Principles of Knowledge Representation and Reasoning

4. Nonmonotonic Reasoning

4.4 Argumentation Theoretic Approaches

Bernhard Nebel

- Motivation
- Stable Extensions
- DL and Poole’s THEORIST
- Admissible and Preferred Extensions
- Upper Bounds for Nonmonotonic Reasoning
- THEORIST: Completeness Results
- DL: Completeness Results
Motivation

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~ Hopefully, such approaches are “more natural” and computationally simpler than ordinary NM logics.
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- $\Delta \subseteq A$ *attacks* $\Delta' \subseteq A$ iff $\Delta$ attacks a $\alpha \in \Delta'$
- $\Delta$ is *closed* iff $\Delta = A \cap \text{Th}(T \cup \Delta)$
Stable Extensions

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- Name comes from von **stable expansions** (AEL) and **stable model semantics** (LP).
Let $(W, D)$ be a DL theory with $D = \{\frac{\alpha_i \cdot \beta_i}{\gamma_i}\}$.
DL and Stable Extensions

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- $A = \{ M\beta_i \mid \frac{\alpha_i : \beta_i}{\gamma_i} \in D \}$
- $M\beta_i = \neg\beta_i$
Let \((W, D)\) be a DL theory with 
\[ D = \{ \frac{\alpha_i}{\gamma_i}, \beta_i, \gamma_i \} \]

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\[ A = \{ M\beta_i | \frac{\alpha_i, \beta_i}{\gamma_i} \in D \} \]

\[ \overline{M\beta_i} = \neg \beta_i \]

**Claim:** \(S = Th(T \cup \Delta)\) (with \(\Delta \subseteq A\)) is a stable extension iff \(E = S - \Delta\) is a Reiter extension of \((W, D)\).
THEORIST and Stable Extensions

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- Then $E$ is a **stable extension** of $(T, A, \overline{\cdot})$ iff $E$ is a **THEORIST extension**.
Admissible and Preferred Extensions

- For a argumentation theoretic frame \( (T, A, \vdash) \), \( Th(T \cup \Delta) \) (with \( \Delta \subseteq A \)) is an **admissible extension** (and \( \Delta \) is called **admissible argument** iff
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- $Th(T \cup \Delta)$ is a **preferred extension** iff it is **admissible** and **set-inclusion maximal**. Then $\Delta$ is called **preferred argument**
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• Corresponds to admissible model semantics [Dung 91] and preferred model semantics [Dung 91] or partial stable model semantics [Sacca and Zaniolo 90] in nonmonotonic logic programming (LP)
Examples

\[ W = \emptyset, \]
\[ D = \left\{ \frac{\neg p}{p}, \frac{\neg q}{r}, \frac{\neg r}{q}, \frac{\neg r}{s} \right\}. \]

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\[ \leadsto \] More *general* . . . stable implies preferred
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Since $E$ is stable, all $\alpha \in A - \Delta$ are attacked by $\Delta$. This implies that $\Delta$ attacks $\Delta'$, hence $\Delta$ is admissible.

Moreover, $\Delta$ is set-inclusion maximal because adding any element from $A - \Delta$ leads to a self-attack!
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