mathcal{OPRA}$ calculus. A good source for realistic data about street networks is the world itself. We are using street networks that have been retrieved from OpenStreetMap[2], make local observations on them and formalize these observations in $\mathcal{OPRA}$. We simplify the OpenStreetMap data in the sense that we abstract from bends in streets. Our streets are just straight lines. With algebraic reasoning global knowledge can be deduced from local observations. For algebraic reasoning we use the tools `GQR` [Gantner *et al.*, 2008] and `SparQ` [Wallgrün *et al.*, 2006, 2009]. Using this overall knowledge, we navigate through the described street network.

[2] `http://www.openstreetmap.org/`

In the rest of this section we are using the street network in Figure 10 as the source for our examples. In our street

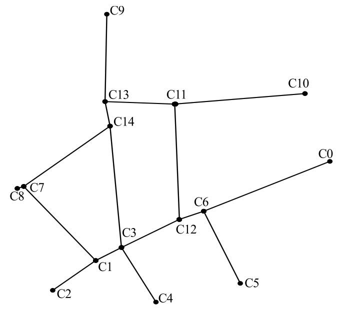


Figure 10: A street network

networks, we label crossings by C_i with $i \in \mathbb{N}$. Please note that our definition of crossings at this point includes dead-ends. In our example these are the dots. The lines depict streets between crossings. We call crossings C_i and C_j with $i \neq j$ that are connected by a street *adjacent*.

4.1 Local Observations

It is our aim to navigate with knowledge that people can make at crossings. When walking to a crossing, you know where you came from and hence your orientation. Further you can see which orientation the other streets at the crossing have with respect to your orientation. And of course you know that streets are streets with a crossing at both ends. You do not know what the situation at any other crossing looks like. This is an abstraction from very short streets.

In the first step of the formalization of our local observations we need to derive oriented points from a given street network. For any point C_i in the network determine the set A of adjacent oriented points. For any $C \in A$ introduce the oriented point $\langle C_i, C \rangle$. For the sake of brevity, we will also write C_iC for such a tuple. As described in Section 2 this representation of an oriented point still contains unnecessary information about the length of the vector from C_i to C, but this does no harm.

Example 5. Consider the network given in Figure 10 and the point C_6. The set of adjacent points to C_6 is $\{C_0, C_5, C_{12}\}$, we hence introduce the set of oriented points $\{\langle C_6, C_0\rangle, \langle C_6, C_5\rangle, \langle C_6, C_{12}\rangle\}$ or written in the short form $\{C_6C_0, C_6C_5, C_6C_{12}\}$.

In the second step, we define the streets. For each oriented point C_iC_j, we define the street via the $\mathcal{OPRA}_m$ relation $C_iC_j\ {}_m\angle_0^0\ C_jC_i$. The oriented point C_jC_i exists, since the streets in our network are not directed and hence if C_j is adjacent to C_i then C_i is adjacent to C_j.

Example 6. For a street in shown in Figure 11. This is the

Figure 11: A street

street between the points C_6 and C_0, hence we have introduced the oriented points C_6C_0 and C_0C_6 in the previous step (see Example 5) at the respective locations to point to each other. So we introduce the relation $C_6C_0 \ {}_m\angle_0^0 \ C_0C_6$.

In the third step, we add the local observations. For each oriented point C_iC_j form the set P of oriented points with for each $p \in P$ the properties $pr_1(p) = C_i$ and $pr_2(p) \neq C_j$ hold. Where pr_1 is the projection to the first component of a tuple and pr_2 to the second one. For each $p \in P$, we form the $\mathcal{OPRA}_m$ relation $C_iC_j \ {}_m\angle_s^{C_iC_j \triangleright p} \ p$. Since $C_i = pr_1(p)$, the first half-relation is clearly s, the computation of the second one will be explained in section Section 4.2.

Example 7. We again refer to Figure 10 and the oriented points introduced in Example 5. Consider the oriented point C_6C_0. For this point we get $P = \{C_6C_5, C_6C_{12}\}$ and the relations

$$C_6C_0 \ {}_m\angle_s^{C_6C_0 \triangleright C_6C_5} \ C_6C_5$$
$$C_6C_0 \ {}_m\angle_s^{C_6C_0 \triangleright C_6C_{12}} \ C_6C_{12}$$

In Algorithm 1 we show a slightly optimized version of the described algorithm where steps two and three are amalgamated.

Algorithm 1 Deriving Observations

1: $\mathcal{C}$ is the set of nodes of a street network
2: $\mathcal{S}$ the set of streets as tuples of start and end points
3: O is the set of oriented points
4: R is the set of relations
5: m is the granularity of the $\mathcal{OPRA}$ calculus
Require: $O = \emptyset$ **and** $R = \emptyset$ **and** $m > 0$
6: Require a correct description of a street network
Require: $\forall C \in \mathcal{C}.\exists s \in \mathcal{S}.C = \mathrm{pr}_1(s) \vee C = \mathrm{pr}_2(s)$
Require: $\forall s \in \mathcal{S}.\exists C_1 \in \mathcal{C}.\exists C_2 \in \mathcal{C}.s = \langle C_1, C_2\rangle \wedge C_1 \neq C_2$
7: Introduction of oriented points
8: **for all** $C \in \mathcal{C}$ **do**
9:   **for all** $s \in \mathcal{S}$ **do**
10:     **if** $\mathrm{pr}_1(s) = C$ **then**
11:       $O := O \cup \{\langle C, \mathrm{pr}_2(s)\rangle\}$
12:     **end if**
13:   **end for**
14: **end for**
15: Definition of streets and local observations
16: **for all** $o \in O$ **do**
17:   $R := R \cup \left\{o_m\angle_0^0\langle \mathrm{pr}_2(o), \mathrm{pr}_1(o)\rangle\right\}$
18:   **for all** $p \in O$ **do**
19:     **if** $\mathrm{pr}_1(o) = \mathrm{pr}_1(p)$ **and** $\mathrm{pr}_2(o) \neq \mathrm{pr}_2(p)$ **then**
20:       $R := R \cup \{o_m\angle_s^{o \triangleright p} p\}$
21:     **end if**
22:   **end for**
23: **end for**
24: **return** R

If we are working with an approach as suggested by Klippel, we add another step that replaces the $\mathcal{OPRA}$-relations by sets of relations as described in Section 4.3.

4.2 Deriving $\mathcal{OPRA}$-relations

For our observations taken in Section 4.1, we need a way to derive $\mathcal{OPRA}$-relations from tuples of points (or line segments). In particular we need is computation in Algorithm 1 Line 20, where $o \triangleright p$ was not determined so far.

By scrutinizing the definitions of the $\mathcal{OPRA}_m$ relations, we see that for any $C_kC_l \ {}_m\angle_i^j \ C_tC_v$, there is little dependence between the i and j. In fact, the only dependence is on i being s or not. We can distinguish these cases easily by determining if $C_k = C_t$ or not. If $C_k = C_t$, we know that $i = s$ and can determine j as a half relation. If $C_k \neq C_t$, there is no dependence between i and j and we can determine both via half relations. We can apply Algorithm 2 to determine $\mathcal{OPRA}$-relations between two oriented points C_kC_l and C_tC_v. The main issue that is still open is the derivation

---

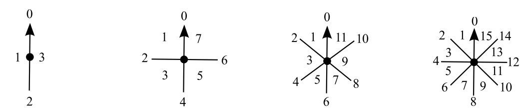
Algorithm 2 Computing $\mathcal{OPRA}$-relations

C_kC_l oriented point
C_tC_v oriented point
m granularity of $\mathcal{OPRA}$
Require: $m > 0$
if $C_k = C_t$ **then**
  return ${}_m\angle_s^{C_kC_l \triangleright C_tC_v}$
else
  return ${}_m\angle_{C_kC_l \triangleright C_tC_v}^{C_tC_v \triangleright C_kC_l}$
end if

---

of the half relations. In fact the needed calculation for the $\mathcal{OPRA}$ relations in Algorithm 1 can be reduced to this step (refer to Algorithm 1 Line 20). All other information in the involved $\mathcal{OPRA}$ relations can already be derived directly in that algorithm.

To determine the $\mathcal{OPRA}_m$ half relations between oriented points C_kC_l and C_tC_v, we determine sectors of the unit circle[3] in the Euclidean plane that correspond to those relations. Then, we compute the angle from C_kC_l to C_tC_v and determine into which sector this angle belongs. This directly yields the half relation. In Figure 12 these sectors are shown for $\mathcal{OPRA}_1$ to $\mathcal{OPRA}_4$. By inspecting the definition of



(a) (b) (c) (d)

Figure 12: Sectors of the circle for $\mathcal{OPRA}_1$ (a), $\mathcal{OPRA}_2$ (b), $\mathcal{OPRA}_3$ (c), and $\mathcal{OPRA}_4$ (d)

$\mathcal{OPRA}$ relations, we also see that half relations with an even identifier are relations on a line, while the ones with an odd identifier are relations in a plane. For an example inspect Figure 12.

The sectioning for $\mathcal{OPRA}_m$ is done by identifying an angle interval with every element of the cyclic group $\mathbb{Z}_{4m}$ as

$$[i]_m = \begin{cases} \left]2\pi\frac{i-1}{4m}, 2\pi\frac{i+1}{4m}\right[& \text{if } i \text{ is odd} \\ \left\{2\pi\frac{i}{4m}\right\} & \text{if } i \text{ is even} \end{cases}$$

[3] In fact the radius of the circle does not matter, since we are disregarding lengths.

Please note that these intervals are normalized to the representation of angles in the interval $[0, 2\pi[$. For an implementation one can create a look-up-table with the borders of the respective intervals and the respective values for i.

To compute the needed angle from $C_k C_l$ and $C_t C_v$, we form the vectors

$$\vec{a} = \begin{pmatrix} (C_k)_x - (C_l)_x \\ (C_k)_y - (C_l)_y \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} (C_k)_x - (C_t)_x \\ (C_k)_y - (C_t)_y \end{pmatrix}$$

if $C_k \neq C_t$ and

$$\vec{a} = \begin{pmatrix} (C_k)_x - (C_l)_x \\ (C_k)_y - (C_l)_y \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} (C_t)_x - (C_v)_x \\ (C_t)_y - (C_v)_y \end{pmatrix}$$

if $C_k = C_t$. The operations $(\_)_x$ and $(\_)_y$ denote the projections to the x and y coordinate of a point. The case distinction takes credit to the fact that in the case of positional equality of oriented points the angle between the orientations is used as an $\mathcal{OPRA}$ half relation. We determine the angle ϕ' from $\vec{a}$ to $\vec{b}$, we use the $\mathrm{atan2}$ function which yields values in the interval $]-\pi, \pi]$ as:

$$\phi' = \mathrm{atan2}(\vec{a}_x \vec{b}_y - \vec{a}_y \vec{b}_x, \vec{a}_x \vec{b}_x + \vec{a}_y \vec{b}_y)$$

we normalize our angles to the interval $[0, 2\pi[$ by

$$\phi = \begin{cases} \phi' + 2\pi & \text{if } \phi' < 0 \\ \phi' & \text{if } \phi' \geq 0 \end{cases}$$

to get an angle that ϕ that is compatible to the intervals in our definition of $[i]_m$. To determine the half relation for ϕ, we just need look up the appropriate interval that has been pre-calculated.

4.3 Factorizing the $\mathcal{OPRA}$-relations to cognitive adequacy

Additionally to investigating navigation with $\mathcal{OPRA}$ relations, we also want to emulate relations as proposed by Klippel [Klippel *et al.*, 2005] in $\mathcal{OPRA}_m$. For this reason, we use n unary predicates p_i with $1 \leq i \leq n$ that partition the set of the $\mathcal{OPRA}_m$ base relations. If an $\mathcal{OPRA}_m$ relation $o_m\angle_s^t q$ has been determined between o and q with Algorithm 2, we form the new relation relations

$$o \left\{ r \mid p_i(r) = p_i({}_m\angle_s^t) \text{ for } 1 \leq i \leq n \right\} q$$

where r is an $\mathcal{OPRA}_m$ relation. We do this for all pairs of oriented points that haven been introduced in Section 4.2. All other pairs are in the universal relation anyways. For this factorization adjacent sectors will be united to a single relation, but the operation involved works for all kinds of predicates, even tough the usefulness might be questionable in many cases.

5 Navigation

Having obtained a description of a street network as an $\mathcal{OPRA}$ constraint network, we are able to apply algebraic closure on them to obtain refined constraint networks. Since we are starting from consistent descriptions, we do not have to fear that algebraic closure detects inconsistencies. In fact, in the descriptions from Section 4.2 and Section 4.3 many

Algorithm 3 Factorization (to Klippel's description)

R set of determined $\mathcal{OPRA}_m$ relations
p_i with $1 \leq i \leq n$ set of predicates
R' set of output relations
Require: $R' = \emptyset$
for all $o_m\angle_s^t q \in R$ **do**
  $R_{\mathrm{tmp}} = \emptyset$
  for all ${}_m\angle_x^y \in \mathcal{OPRA}_m$ **do**
    prop $= true$
    for $1 \leq i \leq n$ **do**
      prop := prop $\wedge\, p_i({}_m\angle_s^t) = p_i({}_m\angle_x^y)$
    end for
    if prop **then**
      $R_{\mathrm{tmp}} := R_{\mathrm{tmp}} \cup \{{}_m\angle_x^y\}$
    end if
  end for
  $R' := R' \cup \{oR_{\mathrm{tmp}}q\}$
end for
return R'

universal relations are contained, since we only made local observations. E.g. the relation between $C_{13}C_9$ and C_6C_5 is universal, since these oriented points cannot be observed together locally at a crossing. Algebraic closure only approximates consistency for $\mathcal{OPRA}$, hence our refined constraint networks might be too big, but this is no issue for our navigation task, it might just lead to detours.

Starting from a refined constraint network of a street network, we want to navigate through it (hopefully without taking too many detours). We are going to apply a least angle strategy for navigation with imprecise and maybe faulty data. We can base the navigation on half relations. Just remember the definition of $\mathcal{OPRA}$ relations. If $C_k C_l \;{}_m\angle_i^j\; C_t C_v$, then $C_t C_v$ is in sector i of $C_k C_l$ with granularity m. The way backwards is of no interest for forward navigation. Based on this we introduce weights on half $\mathcal{OPRA}$ relations. Going forward and taking slight bends is normally good for such a navigation, taking sharp bends and going back is bad. We can assign the weights $w(i)$ to $\mathcal{OPRA}_m$ half relations i as

$$w(i) = \begin{cases} i & \text{if } 0 \leq i \leq 2m \\ 4m - i & \text{if } 2m < i < 4m \end{cases}$$

this yields a weight distribution that assigns the lowest weights to going forward and making slight bends.

Example 8. Consider again the sectors for $\mathcal{OPRA}_4$ in Figure 12d. Applying weights with respect to our formula yields the distribution

$$\begin{array}{rr} w(0) = 0 & w(5) = w(11) = 5 \\ w(1) = w(15) = 1 & w(6) = w(10) = 6 \\ w(2) = w(14) = 2 & w(7) = w(9) = 7 \\ w(3) = w(13) = 3 & w(8) = 8 \\ w(4) = w(12) = 4 & \end{array}$$

which is depicted in Figure 13. We can observe that going forward or taking slight bends has small weights whereas going backwards and taking sharp bends leads to high weights.

In the navigation task, we start at a point *from* and want to reach a point *to*. The current point is *start* initialized

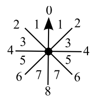


Figure 13: $\mathcal{OPRA}_4$ weight distribution

by $from$. These are point that represent crossings in the street scenario, not oriented points. We determine the set of all $\mathcal{OPRA}$ relations $o_m\angle_i^j p$ with $\mathrm{pr}_1(o) = start$ and $\mathrm{pr}_1(p) = to$. We then form the half relations $o \rhd to$ as

$$o \rhd to = \sum_{\mathrm{pr}_1(p)=to} \left\{ o_m \rhd_i to \mid o_m\angle_i^j p \right\}$$

We then normalize the weights as

$$w = \frac{\sum_{i=o\rhd to} w(i)}{|o \rhd to|} \cdot penalty$$

where $penalty$ is a property of $\mathrm{pr}_2(o)$ that is initialized with 1 and incremented by 1 each time $\mathrm{pr}_2(o)$ is visited on a path. This is introduced to make loops bad ways to go and to get out of dead ends. We now take all o with the minimum w, if there is more than one, we choose by fortune. $\mathrm{pr}_2(o)$ becomes our new point $start$ and its penalty is increased since it is visited. We repeat this, until to is reached. The algorithm for navigation is shown in Line 4.

Algorithm 4 Navigation

1: $from$ start point
2: to end point
3: $start = from$
4: $ROUTE := start$
5: **while** $start \neq to$ **do**
6: $R := \emptyset$
7: $W := \emptyset$
8: **for all** p with $\mathrm{pr}_1(p) = to$ **do**
9: **for all** o with $\mathrm{pr}_1(o) = start$ **do**
10: **if** R contains a relation $o \rhd p$ **then**
11: $R := (R \setminus o \rhd p) \uplus o(\rhd \uplus o_m \rhd_i)p$ if $o_m\angle_i^j p$
12: **else**
13: $R := R \uplus o_m \rhd_i p$ if $o_m\angle_i^j p$
14: **end if**
15: **end for**
16: **end for**
17: **for all** $r \in R$ **do**
18: $W := W \cup (r, \mathrm{weight}(r))$
19: **end for**
20: $cand := r \in R$ with $w(r) = min$
21: $next :=$ random element from cand
22: increase $\mathrm{pr}_2(next).penalty$
23: $start := \mathrm{pr}_2(next)$
24: $ROUTE := ROUTE \circ start$
25: **end while**
26: **return** $ROUTE$

The assignment of weights is shown in Algorithm 5. Please note that we have used disjoint unions of the half relation symbols in Line 4, since those lead to better navigation results in our first experiments, even for low granularities.

Algorithm 5 Weight assignment: weight

1: $o \rhd p$ is given
2: $W := 0$
3: **for all** $r \in \rhd$ **do**
4: **if** $0 \leq r \leq 2m$ **then**
5: $W := W + r$
6: **else**
7: $W := W + 4m - r$
8: **end if**
9: **end for**
10: $W := \frac{W}{|\rhd|} \cdot \mathrm{pr}_2(o).penalty$
11: **return** W

6 Experiments

Finding good data for experiments with navigation based on local observations is a hard task. A big issue is that the maximal size of a street network that we can use for navigation is limited by the number of nodes and by the granularity of the underlying $\mathcal{OPRA}$ calculus. The time needed for applying the algebraic closure algorithm rises steeply with any of these two parameters growing. As a rule of thumb we can say that we can e.g. handle street networks with around 120 to 170 points with $\mathcal{OPRA}_8$ in a reasonable time (2 to 4 hours) when computing algebraic closure with GQR. (However, note that this has to be computed only once, and can then be used for as many navigation tasks as wanted.) On the other hand a network in 170 points in our representation (the reduction of data is described in Figure 4) does not cover big areas in most cases. For example the network shown in Figure 14 that derived from the data on OpenStreetMap (latitude 51.8241200 longitude 9.3117500) for a village with about 1400 inhabitants already has 117 points. Large cities like Paris of course have many more points in our representation and cannot be handled efficiently with the algebraic reasoners. But on the

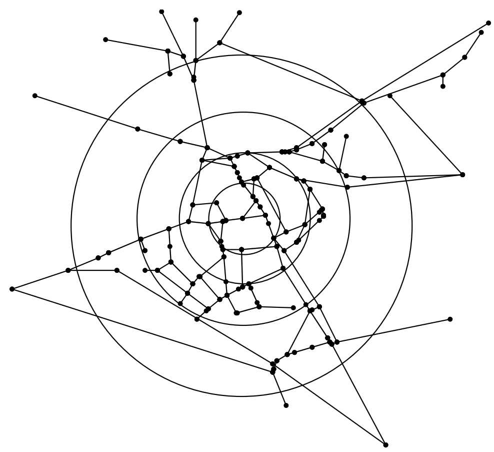


Figure 14: A street network of a village

other hand, we want to observe navigations along paths of very differing lengths, including very long paths to be able to judge the navigation properties of our networks based on local observations under very differing circumstances. For long paths networks of a sufficiently big size are needed. Unfortunately this problem grows even bigger by the fact that the closer we get to boundary of our street network the worse our local observations and refinements will be. For a point in the middle of a street network (as in the inner circle in Figure 14 there are many points around it in all directions with observations being made, putting this point into its place in a qualitative sense. In the middle circle the observations around a certain point already get sparser and information about the points gets less certain, this gets worse in the outer circle. Outside of the outer circle information about the points is very bad. In fact, it turned out, that navigating into the dead ends at the boundary of the map is very alluring, since their position with respect to other points is not very restricted. For meaningful experiments about the navigation performance, the need street networks that are big enough to provide an area in the center for which enough information can be derived.

Our test data has hence to consist of street networks that are small enough to be manageable with qualitative reasoners and that are big enough to yield enough information. For the first requirement networks in no more than 20 points would be nice, for the second one the whole world, since then there would be no boundary problem.

The results of our experiments are available at `http://www.informatik.uni-bremen.de/~till/fuerstenau_K8.html` (for Klippel$_8$) and `http://www.informatik.uni-bremen.de/~till/fuerstenau_O8.html` (for $\mathcal{OPRA}_8$). We have made 66 navigation experiments. The average path length was 16.0 (using $\mathcal{OPRA}_8$ factorized due to Klippel's sectorization of the circle) and 16.2 (using $\mathcal{OPRA}_8$) with our algorithm based on local observations, while that of a shortest path (using the complete map) was 13.2, and that of a uniform shortest path (counting all way lengths as 1) was 11.0. The average length of a random walk was 718.0. As expected, the standard deviation of our algorithm is significantly higher than that of shortest paths:

	Klippel$_8$	$\mathcal{OPRA}_8$	shortest path	uniform shortest path	random walk
mean	16.0	16.2	13.2	11.0	718.0
standard deviation	9.3	9.3	5.1	3.5	519.9

However, our algorithm still performs quite well when compared with shortest paths.

Conclusion

Our experiments show that navigation based on local observations of an agent performs fairly well when compared with shortest paths computed using global map knowledge, and orders of magnitude better than randowm walk.

When making the experiments, we quickly reached the limits of the standard qualitative spatial reasoning tools. The constraint networks generated by our algorithms thus could been seen as a challenge for (further) improving performace of these tools.

Further experiments should be done with different test data. Particularly interesting would be street networks of diverse style. It is e.g. interesting to use layouts of planned and grown cities and villages. Further gyratory traffics (e.g. at Place-Charles-de-Gaulle) of increased interest. With a larger set of experiments, the approach could be used to systematically evaluate street networks with respect to their local navigation quality, and study which features of street networks influence this quality.

References

J. F. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, pages 832–843, 1983.

Z. Gantner, M. Westphal, and S. Wölfl. GQR - A Fast Reasoner for Binary Qualitative Constraint Calculi. In *Proc. of the AAAI-08 Workshop on Spatial and Temporal Reasoning*, 2008.

Alexander Klippel and Daniel R. Montello. Linguistic and nonlinguistic turn direction concepts. In Stephan Winter, Matt Duckham, Lars Kulik, and Benjamin Kuipers, editors, *Spatial Information Theory, 8th International Conference, COSIT 2007, Melbourne, Australia, September 19-23, 2007, Proceedings*, volume 4736 of *Lecture Notes in Computer Science*, pages 354–372. Springer, 2007.

A. Klippel, H. Tappe, L. Kulik, and P. U. Lee. Wayfinding choremes–a language for modeling conceptual route knowledge. *Journal of Visual Languages and Computing*, 16(4):311 – 329, 2005. Perception and ontologies in visual, virtual and geographic space.

T. Mossakowski and R. Moratz. Qualitative reasoning about relative direction on adjustable levels of granularity. *Journal of Artificial Intelligence*, to appear.

D. A. Randell, Z. Cui, and A. G. Cohn. A spatial logic based on regions and connection. In Bernhard Nebel, Charles Rich, and William Swartout, editors, *Proc. of KR-92*, pages 165–176. Morgan Kaufmann, 1992.

J. O. Wallgrün, L. Frommberger, D. Wolter, F. Dylla, and C. Freksa. Qualitative Spatial Representation and Reasoning in the SparQ-Toolbox. In T. Barkowsky, M. Knauff, G. Ligozat, and D. R. Montello, editors, *Spatial Cognition*, volume 4387 of *Lecture Notes in Comput. Sci.*, pages 39–58. Springer, 2006.

J. O. Wallgrün, L. Frommberger, F. Dylla, and D. Wolter. SparQ User Manual V0.7. User manual, University of Bremen, January 2009.

Challenges for Qualitative Spatial Reasoning in Linked Geospatial Data

Manolis Koubarakis and **Kostis Kyzirakos** and **Manos Karpathiotakis** and
Charalampos Nikolaou and **Michael Sioutis** and **Stavros Vassos**
Dept. of Informatics and Telecommunications, National and Kapodistrian University of Athens
Dimitrios Michail
Dept. of Informatics and Telematics, Harokopio University of Athens
Themistoklis Herekakis and **Charalampos Kontoes** and **Ioannis Papoutsis**
Inst. for Space Applications and Remote Sensing, National Observatory of Athens
{koubarak,kkyzir,mk,charnik,sioutis,stavrosv}@di.uoa.gr
{therekak,kontoes,ipapoutsis}@space.noa.gr
{michail}@hua.gr

Abstract

Linked geospatial data has recently received attention, as researchers and practitioners have started tapping the wealth of geospatial information available on the Web. We discuss some core research problems that arise when querying linked geospatial data, and explain why these are relevant for the qualitative spatial reasoning community. The problems are presented in the context of our recent work on the models stRDF and stSPARQL and their extensions with indefinite geospatial information.

1 Introduction

Linked data is a new research area which studies how one can make RDF data available on the Web, and interconnect it with other data with the aim of increasing its value for everybody [Bizer *et al.*, 2009]. The resulting "Web of data" has recently started being populated with geospatial data. A representative example of such efforts is LinkedGeoData[1] where OpenStreetMap data are made available as RDF and queried using the declarative query language SPARQL [Auer *et al.*, 2009]. With the recent emphasis on open government data, some of it encoded already in RDF[2], portals such as LinkedGeoData demonstrate that the development of useful Web applications might be just a few SPARQL queries away.

We have recently developed stSPARQL, an extension of the query language SPARQL for querying linked geospatial data [Koubarakis and Kyzirakos, 2010][3]. stSPARQL has been fully implemented and it is currently being used to query linked data describing sensors in the context of project SemsorGrid4Env[4] [Kyzirakos *et al.*, 2010] and linked earth observation (EO) data in the context of project TELEIOS[5].

In the context of TELEIOS we are developing a Virtual Observatory infrastructure for EO data. One of the applications of TELEIOS is fire monitoring and management led by the National Observatory of Athens (NOA). This application focuses on the development of techniques for real time hotspot and active fire front detection, and burnt area mapping. Technological solutions to both of these cases require the integration of multiple, heterogeneous data sources, some of them available on the Web, with data of varying quality and varying temporal and spatial scales.

In this paper we show how well-known approaches to qualitative spatial representation and reasoning [Renz and Nebel, 2007] can be used to represent and query linked geospatial data using RDF and stSPARQL. Thus, we propose linked geospatial data as an interesting application area of qualitative spatial reasoning techniques, and discuss open problems that might be of interest to the qualitative spatial reasoning community. In particular, we address the problem of representing and querying *indefinite* geospatial information, and discuss the approach we adopt in TELEIOS.

The organization of the paper is as follows. Section 2 introduces the kinds of linked geospatial data that we need to represent in the NOA application of TELEIOS, shows how to represent it in stRDF, and presents some typical stSPARQL queries. Then, Section 3 shows how the introduction of qualitative spatial information in the stRDF data model enables us to deal with the NOA application more accurately. The same section introduces the new model stRDF^i which allows qualitative spatial information to be expressed in RDF and gives examples of interesting queries in the new model. In Section 4 we proceed to discuss some open problems in the stRDF^i framework that require new contributions by the

[1] http://linkedgeodata.org/

[2] http://data.gov.uk/linked-data/

[3] The paper [Koubarakis and Kyzirakos, 2010] presents the language stSPARQL that also enables the querying of *valid times* of triples. Here, we omit time and discuss only the geospatial subset of stSPARQL.

[4] http://www.semsorgrid4env.eu/

[5] http://www.earthobservatory.eu/

qualitative spatial reasoning community. Finally, in Section 5 we discuss related work and in Section 6 we draw conclusions.

The paper is mostly informal and uses examples from the NOA application of TELEIOS. Even in the places where the paper becomes formal, we do not give any detailed technical results for which the interested reader is directed to [Koubarakis *et al.*, 2011].

2 Linked geospatial data in the NOA application

The NOA application of TELEIOS concentrates on the development of solutions for real time hotspot and active fire front detection, and burnt area mapping. Technological solutions to both of these cases require integration of multiple, heterogeneous data sources with data of varying quality and varying temporal and spatial scales. Some of the data sources are streams (e.g., streams of EO images) while others are static geo-information layers (e.g., land use/land cover maps) providing additional evidence on the underlying characteristics of the affected area.

2.1 Datasets

The following datasets are available in the NOA application:

- **Hotspot maps.** NOA operates a MSG/SEVIRI[6] acquisition station and receives raw satellite images every 15 minutes. These images are processed with image processing algorithms to detect the existence of hotspots. The information related to hotspots is stored in ESRI shapefiles and KML files. These files hold information about the date and time of image acquisition, cartographic X, Y coordinates of detected fire locations, the level of reliability in the observations, the fire radiative power assessed, and the observed fire area. NOA receives similar hotspot shapefiles covering the geographical area of Greece from the European project SAFER (Services and Applications for Emergency Response).
- **Burnt area maps.** From project SAFER, NOA also receives ready-to-use accumulated burnt area mapping products in polygon format, projected to the EGSA87 reference system[7]. These products are derived daily using the MODIS satellite and cover the entire Greek territory. The data formats are ESRI shapefiles and KML files with information relating to date and time of image acquisition, and the mapped fire area.
- **Corine Land Cover data.** The Corine Land Cover project is an activity of the European Environment Agency which is collecting data regarding land cover (e.g., farmland, forest) of European countries. The Corine Land Cover nomenclature uses a hierarchical scheme with three levels to describe land cover:
  - The first level consists of five items and indicates the major categories of land cover on the planet, e.g., forests and semi-natural areas.
  - The second level consists of fifteen items and is intended for use on scales of 1:500,000 and 1:1,000,000 identifying more specific types of land cover, e.g., open spaces with little or no vegetation.
  - The third level consists of forty-four items and is intended for use on a scale of 1:100,000, narrowing down the land use to a very specific geographic characterization, e.g., burnt areas.

  The land cover of Greece is available as an ESRI shapefile that is based on the Corine Land Cover nomenclature.
- **Coastline geometry of Greece.** An ESRI shapefile that describes the geometry of the coastline of Greece is available.

[6]MSG refers to Meteosat Second Generation satellites, and SEVIRI is the instrument which is responsible for taking infrared images of the earth.

[7]EGSA87 is a 2-dimensional projected coordinate reference system that describes the area of Greece.

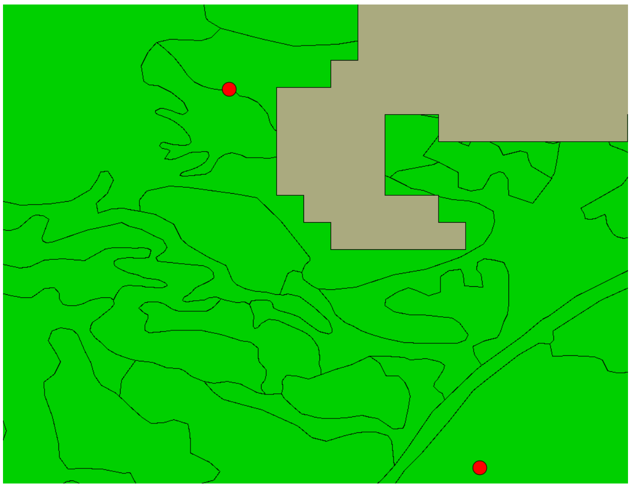


Figure 1: An example of hotspots and burnt area mapping products in the region of Attiki, Greece

Figure 1 presents an example of hotspots and burnt area mapping products, as viewed when layered together over a map of Greece.

2.2 Using semantic web technology

An important challenge in the context of TELEIOS is to develop advanced semantics-based querying of the available datasets along with linked data available on the web. This is a necessary step in order to unlock the full potential of the available datasets, as their correlation with the abundance of data available in the web can offer significant added value. As an introduction to Semantic Web technology, we present a simple example that shows how burnt area data is expressed in the language stRDF, and then proceed to illustrate some interesting queries using the language stSPARQL.

Similar to RDF, in stRDF we can express information using triples of URIs, literals, and blank nodes in the form *"subject predicate object"*. Figure 2 shows four stRDF triples that encode information related to the burnt area that is identified

```
ex:BurntArea_1 rdf:type noa:BurntArea.
ex:BurntArea_1 noa:hasID "1"^^xsd:decimal.
ex:BurntArea_1 geo:geometry "POLYGON((
    38.16 23.7, 38.18 23.7,
    38.18 23.8, 38.16 23.8,
    38.16 23.7));<http://spatialreference
    .org/ref/epsg/4121/>"^^strdf:geometry.
ex:BurntArea_1 noa:hasArea
    "23.7636"^^xsd:double.
```

Figure 2: An example of a burnt area represented in stRDF

by the URI `ex:BurntArea_1`. The prefixes `noa` and `ex` correspond to appropriate namespaces for the URIs that refer to the NOA application and our running example, while `xsd` and `strdf` correspond to the XML Schema namespace and our stRDF namespace, respectively.

In stRDF the standard RDF model is extended with the ability to represent *geospatial data*. In our latest version of stRDF we opt for a practical solution that uses OGC standards to represent geospatial information. We introduce the new data type `strdf:geometry` for modeling geometric objects. The values of this datatype are typed literals that encode geometric objects using the OGC standard *Well-known Text (WKT)* or *Geographic Markup Language (GML)*. Literals of this datatype are called *spatial literals*.

The third triple in Figure 2 shows the use of spatial literals to express the geometry of the burnt area in question. This spatial literal specifies a polygon that has exactly one exterior boundary and no holes. The exterior boundary is serialized as a sequence of its vertices' coordinates. These coordinates are interpreted according to the GGRS87 geodetic coordinate reference system identified by the URI `http://spatialreference.org/ref/epsg/4121/`.

In the case of burnt area maps, these stRDF triples are created by a procedure that processes the relevant shapefiles and produces one stRDF triple for each property that refers to a particular area. Although we are currently doing this manually, in the future we plan to use automated tools as in [Blázquez *et al.*, 2010].

Figure 3 presents a query in stSPARQL that looks for all the URIs of burnt areas that are located in Greece and calculates their area. stSPARQL is an extension of SPARQL in which variables may refer to spatial literals (e.g., variable `?BAGEO` in `?BA geo:geometry ?BAGEO`[8]). stSPARQL provides functions that can be used in filter expressions to express qualitative or quantitative spatial relations. For example the function `strdf:Contains` is used in Figure 3 to encode the topological relation *non-tangential proper part inverse (NTPP^{-1})* of RCC-8 [Cui *et al.*, 1993].

In this query, linked data from DBpedia[9] are used to identify those burnt areas that are located in Greece. DBpedia is an RDF dataset consisting of the contents of Wikipedia that allows you to link other data sets on the Web to Wikipedia

```
select ?BA strdf:Area(?BA)
where {?BA rdf:type noa:BurntArea .
    ?BA geo:geometry ?BAGEO .
    ?C  rdf:type noa:GeographicBound .
    ?C  dbpedia:Country dbpedia:Greece .
    ?C  geo:geometry ?CGEO .
    filter(strdf:Contains(?CGEO,?BAGEO))}
```

Figure 3: An example of a query expressed in stSPARQL

```
select ?BA ?BAGEO
where {?R rdf:type noa:Region .
       ?R geo:geometry ?RGEO .
       ?R noa:hasCorineLandCoverUse ?F .
       ?F rdfs:subClassOf clc:Forests .
       ?CITY rdf:type dbpedia:City .
       ?CITY geo:geometry ?CGEO .
       ?BA rdf:type noa:BurntArea .
       ?BA geo:geometry ?BAGEO .
       filter(strdf:Intersect(?RGEO,?BAGEO)&&
              strdf:Distance(?BAGEO,?CGEO)<2)}
```

Figure 4: A more complex example of a query expressed in stSPARQL

data. The result of this query is a list of URIs that may include `ex:BurntArea_1` of Figure 2.

Figure 4 presents a more complex query in stSPARQL that looks for all burnt areas that were classified as forests according to the Corine Land Cover dataset. These areas must also be located within 2km from a city. This query also uses linked data from DBpedia to retrieve geospatial information about cities.

3 Indefinite geospatial information in the NOA use case

This section motivates our approach towards extending the model stRDF with the ability to represent and query indefinite qualitative spatial information. The new model is named stRDF^{i} where "i" stands for "indefinite".

The infrared imager SEVIRI on board of the MSG satellites has medium resolution, i.e., each image pixel representing a hotspot in the NOA shapefiles corresponds to a 3km by 3km rectangle in geographic space. Thus, a precise representation of the real world situation that corresponds to a hotspot would be to state that there is a geographic region with unknown exact coordinates where a fire is taking place, and that region is included in a known 3km by 3km rectangle. This is captured by the following triples and constraints in stRDF^{i} that introduce the hotspot, the fire corresponding to it and the region corresponding to the fire. This region (`_region1`) is a new kind of literal, called an *unknown literal*, which is asserted to be inside the polygon defined by `"POLYGON((24.81 35.32, 24.84 35.33, 24.84 35.30, 24.81 35.30, 24.81 35.32))"`.

```
noa:hotspot1 rdf:type noa:Hotspot .
noa:fire1 rdf:type noa:Fire .
noa:hotspot1 noa:correspondsTo noa:fire1 .
noa:fire1 noa:occuredIn _region1 .
```

[8] We are assuming that DBpedia offers precise representations of country geometries as values of the predicate `geo:geometry`. This is not the case at the moment since these values are points corresponding to the bounds of a region located in the center of Greece.

[9] `http://www.dbpedia.org/`

```
_region1 strdf:NTPP "POLYGON((24.81 35.32,
  24.84 35.33, 24.84 35.30, 24.81 35.30,
  24.81 35.32));<http://spatialreference.
  org/ref/epsg/4121/>"^^strdf:geometry.
```

Unknown literals are like existentially quantified variables in first-order logic. By convention, identifiers for unknown literals in stRDF^{i} always start with an underscore. In the above example, `strdf:NTPP` is the *non-tangential proper part* relation of RCC-8.

The NOA fire monitoring activities include validating hotspots, i.e., making sure that they do not correspond to false alarms due to the medium resolution of the images, or fires that are not of interest since they do not take place in forested areas. Part of the validation activities of NOA include collecting information about forest fires reported in the Greek Press. Therefore, when fire `noa:fire1` is validated, NOA may want to annotate the relevant hotspot, validated fire and burnt area with information from news sources available on the Web that have reported the corresponding fire. Assuming that Greek newspapers will soon follow the example of New York Times and use tags to annotate news articles, articles reporting fire events may be tagged with the name of the administrative area in which the fire occurred and the word "fire". Then, it is easy to retrieve the geographical coordinates of the place mentioned in the tag and, using standard geometric methods, decide whether the location of the hotspot is near that place.

Alternatively, using techniques from Geographic Information Retrieval and Natural Language Processing [Schockaert *et al.*, 2008; Hoffart *et al.*, 2010] one could harvest qualitative spatial information from the Web. As an example, information related to `noa:fire1` obtained from a regional Greek newspaper available on the Web might say that "there was a fire *north of* the village of Zoniana in the Prefecture of Rethymno, Crete". In this case NOA might choose to produce an annotation which mixes the qualitative spatial information discovered from the newspaper with information that corresponds to the relevant administrative regions of Greece. Of course, such techniques are not always accurate and extracted information has to be accompanied by a confidence level [Hoffart *et al.*, 2010].

The next triples introduce the burnt area corresponding to `noa:fire1` and some details related to the administrative geography of Greece as defined by the recent "Kallikratis Plan"[10]. Since there is already work in encoding the administrative geography of countries, e.g., the UK [Goodwin *et al.*, 2008], in terms of qualitative spatial constraints such as the ones we used above, we expect that such annotations can be a useful source of information for the NOA application. This is stressed by the fact that currently much of this information is or will become available as public open data in portals of the relevant European governments (e.g., see the geodata portal of the Government of Greece[11]).

```
noa:fire1 rdf:type noa:ValidatedFire .
noa:fire1 ex:hasBurntArea _region2 .

kal:Zoniana rdf:type kal:Community .
kal:Mylopotamos rdf:type kal:Municipality .
kal:Rethymno rdf:type kal:Prefecture .

kal:Zoniana kal:occupies _region3 .
kal:Mylopotamos kal:occupies _region4 .
kal:Rethymno kal:occupies _region5 .

kal:Zoniana kal:partOf kal:Mylopotamos .
kal:Mylopotamos kal:partOf kal:Rethymno .

_region3 strdf:NTPP _region4 .
_region4 strdf:NTPP _region5 .
_region1 strdf:northOf kal:Zoniana .
_region2 strdf:northOf kal:Zoniana .
```

In the following, we discuss how to evaluate stSPARQL queries over the stRDF^{i} data given in the beginning of this section. Let us consider the following query: "Find all fires that have occurred in a region which is a non-tangential proper part of the polygon defined by `"POLYGON((24.823 35.308, 24.827 35.308, 24.827 35.305, 24.823 35.305, 24.823 35.308))"`"[12]. In stSPARQL, this query can be expressed as shown in Figure 5. The answer to that query is the one shown in Table 1. Notice, that this answer is conditional. Because the information in the database is indefinite (the exact geometry of `_region1` is not known), we cannot say for sure whether `fire1` satisfies the requirements of the query. These requirements are satisfied under the condition given in the answer.

```
select ?F
where { ?F rdf:type noa:Fire .
   ?F noa:occuredIn ?R .
   filter (strdf:NTPP(?R, "POLYGON((24.823
     35.308, 24.827 35.308, 24.827 35.305,
     24.823 35.305, 24.823 35.308))"))}
```

Figure 5: An example of a query for the stRDF^{i} model expressed in stSPARQL

Table 1: A conditional answer in stRDF^{i}

?F	Condition
noa:fire1	_region1 strdf:NTPP "POLYGON((24.823 35.308, 24.827 35.308, 24.827 35.305, 24.823 35.305, 24.823 35.308))"

Let us consider the query of Figure 5 again. If we rephrase it to "Find fires that have *certainly* occurred in a region which is a non-tangential proper part of the polygon defined by `"POLYGON((24.823 35.308, 24.827 35.308, 24.827 35.305, 24.823 35.305, 24.823 35.308))"`", `fire1` does not satisfy the query. To be able to express such queries over stRDF^{i} data, in [Koubarakis *et al.*, 2011] we have extended

[10] `http://en.wikipedia.org/wiki/Administrative_divisions_of_Greece/`

[11] `http://geodata.gov.gr/`

[12] Notice, that this second polygon is contained in the one mentioned previously.

the semantics of query answering for stSPARQL given in [Koubarakis and Kyzirakos, 2010] using well-known techniques from the literature of incomplete information in relational databases [Imielinski and Lipski, 1984; Grahne, 1991] and constraint databases [Koubarakis, 1997].

4 Open Problems

In Sections 2 and 3 we used the NOA application of TELEIOS as an example to demonstrate how linked geospatial data sets that typically contain geometric objects specified by exact co-ordinates can be enriched with qualitative spatial information to enable better knowledge representation and more expressive query answering.

We expect that various kinds of qualitative spatial information will soon become part of linked geospatial data sets with advances in the automatic extraction of qualitative spatial relations from textual Web sources [Schockaert *et al.*, 2008], images [Mylonas *et al.*, 2009; Hudelot *et al.*, 2008], etc., and the creation of ontologies with a geospatial component such as YAGO2 [Hoffart *et al.*, 2010].

Let us now discuss a few open problems in the $\mathrm{stRDF}^{\mathrm{i}}$ framework that require new contributions by the qualitative spatial reasoning community:

- Checking the consistency of constraint networks that involve qualitative spatial relations among regions identified by a URI and constant ones (e.g., a rectangle or a polygon in the plane $\mathbb{Q}^2$ or in a Cartesian co-ordinate system). This combination of qualitative and quantitative constraints has been studied in detail for temporal constraints [Koubarakis, 2006], but similar results do not exist for spatial constraints.
- Checking the consistency of constraint networks that involve qualitative and quantitative spatial relations among planar regions that are constrained to have certain shapes (e.g., triangles, rectangles, polygons). The case of rectangles has been studied in detail in the past (e.g., see [Balbiani *et al.*, 1999]) and there is some recent work on topological relations among convex planar regions [Li and Liu, 2010].
- Performing variable elimination in constraint networks with qualitative and quantitative spatial constraints or, equivalently, performing quantifier elimination in the associated first-order theory. As shown for the temporal case in [Koubarakis, 1997], variable elimination is needed for answering certainty queries with answer variables (i.e., "What is the region that is on fire and is certainly inside a specific area?"). This cannot be done in the general case even for topological relations [Bennet, 1997] but no detailed results beyond this are known.
- Scalable implementations of constraint network algorithms for qualitative and quantitative spatial constraints. RDF stores supporting linked geospatial data are expected to scale to *billions* of triples like their non-spatial counterparts [Neumann and Weikum, 2008] and recent work in this area is encouraging [Brodt *et al.*, 2010].

  Can this level of scalability be achieved when qualitative spatial relations come into play? A good approach here might to start with algorithms with low polynomial complexity (even if they do not cover the general case) and try to implement them as efficiently as possible. In the temporal case, this approach has been followed successfully by temporal reasoners such as TimeGraph-II and extensions [Gerevini *et al.*, 1994]. In addition, there might be cases where network structure can be exploited (e.g., hierarchical organization of geographical regions).
- There are no publicly available data sets, benchmarks and related implementations. This workshop and the associated QSTR library is an excellent way to bring together the community and make progress in this area. It is also important to liaise with similar efforts in the Semantic Web community.

5 Related Work

Enriching linked data sources with geospatial information is a recent activity. Two representative examples are [Auer *et al.*, 2009; de León *et al.*, 2010]. In [Auer *et al.*, 2009] OpenStreetMap data are made available as RDF and queried using the declarative query language SPARQL. Using similar technologies, [de León *et al.*, 2010] makes available as linked data various heterogeneous Spanish public datasets. In both of these data sources qualitative spatial relations do not appear in the triples. YAGO2 [Hoffart *et al.*, 2010] offers only a part-of relation.

In addition to stSPARQL there have also been other works developing spatial and temporal extensions for RDF and SPARQL [Perry, 2008; Kolas, 2008]. There is also a forthcoming OGC standard [OGC, 2010] for the development of a query language for geospatial data encoded in RDF, called GeoSPARQL.

In contrast to the above works, the area of description logics has studied the representation and reasoning with qualitative spatial relations utilizing data models that are similar to RDF. Racer was the first reasoner to support qualitative spatial relations [Wessel and Moller, 2009]. More recently, [Stocker and Sirin, 2009] has developed an extension of the DL reasoner Pellet [Parsia and Sirin, 2004] that allows reasoning with RCC-8 relations. Finally, [Batsakis and Petrakis, 2010] proposes SOWL, an extension of OWL, to represent spatial qualitative and quantitative information employing the RCC-8 topological relations, cardinal direction relations, and distance relations. To reason about spatial relations a set of SWRL rules are implemented in the Pellet reasoner.

6 Conclusions

In this paper we proposed linked geospatial data on Semantic Web as an interesting application area of qualitative spatial reasoning techniques. In the context of our recent work on the models stRDF and stSPARQL and their extensions with indefinite geospatial information, we discussed some open problems that may be of interest to the qualitative spatial reasoning community. As part of our future work we intend to study the computational complexity of query processing for the languages we have developed.

Acknowledgements

This work has been funded by the FP7 project TELEIOS (257662).

References

[Auer *et al.*, 2009] S. Auer, J. Lehmann, and S. Hellmann. Linkedgeodata: Adding a spatial dimension to the web of data. In *ISWC'09*, pages 731–746, 2009.

[Balbiani *et al.*, 1999] P. Balbiani, J.-F. Condotta, and L. F. del Cerro. A new tractable subclass of the rectangle algebra. In *IJCAI*, pages 442–447, 1999.

[Batsakis and Petrakis, 2010] S. Batsakis and E. G. M. Petrakis. SOWL: spatio-temporal representation, reasoning and querying over the semantic web. In *I-SEMANTICS '10*, USA, 2010.

[Bennet, 1997] B. Bennet. *Logical Representations for Automated Reasoning about Spatial Relationships*. PhD thesis, School of Computer Studies, University of Leeds, 1997.

[Bizer *et al.*, 2009] C. Bizer, T. Heath, and T. Berners-Lee. Linked data-the story so far. *Int. J. Semantic Web Inf. Syst.*, 5(3):1–22, 2009.

[Blázquez *et al.*, 2010] L. M. Vilches Blázquez, B. Villazón-Terrazas, V. Saquicela, A. de León, Ó. Corcho, and A. Gómez-Pérez. Geolinked data and inspire through an application case. In *ACM SIGSPATIAL GIS*, pages 446–449, 2010.

[Brodt *et al.*, 2010] A. Brodt, D. Nicklas, and B. Mitschang. Deep integration of spatial query processing into native rdf triple stores. In *ACM SIGSPATIAL GIS'10*, pages 33–42, USA, 2010.

[Cui *et al.*, 1993] Z. Cui, A. G. Cohn, and D. A. Randell. Qualitative and Topological Relationships in Spatial-Databases. In *SSD'93*, 1993.

[de León *et al.*, 2010] A. de León, V. Saquicela, L. M. Vilches, B. Villazón-Terrazas, F. Priyatna, and O. Corcho. Geographical Linked Data: a Spanish Use Case. In *I-SEMANTICS*, 2010.

[Gerevini *et al.*, 1994] A. Gerevini, L. Schubert, and S. Schaeffer. The temporal reasoning tools TimeGraph I-II. In *ICTAI'94*, pages 513–520, 1994.

[Goodwin *et al.*, 2008] J. Goodwin, C. Dolbear, and G. Hart. Geographical Linked Data: The Administrative Geography of Great Britain on the Semantic Web. *TGIS*, 12:19–30, 2008.

[Grahne, 1991] G. Grahne. *The Problem of Incomplete Information in Relational Databases*, volume 554 of *LNCS*. Springer Verlag, 1991.

[Hoffart *et al.*, 2010] J. Hoffart, F. M. Suchanek, K. Berberich, and G. Weikum. YAGO2: a spatially and temporally enhanced knowledge base from Wikipedia. Technical report, Max Planck Institut Informatik, 2010.

[Hudelot *et al.*, 2008] C. Hudelot, J. Atif, and I. Bloch. Fuzzy spatial relation ontology for image interpretation. *Fuzzy Sets and Systems*, 159(15):1929–1951, 2008.

[Imielinski and Lipski, 1984] T. Imielinski and W. Lipski. Incomplete Information in Relational Databases. *JACM*, 31(4):761–791, 1984.

[Kolas, 2008] D. Kolas. Supporting Spatial Semantics with SPARQL. In *Terra Cognita Workshop*, Germany, 2008.

[Koubarakis and Kyzirakos, 2010] M. Koubarakis and K. Kyzirakos. Modeling and Querying Metadata in the Semantic Sensor Web: The Model stRDF and the Query Language stSPARQL. In *ESWC'10*, pages 425–439, 2010.

[Koubarakis *et al.*, 2011] M. Koubarakis, K. Kyzirakos, B. Nikolaou, M. Sioutis, and S. Vassos. A data model and query language for an extension of rdf with time and space. Deliverable D2.1, TELEIOS project, 2011.

[Koubarakis, 1997] M. Koubarakis. The Complexity of Query Evaluation in Indefinite Temporal Constraint Databases. *TCS*, 171:25–60, January 1997.

[Koubarakis, 2006] M. Koubarakis. Temporal CSPs. In *Handbook of Constraint Programming*. Elsevier, 2006.

[Kyzirakos *et al.*, 2010] K. Kyzirakos, M. Karpathiotakis, and M. Koubarakis. Developing Registries for the Semantic Sensor Web using stRDF and stSPARQL. In *SSN'10*, volume 668, 2010.

[Li and Liu, 2010] S. Li and W. Liu. Topological relations between convex regions. In *AAAI*, 2010.

[Mylonas *et al.*, 2009] P. Mylonas, E. Spyrou, Y. Avrithis, and S. Kollias. Using visual context and region semantics for high-level concept detection. *IEEE Trans. Multi.*, 11:229–243, 2009.

[Neumann and Weikum, 2008] T. Neumann and G. Weikum. RDF-3X: a RISC-style engine for RDF. *VLDB*, 1(1), 2008.

[OGC, 2010] Open Geospatial Consortium Inc OGC. GeoSPARQL - A geographic query language for RDF data, November 2010.

[Parsia and Sirin, 2004] B. Parsia and E. Sirin. Pellet: An OWL DL Reasoner. In *ISWC'04*, 2004.

[Perry, 2008] M. Perry. *A Framework to Support Spatial, Temporal and Thematic Analytics over Semantic Web Data*. PhD thesis, Wright State University, 2008.

[Renz and Nebel, 2007] J. Renz and B. Nebel. Qualitative spatial reasoning using constraint calculi. In *Handbook of Spatial Logics*, pages 161–215. Springer, 2007.

[Schockaert *et al.*, 2008] S. Schockaert, P. D. Smart, A. I. Abdelmoty, and C. B. Jones. Mining topological relations from the web. In *DEXA*, pages 652–656, 2008.

[Stocker and Sirin, 2009] M. Stocker and E. Sirin. Pelletspatial: A hybrid rcc-8 and rdf/owl reasoning and query engine, 2009.

[Wessel and Moller, 2009] M. Wessel and R. Moller. Flexible software architectures for ontology-based information systems. *JAL*, 7(1):75–99, March 2009.

Underwater archaeological 3D surveys validation within the Removed Sets framework

Julien Hué[1], Mariette Serayet[2], Pierre Drap[2], Odile Papini[2], Eric Wúrbel[3]

[1]Institut fr Informatik
Albert-Ludwigs-Universitt Freiburg
Georges-Khler-Allee 52
79110 Freiburg, Germany
hue@informatik.uni-freiburg.de

[2]LSIS-CNRS 6168. Université de la Méditerranée.
ESIL - Case 925. Av de Luminy.
13288 Marseille Cedex 9 France.
serayet,drap,papini@esil.univmed.fr

[3]LSIS-CNRS 6168, Université du Sud Toulon -Var.
BP 132. 83957 La Garde Cedex France.
wurbel@univ-tln.fr

This paper presents the results of the VENUS european project aimed at providing scientific methodologies and technological tools for the virtual exploration of deep water archaeological sites. We focused on underwater archaeological 3D surveys validation problem. This paper shows how the validation problem has been tackled within the Removed Sets framework, according to Removed Sets Fusion (RSF) and to the Partially Preordered Removed Sets Inconsistency Handling (PPRSIH). Both approaches have been implemented thanks to ASP and the good behaviour of the Removed Sets operations is presented through an experimental study on two underwater archaeological sites.

1 Introduction

The VENUS European Project (Virtual ExploratioN of Underwater Sites, IST-034924)[1] aimed at providing scientific methodologies and technological tools for the virtual exploration of deep underwater archaeology sites. In this context, digital photogrammetry is used for data acquisition. The knowledge about the studied objects is provided by both archaeology and photogrammetry. One task of the project was to investigate how artificial intelligence tools could be used to perform reasoning with underwater archaeological 3D surveys. More specifically, this task focused on the validation problem of underwater artefacts 3D surveys. Within this project two different conceptual descriptions of the surveyed artefacts have been proposed leading to two different solutions both developed within the Removed Sets framework. This syntactic approach is more suitable than a semantic one, in order to pinpoint the errors that cause inconsistency. The present paper provides a synthesis of these two solutions. The first solution stems from the Entity Conceptual Model for modeling generic knowledge and uses instantiated predicate logic as representation formalism and Removed Sets Fusion (RSF) with *Sum* strategy for reasoning [9]. The second one is based on an application ontology for modeling generic knowledge and the belief base is represented in instantiated predicate logic equipped with a partial preorder and Partially Preordered Removed Sets Inconsistency Handling (PPRSIH) for reasoning [17]. The paper is organized as follows. After describing in Section 2 the validation problem in the context of the VENUS project, Section 3 gives a brief synthetic presentation of the Removed Sets framework. Section 4 shows how the validation problem is expressed as a RSF problem while Section 5 shows that how the validation problem can be reduced to a PPRSIH problem. Finally, Section 6 discusses the results of the experimental study before concluding.

2 The Validation problem in VENUS

In the context of the VENUS project, digital photogrammetry is used for data acquisition. Usual commercial photogrammetric tools only focus on geometric features and do not deal with the knowledge concerning the surveyed objects. The general goal is the integration of knowledge about surveyed objects into the photogrammetric tool ARPENTEUR [5] in order to provide more "intelligent" 3D surveys. In this project, we investigated how Artificial Intelligence tools can be used for representing and reasoning with 3D surveys information.

Within the context of underwater archaeological surveys, we deal with information of different nature. Archaeologists provide expert knowledge about artefacts, in most of the cases amphorae. Archaeological knowledge takes the form of a characterization of amphorae thanks to a typology hierarchically structured. For each type corresponds a set of features or attributes which we assign an interval representing the expected values for an amphora of this type.

The data acquisition process provides measures coming from the photogrammetric restitution of surveyed amphorae pictures on the underwater site (see ① in figure 1). These observations usually are uncertain, inaccurate or imprecise since the pictures are taken *in situ*, their quality could not be optimal, because of the hostile environment: weather conditions, visibility, water muddying, site not cleaned, ... Moreover, errors could occur during the restitution step. For all these reasons, the archaeological knowledge (see ② in figure 1) and the data coming from the photogrammetric acquisition process could conflict. This special case of inconsistency handling is a validation problem because the measured values of attributes of a surveyed amphora *in situ*, an instance, may not fit with the characterization of the amphorae type it is assumed to belong to. The VENUS project does not use image recognition. The generic knowledge is inserted in the system by the ex-

[1] $http://www.venus-project.eu$

perts. There is no automatic image recognition since the experts recognise the objects in the image during the measuring step thanks to their a priori knowledge.

Example 1 *We illustrate the validation problem with the Pianosa island site [12]. There are* 8 *types of amphorae:* Dressel20*,* Beltran2B*,* Gauloise_3*, ... and each type of amphorae is characterized by* 9 *attributes,* totalHeight*,* totalWidth*,* totalLength*,* bellyDiameter*,* internalDiameter*, ... [14]. However, the only measurable attributes are* totalHeight*,* totalLength[2]*. Default values for these attributes take the form of a range of values* $[v - v.t\%, v + v.t\%]$ *centered around a typical value* v *(expressed in m.) where* t *is a tolerance threshold. For example, the default values for the attributes* totalHeight *and* totalLength *for the Dressel20 type are* [0.5328, 0.7992] *and* [0.368, 0.552]*, while for a Beltran2B type they are* [0.9008, 1.3512] *and* [0.3224, 0.4836]*. Suppose, during the photogrammetric restitution process, the expert focuses on a given amphora, he recognizes as a* Beltran2B*. When the survey provides the values* 1.13 *as* totalHeight *and* 0.27 *as* totalLength*, the question is do these values fit with the characterization of the* Beltran2B*? When the values do not fit, the most probable reason is that the measures are incorrect due to bad conditions of acquisition.*

In order to provide a qualitative representation of this validation problem, a conceptual description of archaeological knowledge is required (see ② in figure 1). Several conceptual descriptions have been used within the VENUS project. At the beginning of the project, we used a object oriented conceptual description, restricted form of the Entity Model approach [16]. The restricted Entity Model is denoted by $\mathcal{E} = \{\mathcal{C},\ \mathcal{V}^d,\ \mathcal{C}^I\}$ where $\mathcal{C}$ is a concept (or a class), $\mathcal{V}^d$ is the set of default values for the attributes, $\mathcal{C}^I$ is a set of constraints on attributes. The concepts are the types of amphorae surveyed on the archaeological site. For each concept, that is each type of amphorae, we represent the measurable attributes. The default values for these attributes take the form of a range of values and $\mathcal{V}^d$ is a set of intervals, each interval corresponding to the possible values of attributes for a given type of amphorae. The set of constraints on the attributes $\mathcal{C}^I$ consists in integrity contraints, domain constraints and conditional constraints which express the compatibility of the measured values of attributes with the default values of attributes for a given type. The belief profile consists of the generic knowledge according to the restricted Entity Model provided by the `typology.xml` file and of the instances of amphorae provided by the `amphora.xml` file.

During the project, we constructed an application ontology [13] from a domain ontology which describes the vocabulary on the amphorae (the studied artefacts) and from a task ontology describing the data acquisition process. This ontology consists of a set of concepts, relations, attributes and constraints like domain constraints. The belief base contains the application and ontology,

[2] For amphorae the attributes *totalWidth* and *totalLength* have the same value since there are revolution objects.

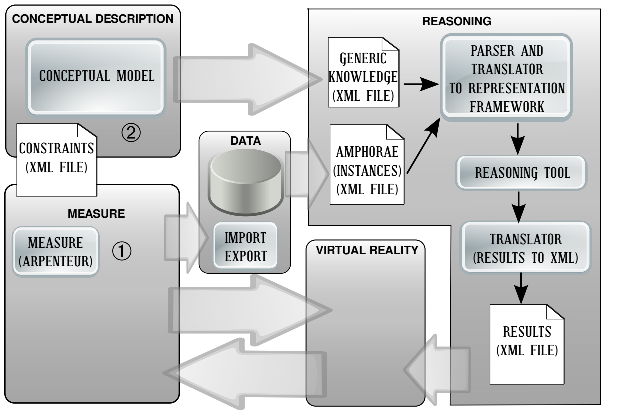


Figure 1: General scheme

constraints and observations. The ontology represents the generic knowlegde which is preferred to observations. Due to the lack of space, we only consider a small part of the ontology (Figure 2).

3 The Removed Sets Framework

The Removed Sets framework provides a syntactic belief change approach for revision and fusion. When dealing with belief change operations since we deal with uncertain, incomplete, dynamic information, inconsistency can result. In order to provide a consistent result of the change operation, the Removed Sets approach focuses on the minimal set of formulae to remove, called *Removed sets*, in order to restore consistency. The Removed Sets operations have been proved to be equivalent to the ones based on maximal consistent subsets [15; 4; 1]. However, in the context of applications where few inconsistencies may occur, the Removed Sets approach seems to be more efficient when implementing large belief bases.

Initially, the Removed Sets approach has been proposed for revising propositional formulae in CNF (RSR [11; 18]). It has then been generalized to arbitary propositional formulae for revision and fusion (RSF [9]). The Removed Sets approach has been extended to totally preordered belief bases (PRSR [2]), (PRSF [8]) and more recently to partially preordered belief bases for revision (PPRSR [17]). A central notion is the one of potential Removed Set[3] which are sets of formulas whose removal restores consistency into the union of belief bases.

Definition 1 *Let* $E = \{K_1, \ldots, K_n\}$ *be a belief profile s.t.* $K_1, \ldots, K_n$ *are propositional belief bases and* $K_1 \sqcup \ldots \sqcup K_n$ *is inconsistent (*$\sqcup$ *denotes set union with accounting for repetitions).* $X \subseteq K_1 \sqcup \ldots \sqcup K_n$ *is a* potential Removed Set *of* E *iff* $(K_1 \sqcup \ldots \sqcup K_n) \backslash X$ *is consistent.*

The collection of potential Removed Sets of E is denoted by $\mathcal{PR}(E)$. Since the number of potential Removed Sets of E is exponential w.r.t. the number of

[3] We give the definitions in the general setting of fusion where revision is a special case.

formulae, we only consider the minimal potential Removed Sets w.r.t. set inclusion. Moreover belief change operations or strategies are formalized in terms of total or partial preorders on potential Removed Sets minimal w.r.t. set inclusion. This strategies can be sorted in two families: majority operators (e.g. $Card$, Sum) which follows the point of view of the majority of the belief bases in E and egalitarist operators (e.g. Max,$GMax$) which tries to satisfy best all the belief bases in E.

3.1 Removed Sets Fusion

For Removed Sets Fusion, the fusion strategies ($Card$, Sum, Max, $GMax$)[9] are formalized thanks to a total preorder over $\mathcal{PR}(E)$. Let X and Y be two potential Removed Sets, for each strategy P a total preorder $\leq_P$ over the potential Removed Sets is defined. $X \leq_P Y$ means that X is preferred to Y according to the strategy P. We define $<_P$ as the strict total preorder associated to $\leq_P$ (i.e. $X <_P Y$ if and only if $X \leq_P Y$ and $Y \not\leq_P X$).

Definition 2 *Let $E = \{K_1, \ldots, K_n\}$ be a belief profile such that $K_1 \sqcup \ldots \sqcup K_n$ is inconsistent. $X \subseteq K_1 \sqcup \ldots \sqcup K_n$ is a* Removed Set *of E according to the strategy P if and only if i) X is a potential Removed Set of E; ii) $\nexists X' \in \mathcal{PR}(E)$ such that $X' \subset X$; iii) $\nexists X' \in \mathcal{PR}(E)$ such that $X' <_P X$.*

The collection of Removed Sets of E according to the strategy P is denoted by $\mathcal{R}_P(E)$. The Removed Sets Fusion operation is defined by:

Definition 3 *Let $E = \{K_1, \ldots, K_n\}$ such that $K_1 \sqcup \ldots \sqcup K_n$ is inconsistent. The merging operation is defined by: $\Delta_P^{RSF}(E) = \bigcup_{X \in \mathcal{R}_P(E)} \{(K_1 \sqcup \ldots \sqcup K_n) \backslash X\}$.*

3.2 Partially Preordered Removed Sets Inconsistency Handling

Let K be a finite set of arbitrary formulae and $\preceq_K$ be a partial preorder on K .$=_K$ denotes the equivalence relation $\preceq_K$ corresponding to $\preceq_K$ i.e. $a =_K b$ iff $(a \preceq_K b) \wedge (b \preceq_K a)$. Restoring the consistency of a partially preordered belief bases involves the definition of a partial preorder on subsets of formulae, called comparators [3; 19]. Several ways have been proposed for defining a preference relation on subsets of formulae of K, from a partial preorder $\preceq_K$. In the VENUS project, we focus on the lexicographic preference [19] which extends the lexicographic preorder initially defined for totally preordered belief bases to partially preordered belief bases. The belief base K is partitionned into $K = E_1 \sqcup \ldots \sqcup E_n$ $(n \geq 1)$ where each subset E_i represents an equivalence class of K w.r.t. $=_K$. A preference relation between the equivalence classes E_i's, denoted by $\prec_s$ is defined by $E_i \prec_s E_j$ iff $\exists \varphi \in E_i$, $\exists \varphi' \in E_j$ such that $\varphi \prec_K \varphi'$. This partition can be viewed as a generalization of the idea of stratification defined for totally preordered belief bases. We rephrase the lexicographic preference defined in [19] as follows:

Definition 4 *Let $K = E_1 \sqcup \ldots \sqcup E_n$ be a finite set of arbitrary formulae partitioned into equivalence class according to $=_K$. Let $\preceq_K$ be a partial preorder on $K = E_1 \sqcup \ldots \sqcup E_n$, $Y \subseteq K$ and $X \subseteq K$. Y is said to be lexicographically preferred to X, denoted by $Y \trianglelefteq_\vartriangle X$, iff $\forall i$, $1 \leq i \leq n$: if $|E_i \cap Y| > |E_i \cap X|$ then $\exists j$, $1 \leq j \leq n$ such that $|E_j \cap X| > |E_j \cap Y|$ and $E_j \prec_s E_i$.*

Let $\mathcal{PR}(K)$ be the set of potential removed sets. Among them, we want to prefer the potential removed sets which allow us to remove the formulae that are not preferred according to $\preceq_K$. Therefore we generalize the notion of Removed Sets to subsets of partially preordered formulae. We denote by $\mathcal{R}_\vartriangle(K)$ the set of removed sets of K.

Definition 5 *Let K be an inconsistent belief base equipped with a partial preorder $\preceq_K$. $R \subseteq K$ is a removed set of K iff i) R is a potential removed set; ii) $\nexists R' \in \mathcal{R}_\vartriangle(K)$ such that $R' \subset R$; iii) $\nexists R' \in \mathcal{R}_\vartriangle(K)$ such that $R' \vartriangleleft_\vartriangle R$.*

Definition 6 *Let K be an inconsistent belief base equipped with a partial perorder $\preceq_K$. The consistency restoration operation is defined by*

$$\Delta^{\Delta}(K) = \bigcup_{X \in \mathcal{R}_\vartriangle(K)} \{K \backslash X\}.$$

3.3 ASP implementation

In order to implement belief change operations within the Removed sets framework, we translate the belief change problem into a logic program with answer set semantics. This method proceeds in two stages. The first stage consists in the translation of E into a logic program Π_E and we have shown that the answer sets of Π_E correspond to the potential removed sets of E [9].

Let E be a belief profile[4]. Each propositional variable a occuring in E is represented by an ASP atom $a \in \mathcal{A}$ in Π_E. The set of all positive, (resp. negative) literals of Π_E is denoted by V^+, (resp. V^-). The set of rule atoms representing formulae is defined by $R^+ = \{r_f \,|\, f \in E\}$ and $F_O(r_f)$ represents the formula of E corresponding to r_f in Π_E, namely $\forall r_f \in R^+, F_O(r_f) = f$. This translation requires the introduction of intermediary atoms representing subformulae. We denote by ρ_f^j the intermediary atom representing f^j which is a subformula of $f \in E$. The first part of the construction has two steps:

1. We introduce rules in order to build a one-to-one correspondence between answer sets of Π_E and interpretations of V^+. For each atom, $a \in V^+$ two rules are introduced: $a \leftarrow not\ a'$ and $a' \leftarrow not\ a$ where $a' \in V^-$ is the negative atom corresponding to a.
2. We introduce rules in order to exclude the answer sets S corresponding to interpretations which are not models of $(E \backslash F)$ with $F = \{f \,|\, r_f \in S\}$. According to the syntax of f, the following rules are introduced: (i) If $f =_{def} a$ then $r_f \leftarrow not\ a$ is introduced; (ii) If $f =_{def} \neg f^1$ then $r_f \leftarrow not\ \rho_{f^1}$ is introduced; (iii) If $f =_{def} f^1 \vee \ldots \vee f^m$ then

[4] In case of inconsistency handling the profile E is reduced to a belief base K.

$r_f \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced; (iv) If $f =_{def} f^1 \wedge \ldots \wedge f^m$ then it is necessary to introduce several rules: $\forall 1 \leq j \leq m$, $r_f \leftarrow \rho_{f^j}$.

This stage is common to any belief change operation while the next one depends on the chosen belief change operation.

In case of fusion the second stage provides, according to selected strategy P, another set of rules that leads to the program Π_E^P and we have shown [9] that the answer sets of Π_E^P correspond to the removed sets of E for a strategy P. In the validation problem since we have to minimize the number of formulae to remove, therefore the number of formulae occuring in a removed set, we select the *Sum* strategy. This strategy is expressed by the *minimize*{} statement and the new logic progam $\Pi_E^{Sum} = \Pi_E \cup minimize\{r_f \mid r_f \in R^+\}$ is such that the answer sets of Π_E^{Sum} which are provided by the CLASP solver [7] correspond to the removed sets of $\Delta_{Sum}^{RSF}(E)$ [9].

In case of partially Preordered Removed Sets Inconsistency Handling the CLASP solver [7] gives the answer sets of Π_E. We then construct a partial preorder between them using the lexicographic comparator $\trianglelefteq_{\Delta}$. We have shown in [17] that the preferred answer sets according to $\trianglelefteq_{\Delta}$ correspond to the removed sets of E. We used a java program to partially preorder the answer sets to obtain the preferred answer sets. Since the lexicographic comparator satisfies the monotony property [19], it is sufficient to compare the answer sets which are minimal according to the inclusion. Moreover, the determination of the minimal answer sets according to this partial preorder does not increase the computationnal cost, since this cost is insignificant compared to the cost of answer sets computation by CLASP.

4 The validation problem wihin RSF

In order to represent the validation problem within the RSF framework and to implement it with ASP, we represented this problem with instantiated predicate logic. The belief profile consists of two belief bases. The first one stems from the restricted Entity Model conceptual description and represents the generic knowledge. We introduce the predicates $type(x, y)$ and $cmp(z, y, x)$ where x is an amphora, y is a type of amphorae and z is an attribute. $type(x, y)$ expresses that an amphora x belongs to a type y and $cmp(z, y, x)$ expresses that an attribute z of an amphora x of type y has a value compatible with the possible values for the type y, as specified in 2. The domain constraints specify that an amphora must have one and only one type. For n types of amphorae, for each amphora there is one disjunction $type(x, y_1) \vee \ldots \vee type(x, y_n)$ and $n(n-1)/2$ mutual exclusion formulae $\neg type(x, y_i) \vee \neg type(x, y_j)$. The conditional constraints specify the compatibility of the attributes values with respect to the type. For each amphora x, for each attribute z and for each type y, there is a formula $type(x, y) \rightarrow cmp(z, y, x)$. Let m be the number of attributes, the incompatibility of type specifies that for each amphora and each type there is a formula $\neg cmp(z_1, y, x) \wedge \ldots \wedge \neg cmp(z_m, y, x) \rightarrow \neg type(x, y)$. The second belief base represents the instances of amphorae: the type the observed amphora belongs to (namely $type(x, y)$) and the compatible attributes with the type (namely $cmp(z, y, x)$). We illustrate the RSF approach with the example 1.

Example 2 *We limit ourselves to only two types of amphorae Beltran2B and Dressel20, respectively denoted by $B2B$ and $D20$ thereafter, and to the survey of one observed amphora (denoted by 4 hereafter). Two attributes are used: totalHeight (denoted by tH) and totalLength (denoted by tL). The first belief base is automatically generated from the typology.xml file and* $K_1 = \{\neg type(4, B2B) \vee \neg type(4, D20), type(4, B2B) \vee type(4, D20), type(4, D20) \rightarrow cmp(tH, D20, 4), type(4, D20) \rightarrow cmp(tL, D20, 4), type(4, B2B) \rightarrow cmp(tH, B2B, 4), type(4, B2B) \rightarrow cmp(tL, B2B, 4), \neg cmp(tH, B2B, 4) \wedge \neg cmp(tL, B2B, 4) \rightarrow \neg type(4, B2B), \neg cmp(tH, D20, 4) \wedge \neg cmp(tL, D20, 4) \rightarrow \neg type(4, D20)\}$. *The second belief base corresponding to the observed amphora is automatically generated from typology.xml and amphora.xml files and* $K_2 = \{type(4, B2B), cmp(tH, B2B, 4)\}$. *The operation $\Delta_{Sum,\top}^{RSF}(E)$ where $E = \{K_1, K_2\}$ is translated into Π_E^{Sum} as follows:*

$cmp(tH, B2B, 4).$
$1\ \{type(4, d20), type(4, B2B)\}\ 1.$
$\leftarrow n\_type(4, d20), type(4, d20).$
$\leftarrow n\_type(4, B2B), type(4, B2B).$
$r(x_0) \leftarrow not\ type(4, B2B).$
$r(x_1) \leftarrow type(4, d20), not\ cmp(tH, d20, 4).$
$r(x_2) \leftarrow type(4, d20), not\ cmp(tL, d20, 4).$
$r(x_3) \leftarrow type(4, B2B), not\ cmp(tH, B2B, 4).$
$r(x_4) \leftarrow type(4, B2B), not\ cmp(tL, B2B, 4).$
$r(x_5) \leftarrow type(4, d20), not\ cmp(tH, d20, 4), not\ cmp(tL, d20, 4).$
$r(x_6) \leftarrow type(4, B2B), not\ cmp(tH, B2B, 4), not\ cmp(tL, B2B, 4).$
$n\_type(4, B2B) \leftarrow r(x_0).$
$n\_cmp(tH, d20, 4) \leftarrow r(x_1).$
$n\_cmp(tL, d2, 4) \leftarrow r(x_2).$
$n\_cmp(tH, B2B, 4) \leftarrow r(x_3).$
$n\_cmp(tL, B2B, 4) \leftarrow r(x_4).$
$minimize\ \{r(x_0), r(x_1), r(x_2), r(x_3), r(x_4) r(x_5), r(x_6)\}.$

Note that the ASP translation uses some shortcuts compared to the translation scheme depicted in section 3.3. Thanks to the cardinality literals by recent ASP solvers, the unique type constraint is reduced to a single rule $1\ \{type(4, d20), type(4, B2B)\}\ 1$. *Also, the generation of the rule corresponding to $type(4, B2B)$ and the mutual exclusion between this atom and its classical negation are compacted into a single rule.*

The only answer set of the above program is $\{cmp(tH, B2B, 4), type(4, B2B), r(x_4), n\_cmp(tL, B2B, 4)\}$ *which corresponds to the removed set* $\{type(4, B2B) \rightarrow cmp(tL, B2B, 4)\}$ *that pinpoints a bad measure for the total length attribute under the hypothesis of an amphora of type Beltran2B.*

5 The validation problem within PPRSIH

The conceptual description in this approach is represented in terms of an application ontology and an extract is illustrated in Figure 2.

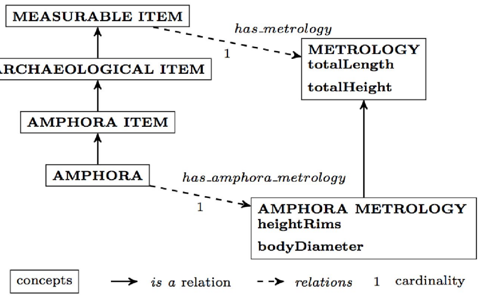


Figure 2: Extract of the application ontology

The belief base consists of the application ontology, the constraints and the instances of amphorae represented in predicate logic. The introduced predicates are shown in an instantiated version in Table 1. The formulae corresponding to the extract of the ontology are given below where $amph$, $amph\_it$, $arch\_it$, $meas\_it$, $metro$, has_metro, tL, tH, $type$ denote $amphora$, $amphora\_item$, $archaeological\_item$, $measurable\_item$, $metrology$, $has\_metrology$, $totalLenght$, $total\ Height$, $typology$ respectively: $\forall x\ arch\_it(x) \rightarrow meas\_it(x)$, $\forall x\ amph\_it(x) \rightarrow arch\_it(x)$, $\forall x\ amph(x) \rightarrow amph\_it(x)$, $\forall x\, meas\_it(x) \rightarrow \exists z\, has\_metro(x,z)$, $\forall x \forall z\, has\_metro(x,z) \rightarrow metro(z)$, $\forall z\ \ metro(z) \rightarrow \exists l\, tL(z,l) \wedge \exists h\, tH(z,h)$, $\forall x\, amph(x) \rightarrow amph\_it(x) \wedge (type(x,y_1) \vee \cdots \vee type(x,y_n))$. The set of constraints consists in integrity constraints which specify that the value of attributes do not exceed a given value, domain constraints are specified by cardinality constraints within the application ontology and conditional constraints express the compatibility of the attribute values with respect to the type. The domain constraints are expressed like in Section 4 by one disjunction $\forall x\, type(x,y_1) \vee \cdots \vee type(x,y_n)$ and $n(n-1)/2$ mutual exclusion formulae $\neg type(x,y_i) \vee \neg type(x,y_j)$. The integrity constraints are expressed by the formulae: $\forall x\ \ meas\_it(x) \rightarrow \exists z \exists h (tH(z,h) \wedge cmpMItH(h,x))$, $\forall x\ \ meas\_it(x) \rightarrow \exists z \exists l (tL(z,l) \wedge cmpMItL(l,x))$, $\forall x\, arch\_it(x) \rightarrow \exists z \exists h (tH(z,h) \wedge cmpARItH(h,x))$, $\forall x\ \ arch\_it(x) \rightarrow \exists z\ \exists l\ (tL(z,l) \wedge cmpARItL(l,x))$, $\forall x\, amph\_it(x) \rightarrow \exists z \exists h (tH(z,h) \wedge cmpAItH(h,x))$, $\forall x\, amph\_it(x) \rightarrow \exists z \exists l (tL(z,l) \wedge cmpAItL(l,x))$. The conditional constraints are expressed by the formulae: $\forall x\, type(x,y_i) \rightarrow \exists z \exists h (tH(z,h) \wedge cmptH(h,y_i))$ $\forall x$, $type(x,y_i) \rightarrow \exists z \exists l (tL(z,l) \wedge cmptL(l,y_i))$. The formulae corresponding to the instances of amphorae are $amph(x)$, $type(x,y)$, $metro(z)$, $meas\_it(x)$, $arch\_it(x)$, $amph\_it(x)$, $has\_metro(x,z)$, $tL(z,l) \wedge cmpMItL(l,x) \wedge cmpARItL(l,x) \wedge cmpAItL(l,x) \wedge \neg cmptL(l,\ \ y_i)$ and $tH(z,h) \wedge cmpMItH(h,x) \wedge cmpARItH(h,x) \wedge cmpAItH(h,x) \wedge \neg cmptH(h,\ \ y_i)$. The belief base is equipped with a partial preorder which reflects the hierarchy of concepts in the ontology. Moreover constraints are preferred to the ontology which is preferred to the instances. We illustrate the PPRSIH approach thanks to example 1.

Example 3 *We limit ourselves to the amphorae types $Beltran2B$ and $Dressel20$ and to the survey of the observed amphora denoted by* 4. *Table 1 presents the instantiated predicates and Figure 3 illustrates the partially preordered belief base.*

predicate	**p**
$meas\_it(4)$	m_i
$amph(4)$	a
$type(4, Beltran2B)$	b
$tH(m,h)$	h
$cmpAItL(l,4)$	c_{AI_l}
$cmpAItH(h,4)$	c_{AI_h}
$cmptL(l, Beltran2B)$	c_{l_b}
$type(4, Beltran2B)$	b
predicate	**p**
$arch\_it(4)$	ar_i
$metro(m)$	m
$has\_metro(4,m)$	h_m
$cmpMItL(l,4)$	c_{MI_l}
$cmpMItH(h,4)$	c_{MI_h}
$cmptL(l, Dressel20)$	c_{l_d}
$cmptH(h, Beltran2B)$	c_{h_b}
$metro(am)$	a_m
predicate	**p**
$amph\_it(4)$	a_i
$type(4, Dressel20)$	d
$tL(m,l)$	l
$cmpARItL(l,4)$	c_{ARI_l}
$cmpARItH(h,4)$	c_{ARI_h}
$cmptH(h, Dressel20)$	c_{h_d}
$amph(4)$	a

Table 1: instantiated predicates and their corresponding proposition p

The validation problem is translated into a logic progam Π_E in the same spirit than the one presented in section 3.3. CLASP provides 1834 *answer sets. However, if only focusing on the minimal answer sets with respect to inclusion we have to partially preorder 320 answer sets. According to the lexicographic comparator $\trianglelefteq_\Delta$, we obtain two uncomparable preferred answer sets S_1 and S_2 such that $F_O(S_1 \cap R^+) = \{a,b\}$ and $F_O(S_2 \cap R^+) = \{l \wedge c_{ARI_l} \wedge c_{AI_l} \wedge c_{MI_l} \wedge \neg c_{l_b}\}$. Therefore, there are two removed sets $R_1 = \{a,b\}$ and $R_2 = \{l \wedge c_{ARI_l} \wedge c_{ARI_l} \wedge c_{MI_l} \wedge \neg c_{l_b}\}$. The removed set R_1 pinpoints the typology while R_2 pinpoints that the value of TotalLength attribute may be wrong. This approach provides 2 removed sets while the RSF one only provides one removed set. The reason is that in PPRSIH approach the typology is only suspected if the value of one of the attributes is incompatible while in RSF approach the typology is suspected if the values of more than one attributes are incompatible.*

6 Concluding discussion

We now present the results of the experimental study, first on the full Pianosa survey which contains 40 amphorae then on the Port-Miou survey which contains 500

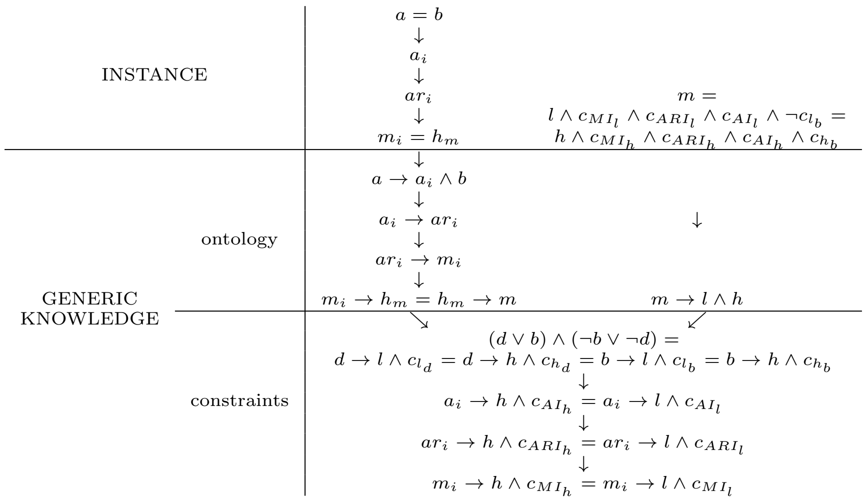


Figure 3: Partial preorder on formulae of the belief base

amphorae. We used 4 different tolerance thresholds t around the typical values of each type: 20%, 10%, 5% and 1% and N denotes the number of inconsistent amphorae. The CPU times $T1$, $T2$, $T3$ and T correspond to the translation from the XML files to the logic program, the ASP implementation of RSF, the translation from ASP to an XML file and the total time $T1+T2+T3$ respectively. The tests were conducted on a Centrino Duo cadenced at 1.73GHz and equipped with 2GB of RAM. The results are summarized in Table 2.

		RSF				PPRSIH			
t	**N**	**T1**	**T2**	**T3**	**T**	**T1**	**T2**	**T3**	**T**
20	5	0.05	0.62	0.95	1.62	0.24	1.12	0	1.36
10	26	0.05	0.60	0.64	1.29	0.27	5.13	0	5.40
5	30	0.05	0.61	0.45	1.11	0.29	5.87	0	6.16
1	36	0.05	0.60	0.33	0.98	0.31	6.91	0	7.22

(a) Pianosa survey (40 amphorae)

	RSF				PPRSIH			
N	**T1**	**T2**	**T3**	**T**	**T1**	**T2**	**T3**	**T**
44	0.43	5.26	0.14	5.83	0.68	9.38	0	10.06
65	0.43	5.06	0.04	5.53	0.75	13.69	0	14.44
72	0.43	4.99	0	5.42	0.81	15.03	0	15.84
81	0.43	5.06	0	5.49	0.88	16.80	0	17.68

(b) Port Miou survey (500 amphorae)

Table 2: CPU times (s) for RSF and PPRSIH on two surveys.

Concerning the knowledge representation aspect the RSF approach stems from the Entity Model conceptual description and uses instantiated predicate logic. It creates a flat knowledge base, with numerous formulae, where all the objects are at the same level. In the full Pianosa survey involving 40 amphorae, the traduction of the problem requires 8462 formulae and 4160 atoms and in the full Port Miou involving 500 amphorae, the traduction of the problem requires 105775 formulae and 52000 atoms. Moreover, it only considers the intrinsic constraints between objects. However, the lack of expressivity and the high number of formulae are compensated by the good computational behaviour of the reasoning tasks expressed in this language. The PPRSIH approach stems from the application ontology and uses instantiated predicate logic equipped with a partial preorder. It creates a more structured belief base, involving less formulae than the first approach. In the full Pianosa survey involving 40 amphorae, the traduction of the problem requires 1080 formulae and 840 atoms and in the full Port-Miou survey involving 500 amphorae and the traduction requires 6021 formulae and 4683 atoms. It allows for representing the intrinsic constraints as well as the taxonomic relations between objects, and relations between objects. The partial preorder defined on the finite set of formulae expresses more structure than the first solution. This approach takes advantage of the good computational behaviour of instantiated predicate logic while expressing, in the same time, a more structured belief base.

Concerning the reasoning aspect, both implementations rely on CLASP which is one of the most efficient current ASP solver. The results obtained on Pianosa as well as on Port Miou survey given in Table 2 clearly show that both approaches deal with the full survey with a very good time. However, the first solution gives the best running times. Moreover, reducing the tolerance intervals increases the number of inconsistencies as illustrated in table 2 and the first solution seems to be not sensitive to this increasing while the running time of the second solution grows with the number of inconsistencies. The consuming task comes from the reading of the answer sets before partially ordering them in order to only select the preferred ones. In order to improve this approach we have to investigate how to directly encode the partial preorder on answer sets within the logic program. Another direction to follow in order to reach a trade-off between representation and reasoning could be to represent the validation problem in Description Logic, since the generic knowledge in expressed in terms of ontology. However, we have to study which low complexity Description Logic could be suitable. Moreover, we have to study to which extent the approach combining Description logic and ASP [6] could be used for implementation as well as the extended ASP solver to first order logic[10].

7 Acknowledgements

Work partially supported by the European Community under project VENUS (Contract IST-034924) of the "Information Society Technologies (IST) program of the 6th FP for RTD". The authors are solely responsible for the content of this paper. It does not represent the opinion of the European Community, and the European Community is not responsible for any use that might be made of data appearing therein.

References

[1] C. Baral, S. Kraus, J. Minker, and V. S. Subrahmanian. Combining knowledge bases consisting of first order theories. In *Proc. of ISMIS*, pages 92–101, 1991.

[2] S. Benferhat, Jonathan Ben-Naim, Odile Papini, and Eric Würbel. An answer set programming encoding of prioritized removed sets revision: application to gis. *Applied Intelligence*, 32(1):60–87, 2010.

[3] S. Benferhat, S. Lagrue, and O. Papini. Revision of partially ordered information : Axiomatization, semantics and iteration. In *Proc. of IJCAI'05*, pages 376–381, Edinburgh, 2005.

[4] G. Brewka. Preferred sutheories: an extended logical framework for default reasoning. In *Proc. of IJCAI'89*, pages 1043–1048, 1989.

[5] P. Drap and P. Grussenmeyer. A digital photogrammetric workstation on the web. *Journal of Photogrammetry and Remote Sensing*, 55(1):48–58, 2000.

[6] T. Eiter, G. Ianni, T. Lukasiewicz, R. Schindlauer, and H. Tompits. Combining answer set programming with description logics for the semantic web. *Artificial Intelligence*, 172(12-13):1495–1539, 2008.

[7] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub. clasp: A conflict-driven answer set solver. In *Proc. of LPNMR'07*, pages 260–265. Springer, 2007.

[8] J. Hué, O. Papini, and E. Würbel. Implementing prioritized merging with asp. In *Proc. of IPMU*, volume 80 of *CCIS*, pages 138–147. Springer, 2010.

[9] J. Hué, E. Würbel, and O. Papini. Removed sets fusion: Performing off the shelf. In *Proc. of ECAI'08*, pages 94–98, 2008.

[10] C. Lefevre and P. Nicolas. A first order forward chaining approach for answer set computing. logic programming and nonmonotonic reasoning. In *Proc. of LPNMR'09*, pages 196–208. Springer, 2009.

[11] O. Papini. A complete revision function in propositionnal calculus. In B. Neumann, editor, *Proc. of ECAI'92*, pages 339–343. John Wiley and Sons. Ltd, 1992.

[12] O. Papini. D3.1 archaeological activities and knowledge analysis. Technical report, Delivrable, VENUS project. http://www.venus-project.eu, january 2007.

[13] O. Papini, O. Curé, P. Drap, B. Fertil, J. Hué, D. Roussel, M. Sérayet, J. Seinturier, and E. Würbel. http://www.venus project.eu. D3.6 reasoning with archaeological ontologies. technical report and prototype of software for the reversible fusion operations. Technical report, Delivrable, VENUS project, july 2009.

[14] O. Papini, E. Würbel, R. Jeansoulin, O. Curé, P. Drap, M. Sérayet, J. Hué, J. Seinturier, and L. Long. D3.4 representation of archaeological ontologies 1. Technical report, Delivrable, VENUS project. http://www.venus-project.eu, July 2008.

[15] N. Rescher and R. Manor. On inference from inconsistent premises. *Theory and Decision*, 1:179–219, 1970.

[16] J. Seinturier. *Fusion de connaissances: Applications aux relevés photogrammétriques de fouilles archéologiques sous-marines.* PhD thesis, Université du Sud Toulon Var, 2007.

[17] M. Sérayet, P. Drap, and O. Papini. Extending removed sets revision to partially preordered belief bases. *International Journal of Approximate Reasoning*, 52(1):110–126, 2011.

[18] E. Würbel, O. Papini, and R. Jeansoulin. Revision: an application in the framework of gis. In *Proc. of KR'00*, pages 505–516, Breckenridge, Colorado, USA, 2000.

[19] S. Yahi, S. Benferhat, S. Lagrue, M. Sérayet, and O. Papini. A lexicographic inference for partially preordered belief bases. In *Proc. of KR'08*, pages 507–516, 2008.

An application-oriented view on graded spatial relations

Steven Schockaert*
Department of Applied Mathematics and Computer Science,
Ghent University, Gent, Belgium
steven.schockaert@ugent.be

Abstract

Approaches for modeling graded spatial relations abound in the literature on image processing and on geographic information systems. In contrast, few proposals have addressed the use of degrees in application contexts where inference plays a central role. We argue that the use of degrees is nonetheless natural in such contexts, and may enhance the application potential of qualitative spatial reasoning in domains where relations between vague regions need to be expressed, as well as domains where the robustness of spatial relations w.r.t. small changes in the underlying configurations is important. In particular, we discuss the interpretation of degrees, contrasting fuzzy regions with formal accounts of spatial vagueness based on supervaluation semantics. We also touch upon the problem of acquiring (fuzzy) spatial relations from the web in an automated way.

1 Introduction

The field of qualitative spatial representation and reasoning (QSR) deals with symbolic representations of spatial configurations [4], modeling for instance how two regions are topologically related [17] (e.g. whether some geographic region overlaps with another) or what the relative position is of a point w.r.t. a vector [29] (e.g. the relative position of a landmark w.r.t. a moving subject). Typically, approaches to QSR are based on a small number of jointly exhaustive and pairwise disjoint (JEPD) relations. In the well-known Region Connection Calculus (RCC [17]), for instance, the topological relationship of two regions is always one of the so-called RCC8 relations: DC (disconnected), EC (externally connected), PO (partially overlapping), EQ (equal), TPP (tangential proper part), NTPP (non-tangential proper part), TPP^{-1} (inverse of TPP), or NTPP^{-1} (inverse of NTPP).

Despite the elegance and conceptual simplicity of such frameworks, the way they discretize a continuum of possible spatial configurations sometimes leads to unintuitive behavior. When spatial relations are derived from images, for instance, one pixel can make the difference between EC and DC, and, even worse, whether an EC or DC relation is found may depend on the resolution of the image. Along similar lines, when most of some region a is contained in a region b, it may be more natural to think of a as being a part of b than to think of a and b as partially overlapping regions. In geography, the situation is further complicated by the fact that many toponyms cannot be characterized by precise boundaries (e.g. downtown Barcelona). It may be difficult to assess the spatial relationship between two vague regions (e.g. *Northern Spain* and *Central Catalonia*), as different views on the delineation of these regions may correspond to different spatial relations.

Many approaches address this kind of problems by assuming spatial relations to be graded (or *fuzzy*). Then it becomes possible to say that Northern Spain and Central Catalonia are overlapping *to some degree*, while Central Catalonia is also considered to be a part of Northern Spain *to some degree*. How these graded relations are defined depends on the application. In this respect, it is useful to note that applications of QSR can roughly be divided in two classes. In a first class of applications, qualitative spatial relations are used to interface between a known quantitative description of a scene and natural language (e.g. to describe a route or to query a geographic information system), or more generally, to abstract away from irrelevant details in quantitative representations (e.g. to recognize types of events [9]). Fuzzy spatial relations have been widely studied in this kind of applications, both from the angle of image processing and understanding (e.g. [12; 11; 25; 14]), and from the angle of geographic information systems (e.g. [18; 15; 21]). The main purpose of using degrees, here, is to induce a total ordering, e.g. to decide which among a given number of situations best satisfies some query. For this first class of applications, one of the main considerations is that relations be cognitively meaningful, more than obeying nice mathematical properties or being easy to compute.

The second class of applications uses qualitative descriptions of spatial scenes as a surrogate for quantitative descriptions, when the latter are not available or require a prohibitive amount of computation. In such applications, symbolic reasoning plays a key role to verify the integrity of qualitative descriptions (e.g. in geo-ontologies [26]) and to augment available knowledge by explicitly deriving its logical consequences. In order to support inference, it is important that spatial relations are defined in such a way that they satisfy important mathematical properties, related to transitivity, re-

*Postdoctoral fellow of the Research Foundation – Flanders.

flexivity, etc., even if the resulting models are somewhat less rich from a cognitive point of view. However, this second class of applications is less prevalent, especially when graded approaches are concerned, as (i) in general, qualitative descriptions are often difficult to obtain in practical applications (other than from quantitative descriptions) and (ii) graded approaches face the added difficulty of obtaining meaningful degrees. The main aim of this paper is to discuss these two issues regarding the application potential of (fuzzy) qualitative spatial reasoning.

The remainder of the paper is structured as follows. In the next section, we discuss a number of ways in which vague regions can be modelled. Then, Section 3 discusses the notion of a fuzzy region, carefully distinguishing it from the related notion of a vague region. In Section 4, we discuss the rationale of fuzzy spatial relations as a compact way of encoding the spatial relationship of fuzzy regions. As a running example, we focus on topological relations, using a fuzzy version of the RCC. Then Section 5 zooms in on the key problem of acquiring (fuzzy) qualitative spatial knowledge. In particular, we review some techniques that have been proposed in the area of geographic information retrieval.

2 Vague regions

Early on, QSR approaches have been extended to cope with the indeterminacy of the boundaries of geographic regions. In the Egg-Yolk calculus [3], and in many related formalisms, regions are represented using two nested sets. The smallest set (called the yolk) corresponds to a lower approximation of the region while the largest set (called the egg) corresponds to an upper approximation.

We may wonder how exactly an (egg,yolk) pair should be interpreted. Often, an epistemic view is assumed in this context. An (egg,yolk) pair then represents our knowledge about where the true boundaries are located. This knowledge may be refined after we have learned more about the region, which is taken into account in the Egg-Yolk calculus by the introduction of a primitive "crisper than" relation. This relation models that one (egg,yolk) pair is a refinement of another one, i.e. a more precise approximation of the same region. However, it should be noted that such a view is at odds with the idea that the spatial relationship between two regions can be determined from their (egg,yolk) representations. Indeed, let $(\overline{a}, \underline{a})$ be the representation of a region a. Clearly, if a has unknown, but precise boundaries, we should always have that $P(a, a)$ holds, where P denotes the part-of relation. Now suppose that $(\overline{a}, \underline{a})$ also models what we know about region b, then typically there are several spatial relations that may still hold between a and b. This means that the spatial relationship between $(\overline{a}, \underline{a})$ and itself may differ depending on whether both occurrences of the pair $(\overline{a}, \underline{a})$ refer to the same or to different entities. In fact, this is strongly related to the well-known observation that no uncertainty calculus can ever be compositional [8]. Moreover, the epistemic view does not conform well with intuition when it comes to geographic regions. The fact that downtown Barcelona does not have precise boundaries is not related to our lack of knowledge about Barcelona, but due to an inherent form of indeterminacy.

Seeing the pair $(\overline{a}, \underline{a})$ not as an imperfect description of a single crisp boundary, but as the perfect description of a vague boundary is close to the supervaluationist view of spatial vagueness [28; 1]. Essentially, under this latter view, the label of a vague region is seen as an under-specified description of a region in space. The possible ways in which the vague region may be interpreted are called the precisifications of the region, and statements (e.g. about which spatial relations hold) may be true for all precisifications of the underlying regions (in which case the statement is *supertrue*), for none of the precisifications (in which case it is *superfalse*), or for some but not all (in which case it lacks a truth value). The pair $(\overline{a}, \underline{a})$ can then be seen as defining the possible precisifications of region a. However, not every region which contains $\underline{a}$ and is contained in $\overline{a}$ is likely to correspond to a plausible precisification. To cope with this, it has recently been proposed [2] to add more structure to the set of precisifications, by linking each precisification to some value of an underlying parameter (or set of parameters). The resulting semantics is called *standpoint semantics*, and the different parameter values are called standpoints. For example, possible precisifications of downtown Barcelona may be linked to parameters that refer to population density, commercial activity, distance to prominent landmarks, etc.

Given this latter view on vague regions, how should we model the spatial relationship between two regions a and b? In principle, this relationship is determined by associating one spatial relation (say an RCC-8 relation) with each pair of precisifications, viz. with each pair of parameter choices. Let $\mathcal{C}$ be the set of all possible choices of all relevant parameters, then we may see the spatial relationship between a and b as a mapping ρ from $\mathcal{C} \times \mathcal{C}$ to $RCC8$, where we write $RCC8$ for the set $\{DC, EC, PO, EQ, TPP, NTPP, TPP^{-1}, NTPP^{-1}\}$. This solution seems to be within the spirit of the standpoint semantics, where the different standpoints regarding the spatial relationship are pairs of standpoints regarding the delineation of regions. However, from an application point of view, this approach is still not entirely satisfactory, as it is not clear how (or whether) the mapping ρ can be represented in a compact way. Note in particular that the set $\mathcal{C}$ itself may be infinite, and we may not even know which parameters are relevant in a given context. As we will see, fuzzy spatial relations can be used as a compact, approximate representation of the mapping ρ, relative to a given context.

3 Fuzzy regions

Assume that $\mathcal{C}$ is equipped with a probability distribution $p_{\mathcal{C}}$, which encodes, in a given context, how likely it is that the elements of $\mathcal{C}$ are actually considered as standpoints. For instance, not every population density value is equally likely to be used as a threshold value in the definition of the boundaries of downtown Barcelona. As each element of $\mathcal{C}$ corresponds to a region, $p_{\mathcal{C}}$ corresponds to a probability distribution on regions. If we moreover see a region as a set of locations (i.e. points), $p_{\mathcal{C}}$ corresponds to a probability distribution on sets of locations, i.e. a random set of locations.

Recall that a random set in a universe U is a probability distribution m on the power set of U. For the ease of presentation, we will restrict ourselves to finite universes[1]. A subset $X \subseteq U$ is called a focal element of a random set m if $m(X) > 0$. It is well-known that fuzzy sets can be seen as special cases of random sets, where the focal elements all belong to a family of nested sets $X_0 \subseteq ... \subseteq X_n \subseteq U$ [7]. The corresponding fuzzy set is usually defined in terms of its membership function A ($u \in U$):

$$A(u) = \sum_{u \in X_i} m(X_i) \tag{1}$$

Note in particular that $A(u) \in [0, 1]$ then reflects the probability that a standpoint is taken for which u is assumed to belong to the region being modeled. In general, the focal elements corresponding to all the possible standpoints $\mathcal{C}$ will not necessarily be nested sets. In that case, a fuzzy set can only represent an approximation of the actual random set model of the spatial region. More details about the relationship between random sets and fuzzy sets can be found in [7].

For applications, the use of fuzzy sets to model the spatial extent of regions has a number of important advantages. First, the random set encoding allows for a compact representation of a fuzzy region as a list of classical regions. In most applications, the number of focal elements can indeed be taken finite and small. In situations where we want a continuum (e.g. to use a gradual boundary for regions such as Central Catalunia), a field-based representation based on the membership function (1) may be more appropriate. Moreover, fuzzy sets are simple and intuitive to use, and do not require access to the set of parameters underlying the possible standpoints. As a result, it becomes possible to estimate fuzzy regions in a purely data-driven manner, e.g. by analyzing web documents [19] or by conducting surveys [15]. Thus, while the degrees underlying a fuzzy set representation may conceptually be linked to meta-standpoints, in practice we do not require access to the distribution $p_{\mathcal{C}}$, or even the set $\mathcal{C}$: fuzzy sets may be directly estimated based on statistical evidence.

It is important to note, however, that in contrast to supervaluationist approaches, fuzzy sets do not actually model any vagueness underlying the boundaries of a region. In fact, the probability distribution $p_{\mathcal{C}}$ which we assumed to exist could be seen as a meta-standpoint regarding the interpretation of a vague region. In other words, fuzzy sets offer a precise, but graded representation of vague regions, which are valid only under a particular view. We refer to [16] for a discussion on the relationship between vagueness and fuzziness.

4 Fuzzy spatial relations

Let A be the fuzzy set (membership function) corresponding to a region a. For each $\lambda \in]0, 1]$ we can consider the λ-cut A_λ defined as

$$A_\lambda = \{x \mid A(x) \geq \lambda\}$$

We can thus characterize the spatial relationship between two regions a and b, modeled by the membership functions A and B, as a $]0, 1]^2 \rightarrow RCC8$ mapping, which maps each $(\lambda, \mu) \in]0, 1]^2$ to the RCC8 relation that holds between A_λ and B_μ. While being expressive, this approach is not suitable in applications, due to the high number of different relationships that can thus be described. Even when we restrict ourselves to a finite subset of the unit interval [0,1], the number of different relations that can be described quickly becomes prohibitively high. For instance, the restriction to $\{0.5, 1\}$ would lead to a calculus which is isomorphic to the Egg-Yolk calculus (based on RCC8 relations in this case).

To cope with the high number of possible relationships, the idea of using fuzzy spatial relations is to group types of configurations which are sufficiently similar for a given purpose. For instance, assume that we restrict ourselves to $\{0.5, 1\}$ and that $PO(A_{0.5}, B_{0.5})$, $PO(A_{0.5}, B_1)$, $DC(A_1, B_{0.5})$, $DC(A_1, B_1)$. This intuitively means that the spatial relationship is PO if we are sufficiently tolerant in the definition of the boundaries of some region and DC otherwise. In this sense, the previous configuration is similar to one where e.g. $DC(A_{0.5}, B_{0.5})$, $DC(A_{0.5}, B_1)$, $PO(A_1, B_{0.5})$, $PO(A_1, B_1)$. In the fuzzy RCC [22], these two configurations are described in the same way, by asserting that $PO(A, B)$ and $DC(A, B)$ both hold to degree 0.5. In fact, in the fuzzy RCC each spatial configuration is described by the degree to which six primitive relations are satisfied (C, O, P, NTP, P^{-1} and NTP^{-1}). From these degrees, the degrees to which each of the RCC8 relations are satisfied can be calculated. As an example, one possible way to define the degree to which $C(A, B)$ holds is:

$$C(A, B) = \max(0, \sup\{\lambda + \mu - 1 \mid C(A_\lambda, B_\mu)\}) \tag{2}$$

where $C(A_\lambda, B_\mu)$ holds if the classical regions A_λ and B_μ are connected in the sense of the RCC. The degree to which A and B are disconnected is then defined as $DC(A, B) = 1 - C(A, B)$.

It is important to note that this use of degrees is fundamentally different from the use of degrees to model fuzzy regions. Indeed, in the latter case, degrees have a clear quantitative interpretation, and can be linked to a random set (although other interpretations are possible, e.g. interpreting fuzzy sets in terms of likelihood functions [6]). In the former case, however, degrees are used as a technical tool which enables us to use a more compact encoding. This is similar to the view put forward by De Finetti [5] that the notion of graded truth in multi-valued logics is (only) useful to allow for more compact descriptions (rather than as a way to reject the principle of bivalence).

As graded relations are thus used to group similar configurations, we have some freedom in how they are defined. In particular, it is desirable that a fuzzy RCC behaves in a way which is similar to the classical RCC, and that important properties related to transitivity among others are satisfied. In [22], it was shown that a fuzzy RCC can be built from an abstract, graded connection relation, in much the same way as the classical RCC is built from an abstract, crisp connection relation. As in the classical RCC, sound and complete inference procedures for the fuzzy RCC can be derived from composition tables, and the overall complexity of the main reasoning problems also remains the same (NP-complete). Prac-

[1] It is important to note, however, that the following discussion generalizes to the infinite case.

tical reasoning with the fuzzy RCC can be done using standard RCC reasoners by virtue of some form of finite model property (Proposition 4 in [22]). As a result, as far as reasoning problems such as satisfiability checking are concerned, it is always possible to represent a fuzzy region as a nested set of crisp region and to express fuzzy spatial relations between the fuzzy regions in terms of disjunctions of classical RCC relations between these crisp regions. In this sense, the fuzzy RCC could be seen as classical qualitative calculus with a large number of spatial relations (depending on the number of different degrees that are used in the input). It would be interesting to see whether techniques that have been developed in the QSR community to cope with large qualitative calculi [13] would help to implement more efficient solvers, taking more of the inherent structure of fuzzy regions into account. Alternatively, most reasoning problems in the fuzzy RCC can straightforwardly be deduced to disjunctive linear programming, for which dedicated solvers exist.

Several models for the fuzzy RCC can be considered, which differ in how the connection relation C is defined. One model is based on the notion of connection presented in (2). It is then assumed that regions are represented as fuzzy sets, but that the type of relations we are interested in remain purely qualitative. Another model was proposed in [20] based on the idea of closeness. In the latter model, two regions are defined to be connected to the extent that they are close to each other, where regions may either be classical or fuzzy sets. Note that although the motivation for making the connection relation graded is quite different in both cases (resp. dealing with fuzzy regions and robustness of transitions such as DC/EC or TPP/NTPP), the resulting calculus is identical. Indeed, the fuzzy RCC inferences are sound regardless of how C is defined, and they are complete w.r.t. both of the aforementioned choices [22].

5 Acquiring spatial information

Approaches for qualitative spatial reasoning are mainly useful in domains where qualitative descriptions of spatial configurations can be obtained, but no quantitative models, and where qualitative results are sufficient. One application where qualitative spatial reasoning is of potential interest is geographic information retrieval. Knowledge about qualitative relations plays a key role, for instance, in query expansion [10]. Moreover, in this domain, there is a strong interest in vernacular places, whose spatial footprint tends to be vague. Indeed, in contrast to administrative regions, usually very little is known about the location of vernacular places.

Consider, for instance, the names of neighborhoods and districts within a given city. For popular neighborhoods, we may be able to find sufficient information on the web to build a useful fuzzy set representation [19; 27]. For lesser-known neighborhoods, however, such a strategy is bound to fail, and we may instead try to derive qualitative models from web documents. However, it turns out that acquiring spatial relations from text documents is hard, due to the inherent ambiguity of spatial propositions such as "in" (which can refer to several relations, including P, P^{-1} and PO), and due to the fact that spatial relations are seldom stated explicitly (e.g. few documents explicitly state that one region is bordering on another one). To some extent, these problems have been addressed in [24], where recall-oriented heuristics are proposed to derive instances of the relations EC and P. Nonetheless, it appears that much more work is needed on the topic of deriving qualitative spatial representations from the web. Although this clearly is a challenging problem, reasonable progress in this area could lead to a widespread use of qualitative spatial reasoning for geographic information retrieval. In this context, it is clear that we should try to take advantage of any quantitative knowledge we have to enrich the available qualitative models. This calls for hybrid reasoning strategies which mix qualitative spatial reasoning with geometric computations. In [23] a heuristic approach along these lines has been proposed, based on a combination of genetic algorithms and ant colony optimization.

Clearly, it is hard to directly extract fuzzy spatial relations from web documents. However, the classical RCC can be seen as a special case of the fuzzy RCC, in which relations are known to hold to degree 1. The role of degrees other than 1 is two-fold. First, when combining quantitative spatial information which is represented using fuzzy sets with qualitative models, degrees will naturally occur when propagating information. The second role is related to the existence of inconsistencies. Qualitative spatial models may be inconsistent for a variety of reasons, and different strategies may be used to cope with them. However, a particularly natural strategy to deal with inconsistency is to gradually weaken those assertions that are involved in an inconsistency. As a simple example, suppose that one piece of evidence leads to $DC(a, b)$ while another leads to $EC(a, b)$. As these two assertions are inconsistent, in the classical RCC we need to choose one (which would be more or less arbitrary) or to replace both assertions by the disjunction $DC(a, b) \vee EC(a, b)$. This latter approach is rather cautious, and would result in a substantial loss of information. In the fuzzy RCC, we may pursue a different strategy, which is especially suitable if both pieces of evidence are rather strong: we assume that both $DC(a, b)$ and $EC(a, b)$ are true to some degree. For instance, the assertions $DC(a, b) \geq 0.5$ and $EC(a, b) \geq 0.5$ are consistent with each other. Especially when some of the regions involved are known to be vague, such a strategy may yield intuitive results; see e.g. [24] for a case study on the neighborhoods of the city of Cardiff. One example from [24] where there was an initial inconsistency in the extracted relations concerns the Cardiff neighborhoods of *Cardiff Bay* and *Butetown*. On the one hand, residents of the wealthy Cardiff Bay tend to consider their neighborhood as being different from the much poorer Butetown. On the other hand, Butetown is also considered to be the new name for the area which used to be called *Tiger Bay*, and which encompasses the Cardiff Bay part of Cardiff. Hence both an adjacency and a part-of relation are intuitively acceptable to describe the relationship between Cardiff Bay and Butetown. Using fuzzy spatial relations, it is possible to express that both of these relations are satisfied to some extent, whereas the classical RCC setting would require us to choose between both relations.

6 Conclusions

In this paper, we have focused on fuzzy models for representing regions and the qualitative spatial relations between them. In particular, we discussed the interpretation of degrees, with the aim of clarifying the use of such models in applications. First, we discussed how a fuzzy set could be linked to a meta-standpoint regarding the delineation of a vague region, and how membership degrees can be given a clear quantitative interpretation in terms of random sets. Then we stressed that the use of degrees in fuzzy spatial relations serves a rather different purpose: enabling a compact representation of the spatial relationship between regions represented as fuzzy sets. Finally, we have briefly looked at geographic information retrieval as a promising application area for (fuzzy) qualitative spatial reasoning.

References

[1] B. Bennett. What is a forest? On the vagueness of certain geographic concepts. *Topoi*, 20(2):189–201, 2001.

[2] B. Bennett. Spatial vagueness. In R. Jeansoulin, O. Papini, H. Prade, and S. Schockaert, editors, *Methods for Handling Imperfect Spatial Information*, volume 256 of *Studies in Fuzziness and Soft Computing*, pages 15–47. Springer Berlin / Heidelberg, 2010.

[3] A. Cohn and N. Gotts. The 'egg-yolk' representation of regions with indeterminate boundaries. In *Geographic Objects with Indeterminate Boundaries (P.A. Burrough and A.U. Frank, eds.)*, pages 171–187. Taylor and Francis Ltd., 1996.

[4] A. G. Cohn and J. Renz. Qualitative spatial representation and reasoning. In F. van Harmelen, V. Lifschitz, and B. Porter, editors, *Handbook of Knowledge Representation*, volume 3 of *Foundations of Artificial Intelligence*, pages 551 – 596. Elsevier, 2008.

[5] B. De Finetti. La logique de la probabilité. In *Actes du Congrès International de Philosophie Scientifique*, pages 565–573, 1935.

[6] D. Dubois, S. Moral, and H. Prade. A semantics for possibility theory based on likelihoods. *Journal of Mathematical Analysis and Applications*, 205:359–380, 1997.

[7] D. Dubois and H. Prade. Fuzzy sets, probability and measurement. *European Journal of Operational Research*, 40(2):135–154, 1989.

[8] D. Dubois and H. Prade. Can we enforce full compositionality in uncertainty calculi? In *Proceedings of the Twelfth National Conference on Artificial Intelligence*, pages 149–154, 1994.

[9] J. Fernyhough, A. G. Cohn, and D. C. Hogg. Constructing qualitative event models automatically from video input. *Image and Vision Computing*, 18(2):81 – 103, 2000.

[10] G. Fu, C. Jones, and A. Abdelmoty. Ontology-based spatial query expansion in information retrieval. In *On the Move to Meaningful Internet Systems*, volume 3761 of *Lecture Notes in Computer Science*, pages 1466–1482. 2005.

[11] C. Hudelot, J. Atif, and I. Bloch. Fuzzy spatial relation ontology for image interpretation. *Fuzzy Sets and Systems*, 159(15):1929 – 1951, 2008.

[12] R. Krishnapuram, J. Keller, and Y. Ma. Quantitative analysis of properties and spatial relations of fuzzy image regions. *IEEE Transactions on Fuzzy Systems*, 1(3):222–233, 1993.

[13] J. J. Li and J. Renz. In defense of large qualitative calculi. In *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence*, pages 315–320, 2010.

[14] P. Matsakis, L. Wendling, and J. Ni. A general approach to the fuzzy modeling of spatial relationships. In R. Jeansoulin, O. Papini, H. Prade, and S. Schockaert, editors, *Methods for Handling Imperfect Spatial Information*, volume 256 of *Studies in Fuzziness and Soft Computing*, pages 49–74. Springer Berlin / Heidelberg, 2010.

[15] D. Montello, M. Goodchild, J. Gottsegen, and P. Fohl. Where's downtown?: behavioral methods for determining referents of vague spatial queries. *Spatial Cognition and Computation*, 3(2-3):185–204, 2003.

[16] H. Prade and S. Schockaert. Handling borderline cases using degrees: An information processing perspective. In P. Cintula, C. Fermüller, L. Godo, and P. Hajek, editors, *Logical Models of Reasoning with Vague Information*, Studies in Logic. College publications, to appear.

[17] D. Randell, Z. Cui, and A. Cohn. A spatial logic based on regions and connection. In *Proceedings of the 3rd International Conference on Knowledge Representation and Reasoning*, pages 165–176, 1992.

[18] V. Robinson. Individual and multipersonal fuzzy spatial relations acquired using human-machine interaction. *Fuzzy Sets and Systems*, 113(1):133–145, 2000.

[19] S. Schockaert and M. De Cock. Neighborhood restrictions in geographic IR. In *Proceedings of the 30th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, pages 167–174, 2007.

[20] S. Schockaert, M. De Cock, C. Cornelis, and E. Kerre. Fuzzy region connection calculus: an interpretation based on closeness. *International Journal of Approximate Reasoning*, 48:332–347, 2008.

[21] S. Schockaert, M. De Cock, and E. Kerre. Location approximation for local search services using natural language hints. *International Journal of Geographical Information Science*, 22(3):315–336, 2008.

[22] S. Schockaert, M. De Cock, and E. E. Kerre. Spatial reasoning in a fuzzy region connection calculus. *Artificial Intelligence*, 173(258-298), 2009.

[23] S. Schockaert, P. Smart, and F. Twaroch. Generating approximate region boundaries from heterogeneous spatial information: An evolutionary approach. *Information Sciences*, 181(2):257–283, 2011.

[24] S. Schockaert, P. D. Smart, A. I. Abdelmoty, and C. B. Jones. Mining topological relations from the web. In *Proceedings of the 19th International Workshop on Database and Expert Systems Applications*, pages 652–656, 2008.

[25] M. Skubic, P. Matsakis, G. Chronis, and J. Keller. Generating multi-level linguistic spatial descriptions from range sensor readings using the histogram of forces. *Autonomous Robots*, 14(1):51–69, 2003.

[26] P. D. Smart, A. I. Abdelmoty, B. A. El-Geresy, and C. B. Jones. A framework for combining rules and geo-ontologies. In *Proceedings of the First International Conference on Web Reasoning and Rule Systems*, pages 133–147, 2007.

[27] F. Twaroch, C. Jones, and A. Abdelmoty. Acquisition of a vernacular gazetteer from web sources. In *Proceedings of the first international workshop on Location and the web*, pages 61–64, 2008.

[28] A. Varzi. Vagueness in geography. *Philosophy & Geography*, 4(1):49–65, 2001.

[29] K. Zimmermann and C. Freksa. Qualitative spatial reasoning using orientation, distance, and path knowledge. *Applied Intelligence*, 6(1):49–58, 1996.

On A Semi-Automatic Method for Generating Composition Tables *

Weiming Liu and Sanjiang Li
Centre for Quantum Computation and Intelligent Systems, Faculty of Engineering and Information Technology,
University of Technology Sydney, Australia
Email: sanjiang.li@uts.edu.au

Abstract

Originating from Allen's Interval Algebra, composition-based reasoning has been widely acknowledged as the most popular reasoning technique in qualitative spatial and temporal reasoning. Given a qualitative calculus (i.e. a relation model), the first thing we should do is to establish its composition table (CT). In the past three decades, such work is usually done manually. This is undesirable and error-prone, given that the calculus may contain tens or hundreds of basic relations. Computing the correct CT has been identified by Tony Cohn as a challenge for computer scientists in 1995. This paper addresses this problem and introduces a semi-automatic method to compute the CT by randomly generating triples of elements. For several important qualitative calculi, our method can establish the correct CT in a reasonable short time. This is illustrated by applications to the Interval Algebra, the Region Connection Calculus RCC-8, the INDU calculus, and the Oriented Point Relation Algebras. Our method can also be used to generate CTs for customised qualitative calculi defined on restricted domains.

1 Introduction

Since Allen's seminal work of Interval Algebra (IA) [1; 2], qualitative calculi have been widely used to represent and reason about temporal and spatial knowledge. In the past decades, dozens of qualitative calculi have been proposed in the artificial intelligence area "Qualitative Spatial & Temporal Reasoning" and Geographic Information Science. Except IA, other well known binary qualitative calculi include the Point Algebra [20], the Region Connection Calculi RCC-5 and RCC-8 [17], the INDU calculus [16], the Oriented Point Relation Algebras $\mathcal{OPRA}$ [14], and the Cardinal Direction Calculus (CDC) [10; 19; 13], etc.

Relations in each particular qualitative calculus are used to represent temporal or spatial information at a certain granularity. For example, The Netherlands is *west* of Germany, The Alps *partially overlaps* Italy, I have today an appointment with my doctor *followed by* a check-up.

Given a set of qualitative knowledge, new knowledge can be derived by using constraint propagation. Consider an example in RCC-5. Given that The Alps *partially overlaps* Italy and Switzerland, and Italy is a *proper part of* the European Union (EU), and Switzerland is *discrete from* the EU, we may infer that The Alps *partially overlaps* the EU. The above inference can be obtained by using composition-based reasoning. The composition-based reasoning technique has been extensively used in qualitative spatial and temporal reasoning, and, when combined with backtracking methods, has been shown to be complete in determining the consistency problem for several important qualitative calculi, including IA, Point Algebra, Rectangle Algebra, RCC-5, and RCC-8. Moreover, qualitative constraint solvers have been developed to facilitate composition-based reasoning [21; 22].

We here give a short introduction of the composition-based reasoning technique. Suppose $\mathcal{M}$ is a qualitative calculus, and $\Gamma = \{v_i \gamma_{ij} v_j\}_{i,j=1}^n$ is a constraint network over $\mathcal{M}$. The composition-based reasoning technique uses a variant of the well-known *Path Consistency Algorithm*,[1] which applies the following updating rule until the constraint network becomes stable or an empty relation appears:

$$\gamma_{ij} \leftarrow \gamma_{ij} \cap \gamma_{ik} \circ_w \gamma_{kj}, \tag{1}$$

where $\alpha \circ_w \beta$ is the *weak composition* (cf. [11; 18]) of two relations α, β in $\mathcal{M}$, namely the smallest relation in $\mathcal{M}$ which contains the usual composition of α and β. Although for OPRA and some other calculi the composition-based reasoning is incomplete to decide the consistency problem, it remains a very efficient method to approximately solve the consistency problem.

The weak composition in a qualitative calculus $\mathcal{M}$ is determined by its *weak composition table* (CT for short). Usually, the CT of $\mathcal{M}$ is obtained by manually checking the consistency of $\{x\alpha y, y\beta z, x\gamma z\}$ for each triple of basic relations $\langle \alpha, \gamma, \beta \rangle$. When $\mathcal{M}$ contains dozens or even hundreds of basic relations, this consistency-based method is undesirable and error-prone. [7] first noticed this problem and identified it as a challenge for computer scientists.

*A complete version of this paper is available via: `http://arxiv.org/abs/1105.4224`. This work was partly supported by an ARC Future Fellowship (FT0990811).

[1] The notion of Path Consistency is usually defined for constraints on finite domains, and not always appropriate for general qualitative constraints, which are defined on infinite domains.

This problem remains a challenge today. We here consider several examples. The Interval Algebra and the RCC-8 algebra contain, respectively, 13 and 8 basic relations. Their CTs were established manually. But if a calculus contains a hundred basic relations, we need to determine the consistency of one million such basic networks. This is manually impossible. The $\mathcal{OPRA}$ calculi and the CDC are large qualitative spatial calculi that have drawn increasing interests. $\mathcal{OPRA}_m$ contains $4m \times (4m+1)$ (i.e. 72, 156, 272 for $m = 2, 3, 4$, respectively) basic relations [14], while the CDC contains 218 basic relations [10]. Sometimes we need ingenious and special methods to establish CT for such a calculus. For the $\mathcal{OPRA}$ calculi, the algorithm presented in the original paper [14] contains gaps and errors. Later, [9] presented the second algorithm, which is quite lengthy and cumbersome. Another simple algorithm has also been proposed recently [15]. Given the huge number of basic relations of $\mathcal{OPRA}_m$, the validity of these algorithms need further verification. As for the CDC, [10] first studied the weak composition. Later, [19] noticed errors in Goyal's method and gave a new algorithm to compute the weak composition. Unfortunately, in several cases, their algorithm does not generate the correct weak composition (see [13]).

In this paper, we respond to this challenge and propose a semi-automatic approach to generate CT for general qualitative calculi. In the remainder of this paper, we first recall basic notions and results about qualitative calculi and weak composition tables in Section 2, and then apply our method to IA, INDU, RCC-8, and $\mathcal{OPRA}_1$ and $\mathcal{OPRA}_2$ in Section 3. Section 4 then concludes the paper.

2 Preliminaries

In this section we recall the notions of qualitative calculi and their weak composition tables. Interested readers may consult e.g. [12; 18] for more information.

Definition 2.1. Suppose U is a universe of spatial or temporal entities, and $\mathcal{B}$ is a set of jointly exhaustive and pairwise disjoint (JEPD) binary relations on U. We call the Boolean algebra generated by $\mathcal{B}$ a *qualitative calculus*, and call relations in $\mathcal{B}$ the *basic relations* of this qualitative calculus.

We consider a simple example.

Example 2.1 (Point Algebra)**.** Suppose $U = \mathbb{R}$. For two points a, b in U, we have either $a < b$, or $a = b$, or $a > b$. Let $\mathcal{B} = \{<, =, >\}$. Then $\mathcal{B}$ is a JEPD set of relations on U. We call the Boolean Algebra generated by $\mathcal{B}$ the Point Algebra.

We next recall the central notion of weak composition.

Definition 2.2. Suppose $\mathcal{M}$ is a qualitative calculus on U, and $\mathcal{B}$ is the set of its basic relations. The *weak composition* of two basic relations α and β in $\mathcal{M}$, denoted as $\alpha \circ_w \beta$, is defined as the smallest relation in $\mathcal{M}$ which contains $\alpha \circ \beta$, the usual composition of α and β.

Usually, a qualitative calculus has a finite set of relations. The weak composition operation of $\mathcal{M}$ can be summarised in an $n \times n$ table, where n is the cardinality of $\mathcal{B}$, and the cell specified by α and β contains all basic relations γ in $\mathcal{B}$ such that $\gamma \cap \alpha \circ \beta \neq \varnothing$. The CT of the Point Algebra is given below.

$\circ$	$<$	$=$	$>$
$<$	$<$	$<$	$*$
$=$	$<$	$=$	$>$
$>$	$*$	$>$	$>$

Definition 2.3. Suppose $\mathcal{M}$ is a qualitative calculus on U with basic relation set $\mathcal{B}$. For basic relations α, β, γ, we call $\langle \alpha, \gamma, \beta \rangle$ a *composition triad*, or *c-triad*, if $\gamma \subseteq \alpha \circ_w \beta$.

We can determine if a 3-tuple is a c-triad as follows.

Proposition 2.1. *A 3-tuple $\langle \alpha, \gamma, \beta \rangle$ of basic relations in $\mathcal{M}$ is a c-triad iff $\gamma \cap \alpha \circ \beta \neq \varnothing$, which is equivalent to saying that the basic constraint network*

$$\{x\alpha y, y\beta z, x\gamma z\} \tag{2}$$

is consistent, i.e. it has a solution in U.

To compute the weak composition of α and β, one straightforward method is to find all basic relations γ such that $\langle \alpha, \gamma, \beta \rangle$ is a c-triad.

3 A General Method for Computing CT

In this section, we propose a general approach to compute the composition table of a qualitative calculus $\mathcal{M}$ with domain U and basic relation set $\mathcal{B}$. The approach is based on the observation that each triple of objects in U derives a valid c-triad.

Proposition 3.1. *Suppose a, b, c are three objects in U. Then $\langle \rho(a,b), \rho(a,c), \rho(b,c) \rangle$ is a c-triad, where $\rho(x,y)$ is the basic relation in $\mathcal{M}$ that relates x to y.*

It is clear that six (different or not) c-triads can be generated if we consider all permutations of a, b, c.

To compute the CT of $\mathcal{M}$, the idea is to choose randomly a triple of elements in U and then compute and record the c-triads related to these objects in a dynamic table. Continuing in this way, we will get more and more c-triads until the dynamic table becomes stable after sufficient large loops. The basic algorithm is given in Algorithm 1, where D is a subdomain of U, Ψ decides when the procedure terminates, TRIAD records the number of c-triads obtained when the procedure terminates, and LASTFOUND records the time when the last triad is first recorded. For a calculus with unknown CT, the condition may be assigned with the form LOOP $\leq 1{,}000{,}000$ (i.e., the algorithm loops one million times), or LOOP $\leq$ LASTFOUND $+ 100{,}000$ (i.e., until no new c-triad is found in the last one hundred thousand loops), or their conjunction. If the CT is known and we want to double-check it, then the boundary condition could be set to TRIAD $< N$ to save time, where N is the number of c-triads of the calculus.

We make further explanations here.

Suppose $\mathcal{M}$ is a qualitative calculus on U. Recall U is often an infinite set. We need first to decide a finite subdomain D of U, as computers only deal with numbers with finite precision. Once D is chosen, we run the loop, say, one million times. Therefore, one million instances of triples of elements in D are generated. We then record all computed c-triads in a dynamic table. It is reasonable to claim that the table is stable if no new entry has been recorded after a long time

Algorithm 1: Computing the Composition Table of $\mathcal{M}$

Input: A subdomain D of $\mathcal{M}$, and a boundary condition Ψ related to $\mathcal{M}$
Output: The Composition Table CT of $\mathcal{M}$

```
Initialise CT;
LOOP ← 0;
TRIAD ← 0;
LASTFOUND ← 0;
while Ψ do
    LOOP ← LOOP + 1;
    Generate triple of objects (a, b, c) ∈ D^3 randomly;
    α ← the basic relation between a and b;
    β ← the basic relation between b and c;
    γ ← the basic relation between a and c;
    α′ ← the basic relation between b and a;
    β′ ← the basic relation between c and b;
    γ′ ← the basic relation between c and a;
    for ⟨r, s, t⟩ ∈ {⟨α, γ, β⟩, ⟨α′, β, γ⟩, ⟨γ, α, β′⟩,
    ⟨β, α′, γ′⟩, ⟨β′, γ′, α′⟩, ⟨γ′, β′, α⟩} do
        if ⟨r, s, t⟩ is not in CT then
            Record triad ⟨r, s, t⟩ to CT;
            TRIAD ← TRIAD + 1;
            LASTFOUND ← LOOP;
        end
    end
end
return CT.
```

(e.g. as long as the time has past to get all recorded c-triads). Because D is finite, Algorithm 1 will generate a stable table after a sufficient large number of iterations.

We observe that a finite subdomain D may restrict the possible c-triads if it is selected inappropriately. We introduce a notion to characterise the appropriateness of a subdomain.

Definition 3.1. Suppose $\mathcal{M}$ is a qualitative calculus defined on the universe U. A nonempty subset D of U is called a *3-complete* subdomain of $\mathcal{M}$ if each consistent basic network as specified in Eq. 2 has a solution in D.

If D is a 3-complete subdomain, then, for each c-triad $\langle\alpha,\gamma,\beta\rangle$, there are a,b,c in D such that $(a,b)\in\alpha$, $(b,c)\in\beta$, and $(a,c)\in\gamma$. Therefore, to determine the CT of $\mathcal{M}$, we need only consider instances of triples in D.

Note that no matter whether the subdomain D is 3-complete, the algorithm always generates 'valid' triads, in the sense that any 3-tuple $\langle\alpha,\gamma,\beta\rangle$ in the CT generated is indeed a c-triad of the calculus. However, the algorithm only converges to the correct CT when the subdomain D is 3-complete.

It is of course important questions to find 3-complete subdomains or to decide if a particular subdomain is 3-complete. However, it seems that there is no general answer for arbitrary qualitative calculi, since the questions are closely related to the semantics of the calculi. For a particular calculus, e.g. IA, this can be verified by formal analysis. Note that a superset of a 3-complete subdomain is also 3-complete. To make sure a chosen subdomain D is 3-complete, we often apply the algorithm on several of its supersets at the same time. If the same number is generated for all subdomains, we tend to believe that D is 3-complete and the generated table is the CT of $\mathcal{M}$. Note a formal proof is necessary to guarantee the 3-completeness of D.

Even if a CT of $\mathcal{M}$ has been somehow obtained, our method can be used to verify its correctness. Double-checking is necessary since computing the CT is error-prone (see the last paragraph of page 1). If there is a c-triad that does not appear in the previously given table, something must be wrong with the table, because the c-triads computed by Algorithm 1 are always valid. It is also possible that the algorithm terminates with a fragment of given composition table. We then can make theoretical analysis to see if the missing c-triads are caused by the incompleteness of the subdomain. If so, we modify the subdomain and run the algorithm again, otherwise, the missing c-triads are likely to be invalid c-triads.

Another thing we should keep in mind is how to generate a triple of elements (a,b,c) from D. Note that if D is small (e.g. in the cases of PA and IA), we can generate all possible triples. If D contains more than 1000 elements, then it will be necessary to generate the triples randomly as there are over a billion different triples. The distribution over D may affect the efficiency of the algorithm. Assuming that we have very limited knowledge of the calculus $\mathcal{M}$, it is natural to take a,b and c independently with respect to the uniform distribution. We note that the better we understand the calculus, the more appropriate the distribution we may choose.

To increase the efficiency of the algorithm, we sometimes use the algebraic properties of the calculus. For example, if the identity relation id is a basic relation, then by $\alpha\circ_w id = \alpha = id\circ_w\alpha$ and $id\subseteq\alpha\circ_w\alpha^\sim$, we need not compute the c-triads involving id, where $\alpha^\sim$ is the converse of α. This is to say, the algorithm only needs to generate pairwise different elements. As another example, suppose that the calculus is closed under converse, i.e. the converse of a basic relation is still a basic relation. Then in Algorithm 1 we need only compute α,β,γ. The other relations and c-triads can be obtained by replacing α',β',γ' in the algorithm by, respectively, $\alpha^\sim,\beta^\sim,\gamma^\sim$. Similar results have been reported in [4].

In the following we examine three important examples. All experiments were conducted on a 3.16 GHZ Intel Core 2 Duo CPU with 3.25 GB RAM running Windows XP. Note the results rely on the random number generator. As our aim is to show the feasibility of the algorithm rather than investigating the efficiency issues, we only provide one group of the results and do not make any statistical analysis.

3.1 The Interval Algebra and the INDU Calculus

We start with the best known qualitative calculus.

Example 3.1 (Interval Algebra)**.** Let U be the set of closed intervals on the real line. Thirteen binary relations between two intervals $x=[x^-,x^+]$ and $y=[y^-,y^+]$ are defined in Table 1. The Interval Algebra [2] is the Boolean algebra generated by these thirteen JEPD relations.

The CT for IA has been computed in 1983 in Allen's famous work. When applying Algorithm 1 to IA, we do not consider all intervals. Instead, we restrict the domain to the

Table 1: Basic IA relations and their converses, where $x = [x^-, x^+], y = [y^-, y^+]$ are two intervals.

Relation	Symbol	Converse	Meaning
before	b	bi	$x^- < x^+ < y^- < y^+$
meets	m	mi	$x^- < x^+ = y^- < y^+$
overlaps	o	oi	$x^- < y^- < x^+ < y^+$
starts	s	si	$x^- = y^- < x^+ < y^+$
during	d	di	$y^- < x^- < x^+ < y^+$
finishes	f	fi	$y^- < x^- < x^+ = y^+$
equals	eq	eq	$x^- = y^- < x^+ = y^+$

Table 2: Implementation for IA, where TRIAD is the number of c-triads recorded by running the algorithm on D_M for $M = 4$ to $M = 20$, LASTFOUND is the loop when the last triad is first recorded

M	4	5	6	7	8	9	10	11	12
TRIAD	139	319	409	409	409	409	409	409	409
LASTFOUND	92	629	1501	878	2111	3517	728	697	932

M	13	14	15	16	17	18	19	20
TRIAD	409	409	409	409	409	409	409	409
LASTFOUND	11212	20249	7335	4343	3632	17862	5533	43875

set of all intervals contained in $[0, M)$ that have integer nodes

$$D_M = \{[p, q] | p, q \in \mathbb{Z}, 0 \leq p < q < M\},$$

and use uniform distribution to choose random intervals. It is easy to see that the size of the domain is $M(M - 1)/2$. Note that to converge fast and generate all entries, we need to choose an appropriate M. If M is too small, then it is possible that some c-triads can not be instantiated. On the other hand, if M is too big, relations that require one or more exact matches (such as m in IA and $\mathsf{m}^=$ in the INDU calculus to be introduced in the next example) is very hard to generate, i.e. the probability of generating such an instance is very small. For a new qualitative calculus, there is no general rules for choosing M. Usually, pilot experiments are necessary to better understand the characteristics of the calculus.

Table 2 summarises the results for $M = 4$ to $M = 20$. In the experiment, we generate one million instances of triples of elements for each domain D_M. In all cases the dynamic table becomes stable in less than 50,000 loops. When the table becomes stable, the numbers of triads computed are not always the correct one (that is 409). This is mainly because the domain is too small. For M bigger than or equal to six, we always get the correct number of triads.[2] The loops needed (i.e. LASTFOUND) vary from less than a thousand to more than 43 thousand (see Table 2). In general, the smaller the domain is the more efficient the algorithm is.

Table 3: Implementation for INDU

M	6	7	8	9	10	11	12	13
TRIAD	1045	1531	1819	1987	2041	2053	2053	2053
LASTFOUND	3766	5753	10417	35201	35891	25031	12512	27728

M	14	15	16	17	18	19	20
TRIAD	2053	2053	2053	2053	2053	2053	2053
LASTFOUND	17223	24578	14758	22491	29034	49693	19772

[2] The 3-completeness of D_6 follows from the fact that each consistent IA network involving three variables has a solution in D_6.

Example 3.2 (INDU calculus)**.** The INDU calculus [16] is a refinement of IA. For each pair of intervals a, b, INDU allows us to compare the durations of a, b. This means, some IA relations may be split into three sub-relations. For example, b is split into three relations $\mathsf{b}^<, \mathsf{b}^=, \mathsf{b}^>$. Similar situations apply to $\mathsf{m}, \mathsf{o}, \mathsf{oi}, \mathsf{mi}$, and bi. The other seven relations have no proper sub-relations. Therefore, INDU has 25 basic relations.

INDU is quite unlike IA. For example, it is not closed under composition, and a path-consistent basic network is not necessarily consistent [3].

Applying our algorithm to INDU, we use the same subdomain D_M as for IA. From Table 3 we can see that D_6 is no longer 3-complete: more than 1000 c-triads do not appear in the stable table. The table becomes complete in D_{11}, which has 2053 c-triads. The 3-completeness of D_{11} is confirmed by the following proposition.

Proposition 3.2. *INDU has at most 2053 c-triads.*

Proof (sketch). For any three INDU relations $\alpha^{\star_1}, \beta^{\star_2}, \gamma^{\star_3}$ ($\star_1, \star_2, \star_3 \in \{<, =, >\}$), it is easy to see that $\langle \alpha^{\star_1}, \gamma^{\star_2}, \beta^{\star_3} \rangle$ is a valid c-triad of INDU only if $\langle \alpha, \gamma, \beta \rangle$ is a valid c-triad of IA and $\langle \star_1, \star_2, \star_3 \rangle$ is a valid c-triad of PA. We note that for IA relations in $\{\mathsf{d}, \mathsf{s}, \mathsf{f}, \mathsf{eq}, \mathsf{si}, \mathsf{fi}, \mathsf{di}\}$, only $\mathsf{d}^<, \mathsf{s}^<, \mathsf{f}^<, \mathsf{eq}^=, \mathsf{si}^>, \mathsf{fi}^>, \mathsf{di}^>$ are valid INDU relations. It is routine to check that there are only 2053 triples of INDU relations that satisfy the above two constraints. We recall that IA has 409 c-triads, and PA has 13 c-triads. $\square$

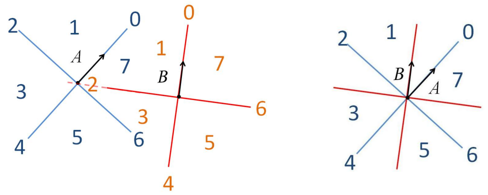
Since 2053 valid c-triads are recorded by running the algorithm on D_{11} for INDU, we know INDU has precisely 2053 c-triads, and D_{11} is 3-complete for INDU. It seems that this is the first time that the CT of INDU has been computed.

3.2 The Oriented Point Relation Algebra

In the $\mathcal{OPRA}_m$ calculus, where m is a parameter characterizing its granularity, each object is represented as an oriented point (*o-point* for short) in the plane. Each o-point has an orientation. Based on which, $2m - 1$ other directions are introduced according to the chosen granularity. Any other o-point is located on either a ray or in a section between two consecutive rays. Each of these rays and sections is assigned an integer from 0 to $4m - 1$. The relative directional information of two o-points A, B is uniquely encoded in a pair of integer numbers (s, t), where s is the ray or section of A in which B is located, and t is the ray or section of B in which A is located. Such a relation is also written as $A_m\angle_s^t B$. In the case that the locations of A and B coincide, the relation between A and B is written as $_m\angle_s B$, where s is the ray or section of A in which the orientation of B is located. Therefore, there are $4m(4m + 1)$ basic relations in $\mathcal{OPRA}_m$.

There are two natural ways to represent o-points. One uses the Cartesian coordinate system, the other use polar coordinate system. We next show the choice of coordinate system will significantly affect the experimental results, which are compared with that of [15].

In the Cartesian coordinate system, an o-point P is represented by its coordination (x, y) and its orientation ϕ.



(a) (b)

Figure 1: o-points in $\mathcal{OPRA}_2$ (a) $_2\angle_7^2$ and (b) $_2\angle_1$.

Definition 3.2. Let M_1 and M_2 be two positive integers. We define a Cartesian based subdomain of $\mathcal{OPRA}_m$ as

$$D_c(M_1, M_2) = \{((x, y), \phi) : x, y \in [-M_1, M_1] \cap \mathbb{Z}, \phi \in \Phi_{M_2}\},$$

where $\Phi_{M_2} \equiv \{0, 2\pi/M_2, \cdots, (M_2 - 1)/M_2 \times 2\pi\}$.

Table 4: Implementation for $\mathcal{OPRA}_1$ on a Cartesian coordinated domain $D_c(M_1, M_2)$

M_2	2	3	4	5	6	8	10	12	16
TRIAD	148	1024	1056	1024	1024	**1440**	1024	1408	**1440**

M_1	2	4	6	8	10
LASTFOUND ($M_2 = 8$)	8082	35932	411893	881787	> 1000000
LASTFOUND ($M_2 = 16$)	18618	295936	174490	> 1000000	> 1000000

Our experimental results show that, for $\mathcal{OPRA}_1$, the algorithm converges and generates the correct CT for subdomains with $M_1 \geq 2$ and $M_2 \in \{8, 16\}$. That is, the smallest 3-complete subdomain is $D_c(2, 8)$.

For $\mathcal{OPRA}_2$, however, the algorithm does not compute the desired CT in ten million loops. Actually, it is impossible to compute the desired CT if we use Cartesian coordination. Consider the following example. Suppose A, B, C are three o-points, such that $\triangle ABC$ is an acute triangle, and the orientation of A is the same as the direction from A to B, the orientations of B and C are similar. In this configuration, we have $A_2\angle_0^1 B$, $B_2\angle_0^1 C$, and $A_2\angle_1^0 C$. This configuration, however, cannot be realised in a Cartesian based subdomain.[3]

Based on the above observation, we turn to the polar coordinated representation. In the polar coordinate system, an o-point P is represented by its polar coordination (ρ, θ) and its orientation ϕ.

Definition 3.3. Let M_1 and M_2 be two positive integers. We define a polar coordinated subdomain of $\mathcal{OPRA}_m$ as

$$D_p(M_1, M_2) = \{((\rho, \theta), \phi) : \rho \in [0, M_1] \cap \mathbb{Z}, \theta, \phi \in \Phi_{M_2}\},$$

where $\Phi_{M_2} \equiv \{0, 2\pi/M_2, \cdots, (M_2 - 1)/M_2 \times 2\pi\}$.

As in Cartesian based subdomains, the parameter M_2 determines if a domain is complete, while M_1 determines the efficiency of the algorithm. For $\mathcal{OPRA}_1$, we have $D(M_1, M_2)$ is a 3-complete subdomain if $M_1 \geq 2$ and $M_2 = 6, 8, 10, 12, 16$ (see Table 6); for $\mathcal{OPRA}_2$, we have $D(M_1, M_2)$ is 3-complete if $M_1 \geq 4$ and $M_2 = 6, 10, 12, 16$ (see Table 7).

[3]The proof of this statement is much involved and omitted in this paper.

Table 5: Implementation for $\mathcal{OPRA}_2$ on a Cartesian coordinated domain $D_c(M_1, M_2)$

M_2	2	4	6	8	10	12	16
TRIAD	2704	2704	21792	23616	21792	21792	35232

Table 6: Implementation for $\mathcal{OPRA}_1$ on a polar coordinated domain $D_p(M_1, M_2)$

M_2	2	3	4	5	6	8	10	12	16
TRIAD	52	1024	1032	1408	**1440**	**1440**	**1440**	**1440**	**1440**

M_1	4	6	8	10	16
LASTFOUND ($M_2 = 8$)	3072	4868	22327	10363	38843
LASTFOUND ($M_2 = 16$)	26219	45831	121542	71205	146536

Table 7: Implementation for $\mathcal{OPRA}_2$ on a polar coordinated domain $D_p(M_1, M_2)$

M_2	2	3	4	6	8	10	12	16
TRIAD	400	24672	2128	**36256**	23616	**36256**	**36256**	**36256**

3.3 The Region Connection Calculus

Our algorithm works very well for simple objects like points and intervals. We next consider a region-based topological calculus RCC-8. It is worth noting that an automated derivation of the composition table was reported in [8] for a similar calculus (the 9-intersection model).

Example 3.3 (RCC-8)**.** Let U be the set of bounded plane regions (i.e. nonempty regular closed sets in the plane). Five binary relations are defined below. The RCC-8 algebra [17] is the Boolean algebra generated by these five relations, the identity relation **EQ**, and the converses of **TPP** and **NTPP**.

Relation	Meaning
DC	$a \cap b = \varnothing$
EC	$a \cap b \neq \varnothing, a^\circ \cap b^\circ = \varnothing$
PO	$a \not\subseteq b, b \not\subseteq a, a^\circ \cap b^\circ \neq \varnothing$
TPP	$a \subset b, a \not\subset b^\circ$
NTPP	$a \subset b^\circ$

Plane regions are much more complicated to represent than intervals or o-points. In most cases they are approximated by polygons or digital regions (i.e., a subset of $\mathbb{Z}^2$). Furthermore, it is natural to take a shot on simple objects at the beginning, since they are easy to deal with and important in applications. For RCC-8, we make experiments over two subdomains: rectangles and disks. The experiments show that these subdomains are good enough for our purpose, but when necessary, we could also consider general polygons or bounded digital regions.

We first consider subdomains whose elements are rectangles sides of which are parallel to the two axes. We introduce one parameter M, and require the four nodes be points in $[0, M) \times [0, M) \cap \mathbb{Z}^2$. The complete RCC-8 CT has 193 table entries. Since $\mathbf{EQ} \circ \mathbf{EQ} = \mathbf{EQ}$, we know $\langle \mathbf{EQ}, \mathbf{EQ}, \mathbf{EQ} \rangle$ is a c-triad. The other 192 c-triads can be confirmed using our algorithm. In Table 8, we show the results of running the algorithm 10 million times and require M vary from 4 to 20. We can see from the table that D_M is a 3-complete subdomain only if $M \geq 6$.

We next consider subdomains consisting of disks (see Table 9). We introduce one parameter M, and require $x, y \in [0, M] \cap \mathbb{Z}$, $r \in [1, M] \cap \mathbb{Z}$, where (x, y) and r are, respectively, the centre and the radius of the closed disk

Table 8: Implementation for RCC-8 using rectangles

M	4	5	6	8	10	15	20
TRIAD	114	177	192	192	192	192	192
LASTFOUND	14776	6513	2332646	56067	198255	261729	1521173

Table 9: Implementation for RCC-8 using disks

M	4	5	6	8	10	15	20
TRIAD	188	192	192	192	192	192	192
LASTFOUND	1759	8913	9489	25955	113757	942914	2961628

$B((x, y), r))$. In this case, $M = 5$ is good enough to generate all c-triads. We notice that the number of loops needed (i.e. LASTFOUND) increases quickly as M increases. For example, when $M = 20$, the dynamic table becomes stable after nearly 3 million loops. This is mainly due to that an instance of the c-triad $\langle \mathbf{NTPP}, \mathbf{NTPP}, \mathbf{NTPP} \rangle$ is very hard to generate. The 'hard' c-triad is, however, easy to prove.

4 Conclusion

In this paper, we introduced a general and simple semi-automatic method for computing the composition tables of qualitative calculi. The described method is a very natural approach, and similar idea was used to derive composition tables for an elaboration of RCC with convexity [6], and for a ternary directional calculus [5]. The table computed in [6] was acknowledged there as incomplete. The table computed in [5] is complete, but its completeness was guaranteed by manually checking all geometric configurations that satisfy the table. Except these two works, very little attention has been given to this natural approach in the literature on composition tables. We think a systematic examination is necessary to discover both the strong and weak points of this approach.

We implemented the basic algorithm for several well-known qualitative calculi, including the Interval Algebra, INDU, $\mathcal{OPRA}_m$ for $m = 1 \sim 4$, and RCC-8. Our experiments suggest that the proposed method works very well for point-based calculi, but not so well for region-based calculi. In particular, we established, as far as we know, for the first time the correct CT for INDU, and confirmed the validity of the algorithm reported for the $\mathcal{OPRA}$ calculi [15]. Our method can be easily integrated into existing qualitative solvers e.g. SparQ [21] or GQR [22]. This provides a partial answer to the challenge proposed in [7].

Recently, Wolter proposes (in an upcoming article [23]) to derive composition tables by solving systems of polynomial (in)equations over the reals. This approach works well for several point-based calculi, but not always generates the complete composition table.

Our method relies on the assumption that the qualitative calculus has a small 'discretised' 3-complete subdomain. All calculi considered in this paper satisfy this property. It is still open whether all interesting calculi appeared in the literature satisfy this property. Future work will also discuss the applications of our method for reasoning with a customised composition table.

References

[1] J. F. Allen. An interval-based representation of temporal knowledge. In *IJCAI*, pages 221–226. William Kaufmann, 1981.

[2] J. F. Allen. Maintaining knowledge about temporal intervals. *Commun. ACM*, 26(11):832–843, 1983.

[3] P. Balbiani, J.-F. Condotta, and G. Ligozat. On the consistency problem for the INDU calculus. *J. Applied Logic*, 4(2):119–140, 2006.

[4] B. Bennett. Some observations and puzzles about composing spatial and temporal relations. In *Proceedings ECAI-94 Workshop on Spatial and Temporal Reasoning*, pages 65–72, 1994.

[5] E. Clementini, S. Skiadopoulos, R. Billen, and F. Tarquini. A reasoning system of ternary projective relations. *IEEE Trans. on Knowl. and Data Eng.*, 22:161–178, February 2010.

[6] A. G. Cohn, D. A. Randell, Z. Cui, and B. Bennett. Qualitative spatial reasoning and representation. In *Qualitative Reasoning and Decision Technologies*, pages 513–522, Barcelona, 1993. CIMNE.

[7] A. G. Cohn. The challenge of qualitative spatial reasoning. *ACM Comput. Surv.*, 27(3):323–325, 1995.

[8] M. J. Egenhofer. Deriving the composition of binary topological relations. *J. Vis. Lang. Comput.*, 5(2):133–149, 1994.

[9] L. Frommberger, J. H. Lee, J. O. Wallgrün, and F. Dylla. Composition in $\mathcal{OPRA}_m$. Technical report, SFB/TR 8, University of Bremen, 2007.

[10] R. K. Goyal. *Similarity assessment for cardinal directions between extended spatial objects*. PhD thesis, 2000. The University of Maine.

[11] S. Li and M. Ying. Region connection calculus: Its models and composition table. *Artif. Intell.*, 145(1-2):121–146, 2003.

[12] G. Ligozat and J. Renz. What is a qualitative calculus? A general framework. In *PRICAI*, pages 53–64. Springer, 2004.

[13] W. Liu, X. Zhang, S. Li, and M. Ying. Reasoning about cardinal directions between extended objects. *Artif. Intell.*, 174(12-13):951–983, 2010.

[14] R. Moratz. Representing relative direction as a binary relation of oriented points. In *ECAI*, pages 407–411. IOS Press, 2006.

[15] T. Mossakowski and R. Moratz. Qualitative reasoning about relative direction on adjustable levels of granularity. *CoRR*, abs/1011.0098, 2010.

[16] A. K. Pujari, G. V. Kumari, and A. Sattar. INDU: An interval and duration network. In *Australian Joint Conference on Artificial Intelligence*, pages 291–303. Springer, 1999.

[17] D. A. Randell, Z. Cui, and A. G. Cohn. A spatial logic based on regions and connection. In *KR*, pages 165–176, 1992.

[18] J. Renz and G. Ligozat. Weak composition for qualitative spatial and temporal reasoning. In *CP*, pages 534–548. Springer, 2005.

[19] S. Skiadopoulos and M. Koubarakis. Composing cardinal direction relations. *Artif. Intell.*, 152(2):143–171, 2004.

[20] M. B. Vilain and H. A. Kautz. Constraint propagation algorithms for temporal reasoning. In *AAAI*, pages 377–382, 1986.

[21] J. O. Wallgrün, L. Frommberger, D. Wolter, F. Dylla, and C. Freksa. Qualitative spatial representation and reasoning in the sparq-toolbox. In *Spatial Cognition*, pages 39–58. Springer, 2006.

[22] M. Westphal, S. Wölfl, and Z. Gantner. GQR: A fast solver for binary qualitative constraint networks. In *AAAI Spring Symposium on Benchmarking of Qualitative Spatial and Temporal Reasoning Systems*, 2009.

[23] D. Wolter. Analyzing qualitative spatio-temporal calculi using algebraic geometry. *Spatial Cognition & Computation*. (to appear).