4. Nonmonotonic Reasoning

4.8 Belief Revision

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- Motivation
- Updates and Revision
- Postulates
- Revision Schemes
- Base Revision
- Connection to NMR
Belief Change

- A dual approach to nonmonotonic reasoning is belief change.
- We start with some belief state $K$. When new information arrives, we change the belief state in order to accommodate the new information.
- In the general case, the changed belief state may not be a superset of the original belief state.
- Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.
Two Scenarios

- We have a theory about the world, and the new information is meant to correct our theory.

  \[\rightarrow\] belief revision: change your belief state minimally in order to accommodate the new information.

- We have a correct theory about the current state of the world, and the new information is meant to record a change in the world.

  \[\rightarrow\] belief update: incorporate the change by assuming that the world has changed minimally.
Updates and Revision are Different

Assume the new information is consistent with our old beliefs.

- In case of *revision*, we would like to add the new information monotonically to our old beliefs.

- For *belief update* this is not necessarily the case.
  - Assume we know that the *door is open or the window is open*.
  - Assume we get the information that after a change in the world, the *door is now closed*.

    → In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that *the window is open*. 
Belief Change Operations

- General assumption: A belief state is modeled by a deductively closed theory, i.e., $K = Cn(K)$ with $Cn$ the consequence operator.
- $\mathcal{L}$: Logical Language (propositional logic).
- $Th_\mathcal{L}$: Set of deductively closed theories (or belief sets) over $\mathcal{L}$.

\[ \leadsto \text{Belief Change Operations:} \]

- Monotonic addition: $+: Th_\mathcal{L} \times \mathcal{L} \rightarrow Th_\mathcal{L}$
  
  $K + \psi = Cn(K \cup \{\psi\}$

- Revision: $\vdash : Th_\mathcal{L} \times \mathcal{L} \rightarrow Th_\mathcal{L}$

- Reasonable revision operations?

\[ \leadsto \text{AGM Revision Postulates} \] (Alchourron, Gärdenfors, Makinson)
AGM Postulates: Constraining the space of Revision Operations

(+1) \( K + \varphi \in Th_L \)
(+2) \( \varphi \in K + \varphi \)
(+3) \( K + \varphi \subseteq K + \varphi \)
(+4) If \( \neg \varphi \notin K \), then \( K + \varphi \subseteq K + \varphi \)
(+5) \( K + \varphi = Cn(\bot) \) only if \( \vdash \neg \varphi \)
(+6) If \( \vdash \varphi \leftrightarrow \psi \) then \( K + \varphi = K + \psi \)
(+7) \( K + (\varphi \land \psi) \subseteq (K + \varphi) + \psi \)
(+8) If \( \neg \psi \notin K + \varphi \),

then \( (K + \varphi) + \psi \subseteq K + (\varphi \land \psi) \).

Note: AGM postulates do not constrain the operation with respect to varying belief sets!
Canonical Revision Operations?

- The postulates *constrain* the space to *fully rational revision operations*, but do not pick a single one.

- Revision operations are closed under intersection, so should we choose the minimum? *NO!*

  This is *full meet revision*, which is known to be useless since
  
  \[ K + \phi = Cn(\phi) \]
  
  for all \( \phi \) that are inconsistent with \( K \).

  What other ways are there to generate a reasonable revision operation?
Belief Revision Schemes

- Preference information (what to keep and what to give up)

- ...may be different for different $K$'s but independent from the new information $\varphi$

compose revision operation pointwise for each $K$

- A belief revision scheme (BRS) is a “recipe” for deriving a revision operation – restricted to a particular set $K$ – from
  - the belief set and
  - preference information over this belief set
Examples

Partial Meet Revision (AGM): Preference information is given by a selection function \( \gamma \) over the sets of maximal consistent subtheories \( (K \downarrow \varphi) \):

\[
K \uplus \varphi \overset{\text{def}}{=} \left( \bigcap \gamma(K \downarrow \neg \varphi) \right) + \varphi
\]

where \( K + \varphi = \text{Cn}(K \cup \{\varphi\}) \).

Cut Revision (GM): Preference information is given by complete preorder \( \preceq \) over all \( \psi \in K \):

\[
K \uplus \varphi \overset{\text{def}}{=} \{ \psi \in K \mid \neg \varphi \prec \psi \} + \varphi
\]

Provided \( \preceq \) satisfies a number of axioms (epistemic entrenchment), cut revisions coincide with the fully rational revision operations.
Revision – Viewed Computationally

- We don’t want to deal with deductively closed theories.

→ Consider belief bases (arbitrary set of props.) as representing belief sets.

- We don’t want to specify an arbitrary amount of preference information.

→ A theory $K$ over the propositional logic $L$ with $n$ propositional atoms can have as much as
  - $2^{2^n}$ different propositions
  - $2^n$ different models

→ Consider ways of specifying preference information in a concise way, i.e., polynomial in the size of the belief base.
Base Revision Schemes

• Starting with the finite belief base $A$ and preference information over the elements of $A$.

→ we want to generate a revision operation (restricted to $Cn(A)$).

∼ Assume a partitioning of $A$ into $n$ priority classes $A_1, \ldots A_n$ such that the elements of $A_i$ are more important or relevant than those of $A_j$ for $j < i$.

∼ Equivalently, a complete preorder $\sqsubseteq$ over $A$ comparing priorities (epistemic relevance).

∼ Define a (base-) revision scheme that keeps as much of the more relevant propositions as possible.

⇒ Base revision schemes generate revision operations in the same way as ordinary schemes do.
Prioritized Meet-Base Revision (PMBR)

Let \((A \downarrow \neg \varphi)\) be the maximal subsets of \(A\) that are consistent with \(\varphi\) and maximize relevant propositions.
Prioritized Meet-Base Revision (PMBR) – Formally

\[ A \oplus \varphi \overset{\text{def}}{=} \left( \bigcap_{B \in (A \downarrow \neg \varphi)} Cn(B) \right) + \varphi. \]

Define a revision operation \( + \) on \( Cn(A) \) (that depends on \( A \) and the priority information) by

\[ Cn(A) + \varphi \overset{\text{def}}{=} A \oplus \varphi. \]
Properties of PMBRs

- Generates *partial meet revision*, but does not satisfy $(\oplus 8)$ in general.
- Deciding whether $A \oplus \varphi \models \psi$ is $\Pi^p_2$-complete, even for one priority class.
- A **revised base** can be represented by
  \[ A \oplus \varphi = Cn \left( \bigvee (A \downarrow \neg \varphi) \right) \land \varphi. \]
- A revised base can become **exponentially large**
  \[ A = \{ p_1, \ldots, p_m, q_1, \ldots, q_m \}, \]
  \[ \varphi = \bigwedge_{i=1}^m (p_i \leftrightarrow \neg q_i). \]
  \((A \downarrow \varphi)\) has size exponential in \(|A|\).
- Worse, in some cases there exist no **concise** representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).
Let \( \widehat{A}_j \overset{\text{def}}{=} \bigcup_{i=j}^n A_i \), then **cut base-revision** \( \otimes \) is defined as:

\[
A \otimes \varphi \overset{\text{def}}{=} Cn(\{ \psi \in A \mid \psi \in A_j, \widehat{A}_j \not\vdash \neg \varphi \}) + \varphi.
\]

- **Natural representation** of revised base.
- **Easy** to compute: in \( P^{NP}[O(\log n)] \).
- Restriction to Horn logic leads to \( O(n \log n) \).
Being less conservative ...

**Idea:** Throw away an entire priority class only if it would lead to a contradiction which cannot be blamed on a lower classes \( \leadsto \text{linear} \) (or **unambiguous**) base-revision \( \circ \).

- Generates **fully rational** revision operations.
- **Complexity:** \( \Delta_2^p \)-complete; \( O(n^2) \) for Horn logic.
- **LBR \( \approx \) CBR**, but a CBR realizing an LBR requires exponentially more priority classes.
Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

- Given $K$ and a revision operation $\vdash$

$\rightsquigarrow$ a nonmonotonic consequence relation can be defined as follows:

$$\phi \rightsquigarrow \psi \text{ iff } \psi \in K \vdash \phi.$$ 

In this case,

- the rationality postulates correspond to principles of NMR (such as cautious monotony etc.);

- in the case of prerequisite-free, normal defaults $D$ (Theorist), the cautious conclusions from $(W, D)$ are simply $D \oplus W$ with one priority level;

- similar relationship between Brewka’s level default theories and PMBRs.
NMR Principles and Rationality Postulates

(±2) \( \varphi \in K \vdash \varphi; \)

\[ \sim \text{ Reflexivity} \]

(±3) \( K \vdash \varphi \subseteq K \vdash \varphi; \)

\[ \sim \text{ Super Classicality} \]

(±6) If \( \vdash \varphi \leftrightarrow \psi \) then \( K \vdash \varphi = K \vdash \psi; \)

\[ \sim \text{ Left Logical Equivalence} \]

(±8) If \( \neg \psi \notin K \vdash \varphi, \)
then \( (K \vdash \varphi) + \psi \subseteq K \vdash (\varphi \land \psi). \)

\[ \sim \text{ Rational Monotonicity} \]
Conclusions from the Correspondence

- NMR can be thought of as the other side of the same coin.
- NMR (at least for default logic) is as hard as revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in polynomial space, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a denser way than classical logic, and with that come higher computational costs.
Outlook & Summary

• While NMR and belief revision seem to be the two sides of the same coin, there are notable **pragmatic differences**:
  ○ Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use **dense representations**.
  ○ A similar argument could be made for the **computational complexity**.

• NMR and Belief Revision can be thought of as **qualitative ways** of dealing with uncertainty in a purely logical setting.

• There exists a strong **correspondence** between NMR and BR.

• Both are computationally expensive and representational problematic.

• There are cases, though, that are tractable and practical.


P. Gärdenfors, Belief Revision and Nonmonotonic Logic: Two Sides of the Same Coin?, In *ECAI-90*, 768-773.