SailAway: Formalizing Navigation Rules

F. Dylla and **L. Frommberger** and **J.O. Wallgrün** and **D. Wolter**[1] and **B. Nebel** and **S. Wölfl**[2]

Abstract. Agents that have to solve navigational tasks need to consider aspects that go far beyond single-agent goal-directed deliberation: What an agent does in a specific situation often interferes with what other agents do at the same time. In order to avoid conflicts or even collisions, situations in space are governed by laws, rules, and agreements between the involved agents. For this reason, artificial agents interacting with humans must be able to process such rule sets, which are usually formulated in natural language. In this paper we present a case study on how to formalize navigation rules in the domain of sea navigation. We present an approach that uses qualitative representations of navigation rules. Qualitative spatial reasoning methods can be applied to distinguish permissible actions in the set of all possible actions. We argue that an agent's spatial representation can be modeled on a qualitative level in a natural way and that this also empowers sophisticated high-level agent control.

1 Introduction

A considerable part of everyday human activities is guided by regulations, for example, regulations on how to behave in traffic scenarios, recommendations on how to use escalators, rules on how to enter subways and buses, or rules of politeness at bottlenecks. Most of these rules have in common that they are usually formulated in natural language and hence extensively use *qualitative terms* to describe spatial situations and actions. For example, in traffic laws qualitative concepts are used to describe relevant situations and also the "correct" behavior of agents in these situations. Another feature is that most of the rules depend on the agent's *role* in a particular situation. What an agent is allowed to do, may depend on whether he is a pedestrian or on the kind of vehicle she is using.

Representations of rule-compliant behavior, of course, are not limited to navigation. Examples of rule sets guiding the behavior of agents can also be found in sports, in games, in expert recommendation systems, and so on. Rule sets need to be made explicit and be formalized at different stages when artificial agents or multi-agent systems are specified or implemented. First, rules can be used to specify the desired behavior of an artificial agent (for instance a mobile robot or an autonomous vehicle) such that an implemented system can be tested against these specifications. Rules may also be used to actively control an artificial agent, for example, when we wish to restrict possible trajectories of a mobile system. Formal encodings of rules are also crucial for implementing control systems that observe and judge the behavior of other agents. Finally, rule sets need to be formalized in order to evaluate them according to given criteria, to find gaps, inconsistencies, or deadlocks. For instance, if a rule set describes how *two* agents have to behave in specific situations, one could investigate how this rule set would perform in more complex situations involving more than two agents: Is the rule set still sound in the sense that its intentions (e.g., collision avoidance) are met if all agent act in compliance with the rules? And, is the rule set complete in the sense that it covers all possible situations?

In this paper, we investigate how rules in sea navigation can be formalized and discuss the benefits of qualitative spatial representation formalisms. Qualitative representations link metrical information perceivable by the agents to more abstract characterization of situations in which rules can or have to be applied. On the basis of these qualitative representations, we show how spatial reasoning techniques can be used to assign rule-compliant actions to each agent in each concrete situation.

2 Approaches to Formalizing Navigation Rules

Most traffic regulations are written down in natural language texts. For making such rules available to a computer implementation, they need to be formalized or encoded in a suitable language. On the basis of this formalization, concrete situations of objects can be classified and permissible actions can be selected. An appropriate formalization is key to an accurate modeling of the rules and essential for empowering effective reasoning. The formalization serves as a double link: It links continuous real-world scenes to discrete classes described by the representations (scene classification) and it links symbolic rule descriptions to possible actions available to agents in a concrete scenario (navigational reasoning).

Navigation rules in sea navigation generally subsume classes of *configurations* (i.e., spatial constellations of agents) in which they assign permissible or obligatory behavior of agents. For example, in a configuration in which two motor boats are in head-on position, one navigational rule prescribes that both boats need to turn ¡starboard[3]. Besides spatial configurations, rules can also depend on other aspects such as types of vehicles used by the involved agents. Sport vessels, for example, have to give way to commercial shipping vessels. However, since knowledge of this kind can be formalized rather easily, the crucial point for formalizing navigation rules is to formally represent spatial configurations in a suitable way (in terms of the considereded rules) and to formally represent the actions prescribed by these rules.

2.1 Logical Framework

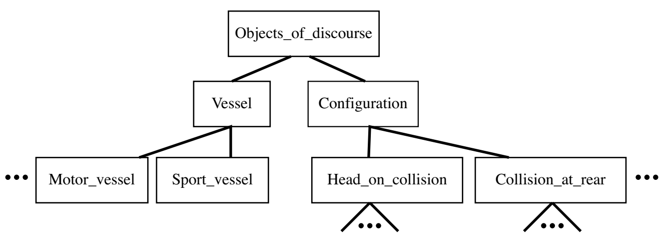
A formalization of navigation rules relates agent types (i.e., classes of vessels) and their spatial constellations as handled by the rules.

[1] Universität Bremen, Germany, email:{dylla, lutz, wallgruen, dwolter}@sfbtr8.uni-bremen.de

[2] University of Freiburg, Germany, email:{nebel, woelfl}@informatik.uni-freiburg.de

[3] *Starboard* is the nautical term that refers to the right side of a vessel with respect to its *bow* (front); *port* refers to the left hand side, *stern* to the back.

This can often be compiled into a small ontology. Using a logical approach representing this information appears most adequate to provide a suitable basis for reasoning. Description logics offer a solid approach to modeling ontological information and provide also the means for formalizing spatial configurations. Agent types and configurations are represented as *concepts*, whereas spatial relations are used as *roles* to interrelate the relative positions of agents. The utilization of qualitative spatial calculi provides us with a suitable set of spatial relations that allows for linking spatial reasoning techniques to the logical framework. Details are discussed in the following sections, at this point, we just assume that a suitable set of spatial relations to model configurations described by rules (e.g. head-on course) exists. We employ one additional role `involves` that relates configurations to agents. For example, if we consider a configuration defined by two agents in head-on course, the role-fillers of `involves` are the specific agents in head-on course. This approach allows us to consider scene classification as ABox-reasoning in description logics: A specific configuration is realized when role fillers for `involves` can be instantiated such that the formula describing the situation is valid. In Fig. 1 we present an overview of the simple ontology employed in this application (a) and give an exemplary logical representation of the exemplary spatial configuration of agents in head-on course (b). It presents the special case of a dangerous configuration of a motor and a sport vessel in head-on collision course.



(a)

$$
\begin{aligned}
\texttt{Motor\_vessel} &\sqsubseteq \texttt{Vessel}\\
\texttt{Sport\_vessel} &\sqsubseteq \texttt{Vessel}\\
\texttt{Head\_on\_coll\_Type1} &\doteq \exists\,\texttt{involves}.(\texttt{Motor\_vessel} \sqcap\\
&\qquad \exists\,\texttt{headon}.\texttt{Sport\_vessel})
\end{aligned}
$$

(b)

Figure 1. Overview of the ontology (a) and exemplary configuration (b)

The advantage of embedding rule formalization in a standard logical framework lies in the possibility of exploiting standard logical reasoning techniques. In principle, it is possible to reason about rule systems themselves (meta-level reasoning) as well as reasoning about rule-compliant actions (navigational reasoning). In any case, fundamental prerequisites are that (a) a finite set of (binary) spatial relations can describe configurations in a sufficiently precise way and that (b) the mapping from natural language to formal representations can be performed in an easy-to-use manner.

In summary, typical rule sets can be formalized using the logical framework of description logics to represent the ontology. The logical framework must incorporate a set of spatial relations that is adequate for representing the rules and for navigational reasoning. Thus, we argue for combining ontological knowledge engineering with appropriate qualitative spatial representation techniques.

2.2 Qualitative Spatial Calculi for Formalizing Configurations of Agents

Qualitative spatial calculi are well-suited to bridge between quantitative scene information observable by an agent and linguistic descriptions of object configurations [8]. Technically speaking, qualitative spatial calculi abstract from metrical data by summarizing similar quantitative states into one qualitative characterization. Qualitative calculi reveal the relative nature of spatial information: properties of objects are compared to one another rather than comparing the properties to some external (measuring) scale.

A binary qualitative calculus defines a set of jointly exhaustive and pairwise disjoint (JEPD) binary relations between objects of some domain D. Usually we are interested in calculi that are closed under converse and composition: The converse operation may be considered a shift of perspective, i.e., it allows us to deduce how object P is related to object Q when we know how Q is related to P. The composition operation yields the set of relations that can hold between objects P and Q if the relations between P and some third object R and the relation between Q and R are known. In other words, composition integrates local knowledge to survey knowledge.

Based on these operations, constraint-based reasoning techniques have been developed in the literature (see, e.g., [1]). In our application, we will apply these methods for infering actions that agents are allowed to perform in a given spatial situation (see section 4).

In the context of sea navigation, position information, i.e., information about direction and distance, is essential. In particular, orientation information is required to differentiate spatial constellations as described by navigation rules. Currently, distance information only plays a subordinate role in our approach: We use such information only to distinguish those boats that are close enough to other boats such that they need to be considered when navigation rules are evaluated. Several calculi for dealing with positional information have been presented in the literature (e.g. [4, 7]). In our context, the $\mathcal{OPRA}_4$ calculus [7] is of particular interest, because this calculus is well-suited for dealing with objects that have an intrinsic front or move in a particular direction.

$\mathcal{OPRA}_4$ is designed for reasoning about relative orientation relations between *oriented points* (points in the plane with an additional direction parameter)[4]. For each pair of oriented points, 4 lines are used to partition the plane into 8 planar and 8 linear regions (see Fig. 2). The orientation of the two points is depicted by the arrows starting at $\vec{A}$ and $\vec{B}$, respectively. The regions are numbered from 0 to 15, where region 0 always coincides with the orientation of the point. An $\mathcal{OPRA}_4$ base relation is a pair (i, j), where i is the number of the region, seen from $\vec{A}$, that contains $\vec{B}$ and j vice versa. These relations are written as $\vec{A}\,{}_4\angle_i^j\,\vec{B}$. Additional base relations describe situations in which both oriented points are at the same position. However, these are not of particular interest in this work, because superpositions of oriented point represent collision situations. It should not go unmentioned that $\mathcal{OPRA}_4$ is not the only calculus rule sets might be modeled with. But since we focus here on translating rules from natural language descriptions to a qualitative formalization for agent control, $\mathcal{OPRA}_4$ is expressive enough for the translation and shows a good run-time behavior in the reasoning process.

[4] $\mathcal{OPRA}_4$ is actually a particular instance of the granulated $\mathcal{OPRA}_m$ calculus in which the granularity parameter m determines the number of base relations. An oriented point $\vec{O}$ can be described by its Cartesian coordinates $x_O, y_O \in \mathbb{R}$ and a direction $\phi_{\vec{O}} \in [0, 2\pi)$ with respect to an absolute frame of reference.

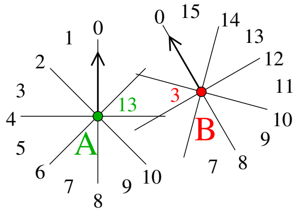


Figure 2. The $\mathcal{OPRA}_4$ relation $\vec{A}\,{}_4\angle_{13}^{3}\,\vec{B}$.

2.3 Modeling Spatio-Temporal Transitions by Conceptual Neighborhoods

Navigation rules restrict the possibilities of agents to act in space. For representing actions in a formal model, we must combine spatial and temporal information. An elegant way for accomplishing this spatio-temporal linkage is provided by so-called *conceptual neighborhoods* [3]. The idea of conceptual neighborhoods is to specify the discrete relation transitions that are possible due to continuous transformation [5]. Two base relations are considered *conceptually neighbored* if there can be a change-over due to an arbitrary small transformation of the objects. We denote the set of relations conceptually neighbored to a relation r by $\mathrm{cn}(r)$. In this context, such transformations are movements or changes of orientation of one or more of the involved objects. Depending on the transformations considered, different conceptual neighborhood structures can be induced [2].

A conceptual neighborhood graph of all base relations can be constructed interpreting the binary relation of "conceptually neighbored" as adjacency in the graph [3]. The neighborhood graph represents continuity aspects on the geometric or physical level of description in a discrete manner: Continuous processes map onto identical or neighboring classes of descriptions.A movement of an agent with respect to another agent can then be traced on the qualitative level as a sequence of neighboring spatial relations which hold for adjacent time intervals. Put differently, actions can be represented on a qualitative level as trajectories in the neighborhood graph. This provides us with an elegant approach to represent actions.

The basic idea underlying our approach is to consider rule-specific *transition systems* that differ from conventional neighborhood graphs in two aspects: First, we label edges in the graph by actions of the involved agents that cause the transition (thus, we obtain a directed graph), and second, we consider only edges that represent rule-compliant (or nearly rule-compliant) behavior of the agents. For example, a neighborhood transition $r_i \xrightarrow{(a_1,a_2)} r_j$ takes place when r_i represents "head-on", a_1 "turn to portside", a_2 "keep course", and r_j "on starboard side".

The starting point for defining these transiton systems is to identify an idealized transition sequence (the *idealized thread*), which may be considered a prototypical rule-compliant plan of maneuvers from a start to an end configuration if we observed the vessels in each point in time.

The idealized thread is not yet a suitable formalization of rule-compliant actions, as it abstracts from alternative action effects that need to be considered: depending on the precise position of the vessels, the same action may lead to different change-overs with respect to the qualitative relations as defined by the neighborhood graph. Therefore, the idealized thread is extended to a transition system that also includes neighbored configurations if they are still within the scope of the traffic rule at hand. For each of these added configurations, we derive actions that lead the vessels closer to the idealized thread. Analogously, we apply this method of neighborhood-based relaxation to start and end configurations.

3 Collision Regulations in Sea Navigation

In our investigations we focus on the domain of sea navigation. Traffic regulations for sea navigation have been defined in the *International Regulations for Preventing Collisions at Sea* (ColRegs) of the International Maritime Organization (IMO). For each pair of vessels, the rules define which one has to give way (burdened vessel) and which is the privileged one (it is possible that both vessels are burdened). The concrete avoidance behavior of burdened vessels is described by specific patterns in supplemental textbooks.

In the following, we will focus on vessels in sight of one another. For each pair of boat types, the conditions "from port", "from starboard", "head-on", and "from rear" must be considered such that, for n different boat types, $4n^2$ cases can be distinguished. However, it is sufficient to first derive transition systems for the general avoidance patterns and then refine these for the concrete boat types and velocities.

In our scenario, every vessel has a goal point where it is directly heading to. If vessels are in danger of collision, they are able to perform one of the three actions: turning starboard (S), turning port (P), or keeping the course (midships, M). These steering actions have a temporary effect: The helm is put for a short period of time and afterwards the helm is put back to midships. In general, the motion of standard vessels can be compared to Ackermann kinematics, i.e., in general turning is not possible without any translational velocity, and sidewards motion is not possible at all. We assume all vessels moving with a constant translational velocity $v_t > 0$. Furthermore, we assume a prototypical velocity for each vessel type. Currently, speed changes are not considered.

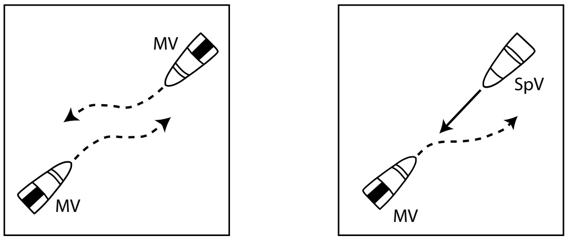
3.1 Kinematic Neighborhood Structure

The kinematics of the vessels induce a neighborhood structure in the underlying qualitative spatial representation (i.e., $\mathcal{OPRA}_4$) that is exploited for constructing the transition system (cf. section 2.3). Since neighborhood transitions must correspond to physically possible behavior, the general neighborhood structure of $\mathcal{OPRA}_4$ takes three different aspects into account: superposition, simultaneous motion, and agent kinematics [2]. The general neighborhood structure for solid objects with unconstrained motion (where objects cannot superpose) $\mathtt{cn}_s$ (subscript s stands for "solid") is defined by

$$\mathtt{cn}_s({}_4\angle_i^j) = \{ {}_4\angle_{i-1}^{j-1}, {}_4\angle_{i-1}^{j}, {}_4\angle_{i-1}^{j+1}, {}_4\angle_{i}^{j-1}, {}_4\angle_{i}^{j+1}, {}_4\angle_{i+1}^{j-1}, {}_4\angle_{i+1}^{j}, {}_4\angle_{i+1}^{j+1} \}$$

But since we assume different prototypical velocities for each vessel type, the neighbohood structure needs to be refined in order to respect to the kinematics of vessels. This means that we have to define a restricted neighborhood function $\mathtt{cn_s}'(r) \subseteq \mathtt{cn}_s(r)$ for each pair of vessel types. Put in other words, we need to capture the relation transitions corresponding to possible actions, i.e., if vessels S_1 and S_2 are in relation r then for every relation $r' \in \mathtt{cn_s}'(r)$ there exists at least one action pair that causes a transition into relation r'.

Due to lack of space we can just outline the general idea how this refined neighborhood structure can be determined. Consider a situation with two vessels S_1 and S_2 of the same type with $S_1\,{}_4\angle_i^j\,S_2$.



(a) Two motor vessels (MVs): both have to alter their course starboard to pass each other on port side

(b) Motor vessel and Sport vessel (SpV): MV has to turn starboard, SpV holds course

Figure 3. Exemplary rule for two different kinds of vessels.

Because of type equality we presume equal translational velocities. According to [2], a turn to port by S_1 results in ${}_4\angle_{i-1}^{j}$, and a turn to starboard in ${}_4\angle_{i+1}^{j}$. A turn by S_2 results in the according changes of j. If both vessels perform a turn, e.g., S_1 to port and S_2 to starboard, the resulting relation can also be ${}_4\angle_{i-1}^{j+1}$.

We now need to determine neighboring relations for any of the $3^2 = 9$ potential action pairs. For the turning actions we assume an arbitrary rotation velocity $v_r > 0$. We denote the actions by (a_1, a_2) where a_1 is the action performed by S_1 and a_2 by S_2. If, for example, in a situation in which $S_1\ {}_4\angle_2^2\ S_2$ holds, one of the action pairs (S, S), (S, M), (M, S), or (M, M) is performed, the resulting configuration is unique, namely $S_1\ {}_4\angle_3^3\ S_2$. The actions (S, P) and (M, P) result in one of the three configurations: $S_1\ {}_4\angle_3^1\ S_2$, $S_1\ {}_4\angle_3^2\ S_2$, or $S_1\ {}_4\angle_3^3\ S_2$, depending on the relative differences in translational and rotational velocity which we do not take into further account here. The actions (P, S) and (P, M) result in the converse: $S_1\ {}_4\angle_1^3\ S_2$, $S_1\ {}_4\angle_2^3\ S_2$, or $S_1\ {}_4\angle_3^3\ S_2$. Only for the action tuple (P, P) the neighboring relations cannot be restricted compared to $\mathtt{cn}_s({}_4\angle_2^2)$.

Another interesting case is $S_1\ {}_4\angle_0^0\ S_2$. Due to $v_t > 0$ it is not possible to end up in $S_1\ {}_4\angle_0^1\ S_2$ if (M, S) is performed, or $S_1\ {}_4\angle_0^{15}\ S_2$ for (M, P). The resulting configuration is definitely $S_1\ {}_4\angle_1^1\ S_2$ or $S_1\ {}_4\angle_{15}^{15}\ S_2$, respectively. The above results need to be determined manually, which is of course a laborious task.

3.2 An Exemplary Rule in Sea Navigation

As mentioned before, different types of vessels require to apply different rules. For example, rule 14(a) of the ColRegs says: "When two power-driven vessels are meeting head-on or nearly head-on courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other." However, if a motor vessel meets a sport vessel, only the motor vessel has to turn starboard, and the sport vessel is the privileged one (these two rules are illustrated in Fig. 3).

We will now give a formalization of the collision avoidance pattern depicted in Fig. 3(a), which is in compliance to the ColRegs and built on the (refined) neigborhood structure presented above.

In the first stage, we generate the idealized thread for the rule, that is, the idealized course describing the transitions from a dangerous into a safe configuration. For this, reconsider the example in Fig. 3(a): First the vessels are head-on, then both must turn starboard. When they are not head-on anymore, they can go midships, and when they are just about side by side, they can turn port, heading to their original course. This idealized thread is depicted in Fig. 4. A box denotes a start configuration and a double circle a safe configuration denoting that the rule is processed and the boats are in no danger of

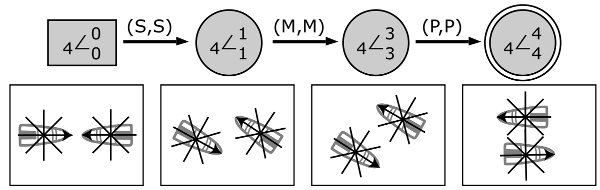


Figure 4. Idealized thread for the rule shown in Fig. 3 (a).

a collision anymore.

The idealized thread is not yet a suitable formalization of rule-compliant actions, as it abstracts from alternative action effects that need to be considered: Depending on the precise position of the vessels, the same action may lead to different change-overs with respect to the qualitative relations as defined by the neighborhood graph. That means, that observed real-world transitions are not necessarily neighborhood transitions (in particular, we can hardly observe transitions form region to line relations as considered in $\mathcal{OPRA}_4$). Therefore, relations that are neighbored to one in the idealized thread are included in the rule transition system as well. Fig. 5 shows the resulting rule transition system generated from the idealized thread. The idealized thread is highlighted by shaded boxes and circles. We note that relation ${}_4\angle_2^2$ has been included in this thread, because ${}_4\angle_1^1$ and ${}_4\angle_3^3$ are no direct neighbors and ${}_4\angle_2^2$ directly links these two relations. These transitions are derived under the premise of prototypical velocity. As we are considering same-type vessels in this rule model we presume the same velocity. But as soon as these actions are executed with different velocities the effects of executing them does not necessarily lead to perceiving the predicted relation. If, for example, ${}_4\angle_3^3$ holds, both vessels should turn port. Ideally, this results in ${}_4\angle_4^4$. But a slight difference in velocity may yield ${}_4\angle_4^3$ or ${}_4\angle_3^4$. Assuming a velocity being *just about the same* for both vessels, we expect that the resulting relation r is at least a conceptual neighbor of the idealized relation. For models concerning different types of vessels with different prototypical assumptions on velocity we need to generalize: If the velocity proportion between two vessels is just about the same as assumed in the prototypical model, $r \in \mathtt{cn}_s(r_p)$ holds.

In this example, our start configuration ${}_4\angle_0^0$ marks linear regions. However, such situations are unlikely to occur and are "unstable", which means that any steering action or difference in velocity may lead to a neighboring relation (in this case, ${}_4\angle_{15}^{15}$ or ${}_4\angle_1^{15}$, e.g.). Therefore we have to consider those situations as start configurations as well (cf. Fig. 5).

Analogously, we add configurations neighbored to the idealized safe configuration. As the side-by-side relation ${}_4\angle_4^4$ is a linear relation (only linear regions occur), it is more likely that a neighboring relation is perceived. For being sure that we cannot go back into a collision situation we end a rule only if one vessel has already completely passed the other vessel, i.e., ${}_4\angle_5^3$, ${}_4\angle_3^5$, and ${}_4\angle_5^5$.

Incorporation of neighboring relations makes our formalization robust against noise in perception and action execution.

4 Reasoning for Agent Control

In the following, we briefly sketch a first concrete application of our formalization of the sea navigation rules. While acting according to the rules will avoid collisions in situations involving two vessels, this is generally not guaranteed when more than two vessels are involved. We therefore investigated how the formalization of the sea navigation rules can be employed to control and coordinate the vessels in order to avoid collisions in more complex situations.

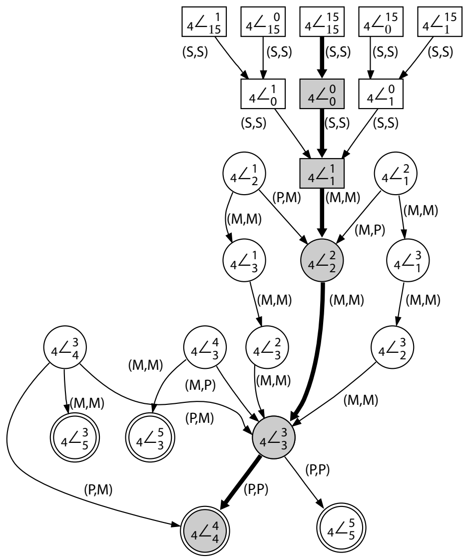


Figure 5. Complete transition system for the rule depicted in Fig. 4

In our approach, we combined the qualitative spatial relations describing the current situation and the relations describing possible future configurations between two boats as provided by the transition systems of the applicable rules to form a constraint network in which consistency corresponds to exemption from collisions. Constraint-based reasoning techniques [6] are then used to find a consistent and thus collision-free solution. The result is then repropagated to determine the suitable actions for the individual vessels that will lead to this particular constellation.

A simple example of the developed SailAway demonstrator depicted in Fig. 6 illustrates how the combination of the formalization with qualitative spatial reasoning techniques achieves collision-free navigation in a situation involving three boats.

5 Conclusion and Outlook

We investigated formalization of navigation rules, focusing on assessing the utility of qualitative representation and reasoning techniques in a real-world application scenario. Our investigation confirmed previous research in that qualitative representations enable mediation between real-world metric information and conceptual knowledge as used in communication or rule descriptions. It provides an effective means to compile rules into a formal representation. Most notably, a qualitative representation links to formal logic frameworks and enables a tight integration of all components in a complex agent control application.

Currently, we only make use of comparatively simple reasoning techniques as we only aim at determining *some* action that is compliant with the rules. A more sophisticated approach would involve a planning component. So it appears promising to extend qualitative representation and reasoning to become an integral part of frameworks for reasoning about action and change and to be integrated into high-level agent control languages. We aim at advancing our approach in that direction.

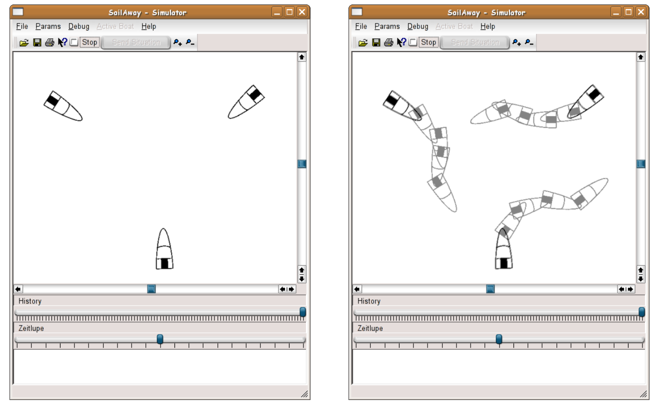


Figure 6. Configurations in the SailAway simulator window: left a situation with three motor vessels, right the resulting trajectories

6 Acknowledgments

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