4. Nonmonotonic Reasoning

4.1 Introduction

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- Motivation
- Different Kinds of Nonmonotonic Reasoning
- Different Formal Approaches
A Motivating Example: Common Sense Reasoning

1. Tweety was a bird like other birds.
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2. He enjoyed his life and built a nest every summer.
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3. During the summer he stayed in the *northern hemisphere*, in the winter he stayed in *Africa*. 
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How would you formalize the example in formal logic so that you get the expected answers?
A Formalization . . .

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〜 what if Tweety is a **Emu**?

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〜 Tweety could be a canary **traveling with** a rich woman each year
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**Reasoning about Actions:** When reasoning about actions, it is customary to assume that a property changes only if it has to change, i.e., by default properties do not change.
Default Reasoning: Jump to a conclusion if there is no information that contradicts the conclusion
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**Defeasible Reasoning:** Reasoning based on assumptions that can turn out to be wrong – i.e., *conclusions are defeasible*. In particular, default reasoning is defeasible.

**Nonmonotonic Reasoning:** In classical logic, the set of consequence grows monotonically with the set of premises. If reasoning becomes defeasible, then reasoning becomes *non-monotonic*. 
Approaches to Non-Monotonic Reasoning

- **Consistency-based:** Extend classical theory using rules that are consistently applicable

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- **Belief Revision**: Revise your beliefs if evidence to the contrary comes up
  - The axiomatic AGM approach to belief revision
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- **Typically** bird$(x)$ implies can-fly$(x)$
- $\forall x: \text{emu}(x) \rightarrow \text{bird}(x)$
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NM Logic – Normal Models

If \( \varphi \) typically implies \( \psi \), then the models satisfying \( \varphi \land \psi \) should be more normal than those satisfying \( \varphi \land \neg \psi \). Similarly, try to minimize the extension of a chosen “Abnormality” predicate.

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$\sim \rightarrow \text{can-fly(tweety)}$

+ $\text{emu(tweety)}$
- Now in all models (also the normal ones) $\sim \rightarrow \neg \text{can-fly(tweety)}$
• If \( \varphi \) typically implies \( \psi \), then let us assume \( \varphi \rightarrow \psi \) – provided there is no argument against it that is not itself defeated!
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→ Different notion of being against an argument and defeating an argument!
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⇒ When it becomes known that Tweety is a penguin, modify the universally quantified implication.