Principles of Knowledge Representation and Reasoning

4. Nonmonotonic Reasoning

4.4 Argumentation Theoretic Approaches

Bernhard Nebel

- Motivation
- Stable Extensions
- DL and Poole’s THEORIST
- Admissible and Preferred Extensions
- Upper Bounds for Nonmonotonic Reasoning
- THEORIST: Completeness Results
- DL: Completeness Results
Motivation

- With conventional, “stable” extensions, one always has to consider all assumptions, when a particular formula should be proven.
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→ Hopefully, such approaches are “more natural” and computationally simpler than ordinary NM logics.
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- $\Delta \subseteq A$ **attacks** $\Delta' \subseteq A$ iff $\Delta$ attacks a $\alpha \in \Delta'$
- $\Delta$ is **closed** iff $\Delta = A \cap \text{Th}(T \cup \Delta)$
Stable Extensions

- For a argumentation-theoretic frame \((T, A, \bar{\cdot})\), \(\text{Th}(T \cup \Delta)\) (with \(\Delta \subseteq A\)) is a stable extension (and \(\Delta\) is called stable argument) iff

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- Name comes from von stable expansions (AEL) and stable model semantics (LP).
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DL and Stable Extensions

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• \(A = \{M\beta_i \mid \frac{\alpha_i:\beta_i}{\gamma_i} \in D\}\)

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**Claim:** \(S = \text{Th}(T \cup \Delta)\) (with \(\Delta \subseteq A\)) is a **stable extension** iff \(E = S - \Delta\) is a **Reiter extension** of \((W, D)\).
THEORIST and Stable Extensions

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$\Rightarrow$ Then $E$ is a *stable extension* of $(T, A, \overline{\beta})$ iff $E$ is a **THEORIST extension**
Admissible and Preferred Extensions

- For an argumentation theoretic frame \((T, A, \cdot)\), \(E = \text{Th}(T \cup \Delta)\) (with \(\Delta \subseteq A\)) is an **admissible extension** (and \(\Delta\) is called **admissible argument**) iff

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- \(\text{Th}(T \cup \Delta)\) is a **preferred extension** iff it is **admissible** and **set-inclusion maximal**. Then \(\Delta\) is called **preferred argument**.
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• $\text{Th}(T \cup \Delta)$ is a preferred extension iff it is admissible and set-inclusion maximal. Then $\Delta$ is called preferred argument.

• Corresponds to admissible model semantics [Dung 91] and preferred model semantics [Dung 91] or partial stable model semantics [Sacca and Zaniolo 90] in nonmonotonic logic programming (LP).
Examples

\[ W = \emptyset, \]
\[ D = \left\{ \frac{\neg p}{p}, \frac{\neg q}{r}, \frac{\neg r}{q}, \frac{\neg r}{s} \right\}. \]

1. Reiter extensions = stable extensions?
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\( \rightsquigarrow \) More general \( \ldots \) stable implies preferred
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**Theorem.** Stable extensions are preferred.

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Since $E$ is stable, all $\alpha \in A - \Delta$ are attacked by $\Delta$. This implies that $\Delta$ attacks $\Delta'$, hence $\Delta$ is admissible.

Moreover, $\Delta$ is set-inclusion maximal because adding any element from $A - \Delta$ leads to a self-attack!
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**Stable and Preferred Extensions**

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Moreover, $\Delta$ is set-inclusion maximal because adding any element from $A - \Delta$ leads to a self-attack! Hence $\Delta$ is a preferred argument and for this reason $E$ must be a preferred extension.
Normal, and Flat Frameworks

An argumentation theoretic framework is called normal if all preferred extensions are stable.
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**Proposition.** In flat frameworks there exists at least one admissible (and one preferred) argument: $\emptyset$. 
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• **Flat Frameworks** can simplify life a lot. For instance, **sceptical reasoning** under the **admissibility semantics** reduces to the **monotonic background logic**... because the empty argument is always admissible
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→ Simple Frameworks

→ Same as flat for sceptical reasoning under the admissibility semantics.
Computational Complexity

We consider the following problems:

- **Credulous reasoning**: Is there an extension that contains a given formula $\alpha$?
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*⇒* Generic Results? Unfortunately only upper bounds.
Upper Bounds

Let $C$ be the complexity class of the entailment problem of the underlying monotonic logic.
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<table>
<thead>
<tr>
<th></th>
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<tr>
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Results for stable semantics are also lower bounds.
Upper Bounds for Argument Verification

**Theorem.** For general argumentation theoretic frameworks, argumentation verification is in the following complexity classes:

1. $P^c$ for stable semantics
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  - Admissibility Semantics:
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  we simply cannot avoid the problem of being forced to consider all supersets of an admissible set.
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- Nevertheless, the argumentation theoretic approach seems to be reasonable for some applications in an LP context
Literature

