Published in *Proceedings of the 10th European Conference on Artificial Intelligence (ECAI'92)*,
p. 349–353, Vienna, Austria, August 1992. Wiley.

On the Computational Complexity of Planning and Story Understanding*

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Abstract. Other authors have shown that *temporal projection*—the computation of the consequences for a set of events—is intractable even for severly restricted cases. They have also suggested that temporal projection is the basic problem underlying planning, plan validation, and story understanding. We have earlier shown that plan validation is actually tractable for a broad and important class of plans, thus indicating that temporal projection and plan validation are not as closely related as was believed. In this paper, we show that also planning and story understanding is sometimes tractable when temporal projection is intractable. This means that temporal projcetion is hardly a necessary ingredient of these tasks either.

1 Introduction

Dean and Boddy [4] have earlier analyzed the computational complexity of *temporal projection* (i.e. the problem of computing the consequences of a set of events) in a propositional STRIPS-like [5] language. They found that even severly restricted cases are NP-hard, which motivated them to develop an approximation algorithm. Their main motivations for studying this problem was that temporal projection appeared to be the basis of *plan validation*, *planning*, and *story understanding*. Hence, the complexity results for temporal projection were assumed to carry over to these problems.

Contrary to their assumption, we have found that plan validation is polynomial for the broad and important class of unconditional plans [10, 9]. Furthermore, we also found that even *planning* is polynomial for a severly restricted case where temporal projection is NP-hard. Considering these results, there are two obvious questions.

The first question is to what extent the result on polynomial planning can be extended to less restricted problems. It follows immediately from Bylander's result [3] that this does not extend to the class of all unconditional plans, as was the case with plan validation. However, by re-expressing the restrictions of the SAS-PUS planning problem [1] in the same notation as Dean and Boddy use it is possible to show that derivation of optimal plans is polynomial for a more interesting problem.

The second question is whether the complexity results for temporal projection carry over to story understanding. Once again, this turns out not to be the case. Story understanding is polynomial under the restrictions for which we first found planning to be polynomial, if we add the reasonable assumptions that stories are *coherent* and *non-repeating*. Story understanding for the class of all unconditional plans is NP-hard, however.

2 Temporal Projection

The formalization of the temporal projection problem for partially ordered events given below closely follows the presentation by Dean and Boddy [4, Sect. 2]. The problem of temporal projection is to decide whether a given propositional atom holds, possibly or necessarily, after a given event in an event system.

Definition 1 *A* **causal structure** *is given by a tuple* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$, *where*

- $\mathcal{P} = \{p_1, \ldots, p_n\}$ *is a set of propositional atoms, the* **conditions**,
- $\mathcal{E} = \{\epsilon_1, \ldots, \epsilon_m\}$ *is a set of* **event types**,

*This work was supported by the German Ministry for Research and Technology (BMFT) under contract ITW 8901 8 and the Swedish National Board for Technology Development (STU) under grant STU 90-1675.

- $\mathcal{R} = \{r_1, \ldots, r_o\}$ *is a set of* **causal rules** *of the form* $r_i = \langle \epsilon_i, \varphi_i, \alpha_i, \delta_i \rangle$*, where*
  - $\epsilon_i \in \mathcal{E}$ *is the* **triggering event type**,
  - $\varphi_i \subseteq \mathcal{P}$ *is a set of* **preconditions**,
  - $\alpha_i \subseteq \mathcal{P}$ *is the* **add list**,
  - *and* $\delta_i \subseteq \mathcal{P}$ *is the* **delete list**.

In order to talk about sets of concrete events and temporal constraints over them, the notion of a *partially ordered event set* is introduced.[1]

Definition 2 *Assuming a causal structure* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$*, a* **partially ordered event set** *(POE) over* Φ *is a pair* $\Delta_\Phi = \langle \mathcal{A}_\Phi, \prec \rangle$ *consisting of a set of* **actual events** $\mathcal{A}_\Phi = \{e_1, \ldots, e_p\}$ *such that* $type(e_i) \in \mathcal{E}$*, and a strict partial order*[2] $\prec$ *over* $\mathcal{A}_\Phi$.

POEs denote sets of possible sequences of events satisfying the partial order. A **partial event sequence** of length m over such a POE $\langle \mathcal{A}, \prec \rangle$ is a sequence $\mathbf{f} = \langle f_1, \ldots, f_m \rangle$ such that (1) $\{f_1, \ldots, f_m\} \subseteq \mathcal{A}$, (2) $f_i \neq f_j$ if $i \neq j$, and (3) for each pair f_i, f_j of events appearing in $\mathbf{f}$, if $f_i \prec f_j$ then $i < j$. If the event sequence is of length $|\mathcal{A}|$, it is called a **complete event sequence** over the POE. The set of all complete event sequences over a POE Δ is denoted by $CS(\Delta)$. If $\mathbf{f} = \langle f_1, \ldots, f_k, \ldots, f_m \rangle$ is an event sequence, then $\mathbf{f}/f_k$ denotes $\langle f_1, \ldots, f_k \rangle$ Further, we write $\mathbf{f}; g$ to denote $\langle f_1, \ldots, f_m, g \rangle$.

Each event maps states (subsets of $\mathcal{P}$) to states. Let $S \subseteq \mathcal{P}$ denote a state and let e be an event. Then we say that the causal rule r is **applicable** in state S iff $r = \langle type(e), \varphi, \alpha, \delta \rangle$ and $\varphi \subseteq S$. Given e and S, $app(S, e)$ denotes the set of all **applicable rules** for e in state S. An event e is said to be **applicable** in a state S iff $app(S, e) \neq \emptyset$. In order to simplify notation, we write $\varphi(r)$, $\alpha(r)$, $\delta(r)$ to denote the sets φ, α, and δ, respectively, appearing in the rule $r = \langle \epsilon, \varphi, \alpha, \delta \rangle$. Based on this notation, we define what we mean by the *result* of a sequence of events relative to a state S.

Definition 3 *The result function "R" from states and event sequences to states is defined recursively by:*

$$\begin{aligned} &R(S, \langle\rangle) = S \\ &R(S, (\mathbf{f}; g)) = \\ &\qquad (R(S, \mathbf{f}) - \{\delta(r) |\, r \in app(R(S, \mathbf{f}), g)\}) \cup \\ &\qquad\qquad \{\alpha(r) |\, r \in app(R(S, \mathbf{f}), g)\}. \end{aligned}$$

[1] This notion is similar to the notion of a *nonlinear plan.*

[2] A strict partial order is a transitive and irreflexive relation.

Given a state S, we will often restrict our attention to event sequences such that all events are applicable in the states in which they are applied. These sequences are called **admissible event sequences** relative to the state S. The set of all complete event sequences over Δ that are admissible relative to S are denoted by $ACS(\Delta, S)$. If $CS(\Delta) = ACS(\Delta, S)$, we will say that Δ is **coherent** relative to S.

In the following, we will often talk about which consequences a POE will have on some initial state. For this purpose, the notion of an *event system* is introduced.

Definition 4 *An* **event system** Θ *is a pair* $\langle \Delta_\Phi, \mathcal{I} \rangle$*, where* Δ_Φ *is a POE over the causal structure* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$*, and* $\mathcal{I} \subseteq \mathcal{P}$ *is the* **initial state**.

In order to simplify notation, the functions CS and ACS are extended to event systems with the obvious meaning, i.e., $CS(\langle \Delta, S \rangle) = CS(\Delta)$ and $ACS(\langle \Delta, S \rangle) = ACS(\Delta, S)$. Further, if $CS(\Theta) = ACS(\Theta)$, Θ is called coherent.

The problem of temporal projection as formulated by Dean and Boddy [4] is to determine whether some condition holds, *possibly* or *necessarily*, after a particular event of an event system.

Definition 5 *Given an event system* Θ*, an event* $e \in \mathcal{A}$*, and a condition* $p \in \mathcal{P}$*:*

$$\begin{aligned} p \in Poss(e, \Theta) \quad &\textit{iff} \quad \exists \mathbf{f} \in CS(\Theta)\colon p \in R(\mathcal{I}, \mathbf{f}/e) \\ p \in Nec(e, \Theta) \quad &\textit{iff} \quad \forall \mathbf{f} \in CS(\Theta)\colon p \in R(\mathcal{I}, \mathbf{f}/e). \end{aligned}$$

In the general case, temporal projection is quite difficult. Dean and Boddy [4] show that the decision problems $p \in Poss(e, \Theta)$ and $p \in Nec(e, \Theta)$ are NP-complete and co-NP-complete, respectively, even under some severe restrictions, such as restricting α or δ to be empty for all rules, or requiring that there is only one causal rule associated with each event type.

Definition 6 *A causal structure* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$ *is* **unconditional** *iff for each* $\epsilon \in \mathcal{E}$*, there exists only one causal rule with the triggering event type* ϵ*. An event system* $\langle \Delta_\Phi, \mathcal{I} \rangle$ *is unconditional iff* Δ_Φ *is unconditional. An event system is called* **simple** *iff it is unconditional,* $\mathcal{I}$ *is a singleton, and for each causal rule* $r = \langle \epsilon, \varphi, \alpha, \delta \rangle$*, the sets* φ*,* α*, and* δ *are singletons and* $\varphi = \delta$*.*

Dean and Boddy conjectured that temporal projection is easy for simple event systems. This turns out to be false, however.

Theorem 1 *For simple event systems Θ, deciding $p \in Poss(e, \Theta)$ is* NP*-complete and deciding $p \in Nec(e, \Theta)$ is co-*NP*-complete.*[3]

This result only strengthens the claim that temporal projection is a hard problem. On the other hand, applying the same restrictions to the planning problem turns out to give a more surprising result, as will be shown below.

In the context of planning, events as introduced above are usually called **actions** and POEs are called **nonlinear plans**, or simply **plans**. In the following, we use these terms interchangeably.

Definition 7 *A* **planning task** Π *is given by* $\langle \Phi, \mathcal{I}, \mathcal{G} \rangle$, *where* $\Phi = \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$ *is a causal structure as defined above, and* $\mathcal{I} \subseteq \mathcal{P}$ *and* $\mathcal{G} \subseteq \mathcal{P}$ *are the* **initial state** *and* **goal state**, *respectively. A* **plan** Δ_Φ **solves** Π *iff (1)* $\mathcal{G} \subseteq R(\mathcal{I}, \mathbf{f})$ *for all* $\mathbf{f} \in CS(\Delta_\Phi)$, *and (2)* $ACS(\Delta_\Phi, \mathcal{I}) = CS(\Delta_\Phi)$. *A plan* $\langle \mathcal{A}_\Phi, \prec \rangle$ *is* **unconditional** *iff* Φ *is unconditional. A solution* $\Delta = \langle \mathcal{A}, \prec \rangle$ *for* Π *is* **minimal** *iff for all other solutions* $\Delta' = \langle \mathcal{A}', \prec' \rangle$, *it holds that* $|\mathcal{A}| \leq |\mathcal{A}'|$.

Furthermore, we say that a planning problem is simple if it obeys the same restrictions as simple event systems. Using Bylander's [3] Theorem 8, the tractability of the solution existence problem for the simple planning problem follows immediately. In this case, also plan derivation is tractable, however.

Proposition 2 *For simple planning tasks, it can be decided in polynomial time whether there exists a solution. Further, a minimal valid plan can be derived in polynomial time.*[4]

This result is somewhat surprising since temporal projection, which was supposed to be the underlying problem in plan validation and planning, is harder than planning itself in this case. Starting from this observation, we have earlier found that plan validation is, in fact, solvable in polynomial time for the broad and important class of unconditional plans [9, 10]. The reason for this is that a planner has no reason to construct a plan that is not coherent. That is, plan validation is more realistically defined as first testing whether the plan is coherent, and reject if it is not, and then test whether it achieves its goal. In contrast to temporal projection as defined by Dean and Boddy [4], this task is tractable for unconditional plans.

[3]The proof [9, pp. 7–10] uses a transformation from the *path with forbidden pairs* problem [6, p. 203].

[4]The proof [9, p. 11] shows that the problem can be transformed to finding a shortest path in a graph of size polynomial in the size of the original problem.

3 Planning

Starting with the observation that planning is tractable for simple problems it is interesting to ask the question whether there are other, less restricted, planning problems that are also tractable. The results of Bylander [3] show that this does not hold for all unconditional plans, for example. However, he has found three planning problems for which the plan existence problem is tractable. On the other hand, Bäckström and Klein has reported a planning problem called the SAS-PUS problem [1] for which optimal plans can be derived in polynomial time. Any direct comparisons with the simple problem or Bylanders tractable problems is, unfortunately, non-trivial since the SAS-PUS problem is defined using another formalism called the *simplified action structures (SAS)* [1, 2].

The purpose of this section is to re-express the restrictions of the SAS-PUS problem in the formalism from Section 2 in order to facilitate such a comparison.

Definition 8 *A planning task* $\Pi = \langle \langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle, \mathcal{I}, \mathcal{G} \rangle$ *is* **SAS-PUS convertible** *iff it satisfies the following restrictions:*

1. $\langle \mathcal{P}, \mathcal{E}, \mathcal{R} \rangle$ *is unconditional;*
2. $\mathcal{P}$ *can be partitioned into* m *disjoint subsets* $P_1, \ldots, P_m$ *s.t.* $|P_i| > 1$ *for* $1 \leq i \leq m$ *and for all causal rules* $\langle \epsilon, \varphi, \alpha, \delta \rangle \in \mathcal{R}$
   (a) $\delta \subseteq \varphi$,
   (b) $|\delta| = 1$;
   (c) $|\varphi \cap P_i| \leq 1$ *for all* i,
   (d) $|\alpha \cap P_i| = |\delta \cap P_i| \leq 1$ *for all* i,
   (e) $\alpha \cap \delta = \emptyset$, *and*
   (f) $|\mathcal{I} \cap P_i| = |\mathcal{G} \cap P_i| = 1$ *for all* i.
3. *for all pairs of causal rules* $\langle \epsilon, \varphi, \alpha, \delta \rangle, \langle \epsilon', \varphi', \alpha', \delta' \rangle \in \mathcal{R}$
   (a) *if* $\varphi = \varphi'$, $\alpha = \alpha'$, *and* $\delta = \delta'$, *then* $\epsilon = \epsilon'$;
   (b) *if* $\epsilon \neq \epsilon'$, *then* $\alpha \cap \alpha' = \emptyset$; *and*
   (c) *for all* $1 \leq i \leq m$, *if* $(\varphi - \delta) \cap P_i \neq \emptyset$ *and* $(\varphi' - \delta') \cap P_i \neq \emptyset$ *then* $(\varphi - \delta) \cap P_i = (\varphi' - \delta') \cap P_i$.

The restrictions can be understood as follows. Each partition P_i can be viewed as the value domain of a state variable x_i, an action can change the value of a state variable only if it already has a defined value, an action can only change the value of one state variable, there must be no two different action types changing the same state variable to the same value, and the initial state and the goal state are fully specified. Finally, restriction (3c) captures the notion of *single-valuedness* [1, 2].

Theorem 3 *Minimal nonlinear plans for* SAS-PUS *convertible planning tasks can be derived in polynomial time.*

Proof Sketch. Define a transformation between sets of propositions and partial states in the SAS formalism and also map action conditions in the obvious way. Prove that a SAS-PUS convertible problem Π can be transformed into a SAS-PUS problem Π' in this way s.t. the solutions for Π' are exactly the solutions for Π. ■

Although the SAS-PUS convertible problem does not properly subsume the simple problem[5] it is most likely of more practical interest. For example, the blocks world problem which Bylander [3, Theorem 10] proved tractable can be encoded as a SAS-PUS convertible planning task, using the same encoding as Bylander, if restricted to total goal states. Hence, not only plan existence but also plan generation is tractable for this problem.

4 Story Understanding

Besides plan validation, Dean and Boddy [4, p. 375] also mention story understanding as one domain where temporal projection is important:

> "...an author may not provide the reader with the exact time of all events mentioned in a narrative, knowing that it is not critical that the reader have such information in order to follow the story."

Theorem 1, however, tells us that we are lost, as authors or readers. Even in the simplest case, the author has better to provide complete information or there is the danger that the reader gets lost in figuring out what is the case.[6] However, if we place some reasonable restrictions on the problem, the computational problems vanish.

First of all, it seems reasonable that we consider only admissible event sequences. It simply makes no sense that an author tells a reader that an event takes place that does not have any effect on the world. Conversely, one could argue that an author does not tell the exact time of events if the reader is able to recover the sequential information by other means, for instance, by the *coherence* of the events. Secondly, we will assume that a story is *non-repeating*, i.e., all states are different. Otherwise, the story would contain more than once the same situation—which is rather unlikely. In order to capture this formally, we introduce the notion of **non-repeating sequences** of an event system, written $NCS(\langle\Delta,\mathcal{I}\rangle)$, with the intention that for all events g, h, where $g \neq h$, appearing in an event sequence $\mathbf{f}$, we have $R(\mathcal{I},\mathbf{f}/g) \neq R(\mathcal{I},\mathbf{f}/h)$. Evidently, it is the case that $NCS(\Theta) \subseteq ACS(\Theta)$ because the occurrence of an event e that is not applicable leads to the same state as before the occurrence of e. Using this formalization of story-understanding, a variant of temporal projection is defined.

Definition 9 *Given an event system Θ, an event $e \in \mathcal{A}$, and a condition $p \in \mathcal{P}$:*

$$\begin{aligned} p \in Poss^{+}(e,\Theta) \quad &\textit{iff} \quad \exists \mathbf{f} \in NCS(\Theta)\colon p \in R(\mathcal{I},\mathbf{f}/e) \\ p \in Nec^{+}(e,\Theta) \quad &\textit{iff} \quad \forall \mathbf{f} \in NCS(\Theta)\colon p \in R(\mathcal{I},\mathbf{f}/e). \end{aligned}$$

Proposition 4 *For simple event systems Θ, $p \in Nec^{+}(e,\mathcal{I})$ and $p \in Poss^{+}(e,\Theta)$ can be decided in polynomial time.*

Proof Sketch. The restriction to non-repeating sequences over simple event systems implies that the effects of all events are unique, and it follows that $|NCS(\Theta)| \leq 1$. Reconstructing the (only) admissable event sequence, or finding out that there is none, can be done in polynomial time. ■

Thus story understanding (in the highly abstract form as defined here) is easier than temporal projection in the case of simple event systems. The question is, in how far this result can be generalized.

If we remove the restriction that the event sequence is non-repeating and require only that the course of events is admissible, the complexity of story understanding for simple event systems is not obvious [9, p. 31]. However, as we remarked above, the *non-repeating* restriction seems to be quite reasonable.

Generalizing the problem to general conditional event systems leads immediately to NP-completeness because we can design the causal rules in a way such that all sequences are non-repeating. A more interesting question is, whether we can solve the problem for general unconditional event systems. Because plan-validation is easy in this case, one may suspect that

[5] The simple problem does not necessarily satisfy restriction 3b.

[6] Note that NP-completeness means that we (most probably) cannot hope to solve the problem effortlessly. Instead, "puzzle mode" reasoning is necessary to arrive at a conclusion [7].

this also holds for temporal projection in an story understanding context. Unfortunately, this is not true, though.

Theorem 5 *For unconditional event systems Θ, deciding $p \in Poss^{+}(e, \Theta)$ is* NP*-complete.*

Proof Sketch. Membership in NP is obvious. For the hardness part we use the problem of *directed Hamilton path*, which is NP-complete [6, p. 199]. ■

Assuming that story understanding is an easy (i.e., tractable) task, this result implies that the formalization of the problem is still too general to account for the structure of the domain. It is desirable to identify restrictions that lead to polynomial algorithms for temporal projections in this domain, but there do not seem to be natural and obvious such conditions.

However, it should be noted that story understanding most probably involves more than can be expressed in our formalism. It seems plausible that plan recognition is one crucial part in story understanding and that abduction in general plays a vital role in such a task. Since we cannot express any of these phenomena, it seems to make not much sense to speculate about the complexity of this task. What seems to be clear, however, is that story understanding is more than temporal projection and that most probably other mechanisms than temporal projection are responsible for inferring the outcome of a story.

5 Conclusions

We have previously observed that plan validation is polynomial for the class of coherent, unconditional plans although temporal projection is NP-hard for such plans. We have also observed that planning is also polynomial in a severly restricted case where temporal projection is NP-hard. Continuing from these results we have shown that there is at least one more interesting planning problem for which optimal plans can be derived in polynomial time, namely the SAS-PUS convertible problem. This also implies that optimal planning is tractable for a simple setting of the blocks world problem. We have also found that story understanding is polynomial in some cases where temporal projection is NP-hard. Although this positive result does not apply to important generalizations like unconditional event systems, it is most likely the case that there is more to story understanding than just temporal projection. Consequently, adding to our previous findings that temporal projection does not seem to be the basis of plan validation this seems to be equally true of planning and story understanding.

As a final remark, it should be noted that the criticisms expressed in this paper are possible only because Dean and Boddy [4] made their ideas and claims very explicit and formal.

Acknowledgements

We would like to thank Bart Selman, who provided helpful comments on an earlier version of this paper.

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