Principles of Knowledge Representation and Reasoning

4. Nonmonotonic Reasoning

4.8 Belief Revision

Bernhard Nebel

- Motivation
- Updates and Revision
- Postulates
- Revision Schemes
- Base Revision
- Connection to NMR
Belief Change

- A dual approach to nonmonotonic reasoning is belief change
Belief Change

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• We start with some belief state $K$. When new information arrives, we change the belief state in order to accommodate the new information.
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• Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.
Two Scenarios

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→ belief revision: change your belief state minimally in order to accommodate the new information

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→ belief update: incorporate the change by assuming that the world has changed minimally
Updates and Revision are Different

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  - Assume we know that the *door is open or the window is open*.
  - Assume we get the information that after a change in the world, the *door is now closed*.

   In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that *the window is open*. 
Belief Change Operations

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- *Reasonable* revision operations?

\( \sim \) **AGM Revision Postulates** (Alchourron, Gärdenfors, Makinson)
AGM Postulates: Constraining the space of Revision Operations

(\dag 1) \ K + \varphi \in Th_\mathcal{L};
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Constraining the space of Revision Operations

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(\(\dagger 2\)) \(\varphi \in K \dagger \varphi\);
AGM Postulates: Constraining the space of Revision Operations

(+$1$) $K + \varphi \in Th_L$;

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AGM Postulates: Constraining the space of Revision Operations

(+1) \( K + \varphi \in Th_L; \)
(+2) \( \varphi \in K \hat{+} \varphi; \)
(+3) \( K \hat{+} \varphi \subseteq K + \varphi; \)
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(+1) \( K \vdash \phi \in Th_L \);
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(\pm 5) \ K \vdash \varphi = \text{Cn}(\bot) \text{ only if } \vdash \neg \varphi;
(\pm 6) \text{If } \vdash \varphi \leftrightarrow \psi \text{ then } K \vdash \varphi = K \vdash \psi;
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    then \( (K + \varphi) + \psi \subseteq K + (\varphi \land \psi) \).

Note: AGM postulates do not constrain the operation with respect to varying belief sets!
Canonical Revision Operations?

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\[ \therefore \text{ NO!} \]. This is *full meet revision*, which is known to be useless since
\[ K \triangledown \phi = Cn(\phi) \] for all \( \phi \) that are inconsistent with \( K \).
The postulates constrain the space to fully rational revision operations, but do not pick a single one.

Revision operations are closed under intersection, so should we choose the minimum?

\[ \forall \phi \text{ that are inconsistent with } K. \]

\[ K \cup \phi = Ch(\phi) \]

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\[ \rightarrow \text{ What other ways are there to generate a reasonable revision operation?} \]
Belief Revision Schemes

- Preference information (what to keep and what to give up)
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- A belief revision scheme (BRS) is a “recipe” for deriving a revision operation – restricted to a particular set $K$ – from
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  - the belief set and
  - preference information over this belief set
Examples

Partial Meet Revision (AGM): Preference information is given by a selection function $\gamma$ over the sets of maximal consistent subtheories $(K \downarrow \varphi)$:

$$K + \varphi \overset{\text{def}}{=} \left( \bigcap \gamma(K \downarrow \neg \varphi) \right) + \varphi$$

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Cut Revision (GM): Preference information is given by complete preorder $\preceq$ over all $\psi \in K$:

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Provided $\preceq$ satisfies a number of axioms (epistemic entrenchment), cut revisions coincide with the fully rational revision operations.
Revision – Viewed Computationally

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Revision – Viewed Computationally

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\sim \textbf{Consider belief bases} (arbitrary set of props.) as \textit{representing} belief sets.

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A theory $K$ over the propositional logic $\mathcal{L}$ with $n$ propositional atoms can have as much as

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\[ \leadsto \] Consider ways of specifying preference information in a concise way, i.e., polynomial in the size of the belief base.
Base Revision Schemes

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$\sim$ Assume a partitioning of $A$ into $n$ priority classes $A_1, \ldots, A_n$ such that the elements of $A_i$ are more important or relevant than those of $A_j$ for $j < i$. 
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  ⇒ Base revision schemes generate revision operations in the same way as ordinary schemes do.
Prioritized Meet-Base Revision (PMBR)

Let \((A \downarrow \neg \varphi)\) be the maximal subsets of \(A\) that are consistent with \(\varphi\) and maximize relevant propositions.
Prioritized Meet-Base Revision (PMBR)

Let $(A \Downarrow \neg \varphi)$ be the maximal subsets of $A$ that are consistent with $\varphi$ and maximize relevant propositions.

![Diagram with levels and steps]
Prioritized Meet-Base Revision (PMBR) – Formally

\[ A \oplus \varphi \overset{\text{def}}{=} \left( \bigcap_{B \in (A \downarrow \neg \varphi)} Cn(B) \right) + \varphi. \]
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\[ A \oplus \varphi \overset{\text{def}}{=} \left( \bigcap_{B \in (A \downarrow \neg \varphi)} Cn(B) \right) + \varphi. \]

Define a revision operation \( + \) on \( Cn(A) \) (that depends on \( A \) and the priority information) by

\[ Cn(A) \dot{+} \varphi \overset{\text{def}}{=} A \oplus \varphi. \]
Properties of PMBRs

- Generates partial meet revision, but does not satisfy $(\vee 8)$ in general.
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• Deciding whether \( A \oplus \varphi \vdash \psi \) is \( \Pi^p_2 \)-complete, even for one priority class.
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\begin{align*}
A &= \{p_1, \ldots, p_m, q_1, \ldots, q_m\} \\
\varphi &= \bigwedge_{i=1}^{m} (p_i \leftrightarrow \neg q_i)
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  \((A \downarrow \varphi)\) has size exponential in \(|A|\).
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A \oplus \varphi = Cn\left(\bigvee (A \Downarrow \neg \varphi) \land \varphi\right).
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\((A \Downarrow \varphi)\) has size exponential in \(|A|\).

• Worse, in some cases there exist no *concise* representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).
Let $\widehat{A}_j \overset{\text{def}}{=} \bigcup_{i=j}^n A_i$, then cut base-revision $\otimes$ is defined as:

$$A \otimes \varphi \overset{\text{def}}{=} \text{Cn}(\{\psi \in A \mid \psi \in A_j, \widehat{A}_j \not\vdash \neg \varphi\}) + \varphi.$$
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A \otimes \varphi \overset{\text{def}}{=} \mathcal{C}_n(\{\psi \in A \mid \psi \in A_j, \hat{A}_j \not\vdash \neg \varphi\}) + \varphi.
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- **Natural representation** of revised base.
Cut Base-Revision

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- **Natural representation** of revised base.
- **Easy** to compute: in $P^{NP}[O(\log n)]$. 

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<table>
<thead>
<tr>
<th>Level n</th>
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<tbody>
<tr>
<td>Level n-1</td>
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<tr>
<td>Level n-2</td>
</tr>
<tr>
<td>Level n-3</td>
</tr>
<tr>
<td>Level 1</td>
</tr>
</tbody>
</table>
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- go downwards
- as long as $\phi$
- is consistent with
- Level $n - Level i$
- cut
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- Natural representation of revised base.
- Easy to compute: in $P^{NP}[O(\log n)]$.
- Restriction to Horn logic leads to $O(n \log n)$. 
Being less conservative . . .

**Idea:** Throw away an entire priority class only if it would lead to a contradiction which cannot be blamed on a lower classes $\leadsto$ *linear* (or *unambiguous*) base-revision $\odot$. 
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- **Complexity:** \( \Delta^p_2 \)-complete; \( O(n^2) \) for Horn logic.
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**Idea:** Throw away an entire priority class only if it would lead to a contradiction which cannot be blamed on a lower classes $\leadsto$ **linear** (or **unambiguous**) base-revision $\odot$.

- Generates **fully rational** revision operations.
- **Complexity:** $\Delta^p_2$-complete; $O(n^2)$ for Horn logic.
- $LBR \approx CBR$, but a CBR realizing an LBR requires exponentially more priority classes.
Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection.
Revision vs. Nonmonotonic Reasoning

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- Given $K$ and a revision operation $\dot{\cup}$

$\sim$ a nonmonotonic consequence relation can be defined as follows:

$\phi \sim \psi$ iff $\psi \in K \dot{\cup} \phi$. 
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In this case,

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Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

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\[ \phi \not\sim \psi \text{ iff } \psi \in K + \phi. \]

In this case,

- the rationality postulates correspond to principles of NMR (such as cautious monotony etc.);
- in the case of prerequisite-free, normal defaults $D$ (Theorist), the cautious conclusions from $(W, D)$ are simply $D \oplus W$ with one priority level;
- similar relationship between Brewka's level default theories and PMBRs.
NMR Principles and Rationality Postulates

(2) \( \varphi \in K \vdash \varphi; \)
NMR Principles and Rationality Postulates

(\textastrel{\textdagger}{2}) \varphi \in K \dagger \varphi;

\sim \textbf{Reflexivity}
NMR Principles and Rationality Postulates

(★2) \( \varphi \in K \vdash \varphi; \)

\( \rightsquigarrow \textbf{Reflexivity} \)

(★3) \( K \vdash \varphi \subseteq K + \varphi; \)
NMR Principles and Rationality Postulates

(\#2) \( \varphi \in K \uparrow \varphi; \)

\( \therefore \text{ Reflexivity} \)

(\#3) \( K \uparrow \varphi \subseteq K + \varphi; \)

\( \therefore \text{ Super Classicality} \)
NMR Principles and Rationality Postulates

(\(\ddagger 2\)) \(\varphi \in K \vdash \varphi\);

\(\sim\) Reflexivity

(\(\ddagger 3\)) \(K \vdash \varphi \subseteq K + \varphi\);

\(\sim\) Super Classicality

(\(\ddagger 6\)) If \(\vdash \varphi \leftrightarrow \psi\) then \(K \vdash \varphi = K \vdash \psi\);
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\sim \textbf{Left Logical Equivalence}

(</sup>8) If \( \neg \psi \notin K \vdash \varphi, \)

then \( (K \vdash \varphi) + \psi \subseteq K \vdash (\varphi \land \psi). \)
NMR Principles and Rationality Postulates

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\( \leadsto \text{Rational Monotonicity} \)
Conclusions from the Correspondence

- NMR can be thought of as the other side of the same coin.
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- NMR can be thought of as the other side of the same coin.
- NMR (at least for default logic) is as hard as revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in polynomial space, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a denser way than classical logic, and with that come higher computational costs.
Outlook & Summary

• While NMR and belief revision seem to be the two sides of the same coin, there are notable **pragmatic differences**
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• There exists a strong correspondence between NMR and BR
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- NMR and Belief Revision can be thought of as qualitative ways of dealing with uncertainty in a purely logical setting.
- There exists a strong correspondence between NMR and BR.
- Both are computationally expensive and representational problematic.
- There are cases, though, that are tractable and practical.


P. Gärdenfors, Belief Revision and Nonmonotonic Logic: Two Sides of the Same Coin?, In *ECAI-90*, 768-773.