Lecture 26: Planning in multiagent setting

- Nash equilibria
- Games with incomplete information
Randomized actions

Let $R_i(a_1, \ldots, a_n)$ for every $i \in N = \{1, \ldots, n\}$ be a function that maps the actions $a_j \in A(j)$ of agents $j \in \{1, \ldots, n\}$ to the rewards of agent $i$.

Randomized actions $\alpha$ for agent $i \in N$ are probability distributions over $A(i)$.

$R_i(\alpha_1, \ldots, \alpha_n)$ is the expected reward for agent $i \in N$ under the randomized actions $\alpha_1, \ldots, \alpha_n$. 
Nash equilibrium

Let \((\alpha_1, \ldots, \alpha_n)\) be randomized actions for agents 1, \ldots, n.

This \(n\)-tuple is a \textit{Nash equilibrium} if for every \(i \in N\), there is no randomized action \(\alpha_i'\) so that

\[ R_i(\alpha_1, \ldots, \alpha_i', \ldots, \alpha_n) > R_i(\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n) \]

That is, no agent can do better by using some other (randomized) action.

THEOREM Every finite game has at least one Nash equilibrium.
Nash equilibria: examples

Battle of the sexes: Ballet vs. Soccer

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>S</td>
<td>0,0</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Deterministic Nash equilibria are \((B, B), (S, S)\).

Nash equilibrium with randomized actions is \((\alpha_1, \alpha_2)\) with \(\alpha_1(B) = \frac{2}{3}, \alpha_1(S) = \frac{1}{3}, \alpha_2(B) = \frac{1}{3}, \alpha_2(S) = \frac{2}{3}\) and rewards \(\langle \frac{6}{9}, \frac{6}{9} \rangle\).
Nash equilibria: examples

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>1,1</td>
<td>0,0</td>
<td>0,0</td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>C</td>
<td>0,0</td>
<td>0,0</td>
<td>3,3</td>
</tr>
</tbody>
</table>

Three deterministic Nash equilibria: (A,A), (B,B), (C,C)

Randomization in any subset of \{A, B, C\}
Nash equilibria: examples

Paper, Rock, Scissors

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0,0</td>
<td>1,-1</td>
<td>-1,1</td>
</tr>
<tr>
<td>R</td>
<td>-1,1</td>
<td>0,0</td>
<td>1,-1</td>
</tr>
<tr>
<td>S</td>
<td>1,-1</td>
<td>-1,1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

One Nash equilibrium: play every action with probability $\frac{1}{3}$. 

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Games with partial observability

Several formalizations

- Stochastic games: states split to observational classes
- Stochastic games: in state $s \in S$, observation $o$ is made with probability $p$
- Game trees with indistinguishable tree nodes
Example: simplified Poker

p=0.5
chance
p=0.5

bad cards

A
resign
hold
−10
B
−10
resign
see
10
−20

good cards

A
resign
hold
B
10
20
resign
see

# Plans/strategies

<table>
<thead>
<tr>
<th></th>
<th>plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1hold</td>
<td>hold even when cards are bad</td>
</tr>
<tr>
<td>A2resign</td>
<td>hold only when cards are good</td>
</tr>
<tr>
<td>B1see</td>
<td>always see</td>
</tr>
<tr>
<td>B2resign</td>
<td>always resign</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B1see</th>
<th>B2resign</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1hold</td>
<td>0</td>
</tr>
<tr>
<td>A2resign</td>
<td>5</td>
</tr>
</tbody>
</table>
Values of the plans/strategies

expected reward for A

p(A1hold)=0.0  p(A1hold)=1.0

B1see

B2resign

p(A1hold)=0.0  p(A1hold)=1.0
Interesting optimal strategies

- Bluffing: raise the bets, when your hand is bad.
- Sand-bagging: do not raise the bets, when your hand is good.
- These show up also in many real world situations: one can often benefit by keeping the opponents uncertain.