Multi-Agent Systems

Decentralized Multi-Agent Path Finding

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⇒ How do we define the *joint execution* of such plans?

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- How do we define joint execution of such conflicting plans?

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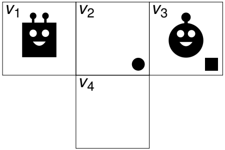

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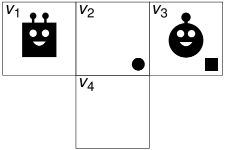

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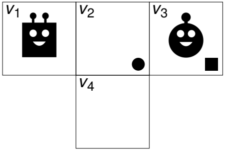

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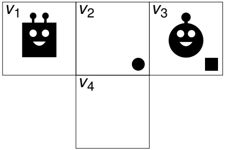

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\end{aligned}
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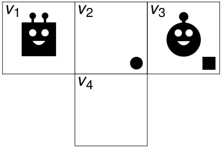
Joint execution

- Let us assume, all agents have planed and a subset of them came up with a *family of plans* $(\pi_i)_{i \in A}$.
- Among the agents that have a plan with their own action as the next action to execute, one is chosen.
- The action of the chosen agent is executed.
- Agents, which have anticipated the action, track that in their plans.
- All other agents have to *replan* from the new state.
- Since everybody has a successful plan, no acting agent will ever execute an action that leads to a dead end.

Example execution



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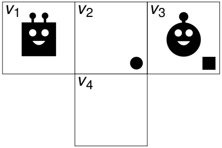

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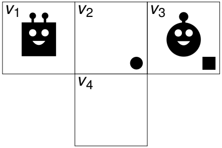

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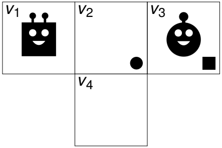

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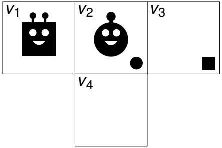

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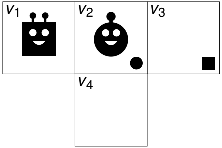

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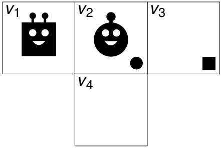

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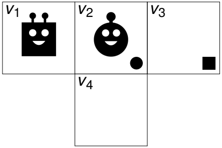

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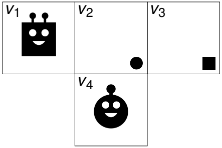

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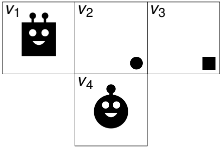

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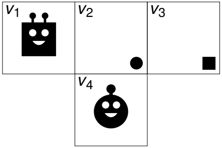

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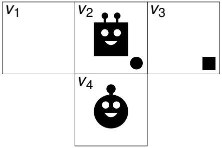

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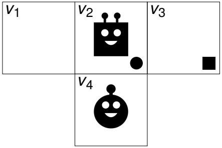

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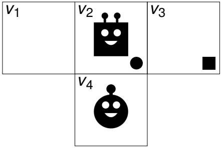

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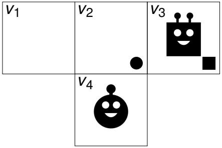

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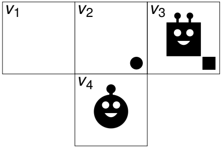

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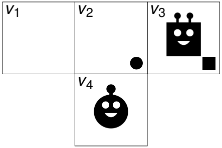

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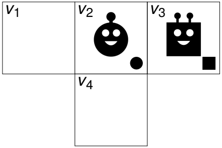

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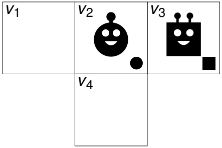

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Done!

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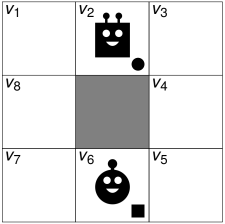
- Agents could be *eager*: If agents could act (without creating a cycle or a dead end), they choose to act.

→ Agents might create cyclic executions (without creating plans that are cyclic), leading to *infinite executions*.

Example for infinite execution



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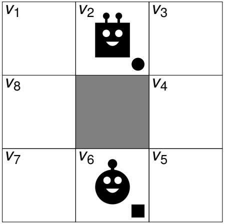

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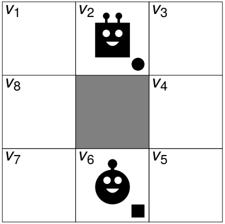

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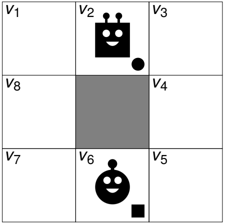

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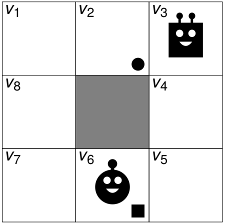

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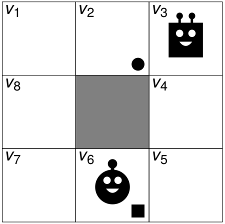

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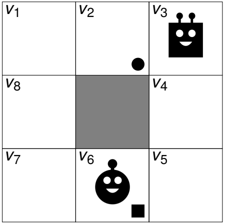

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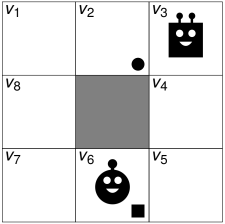

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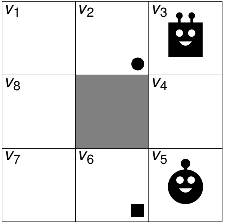

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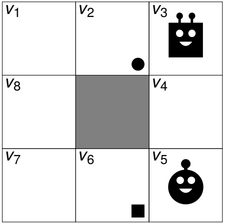

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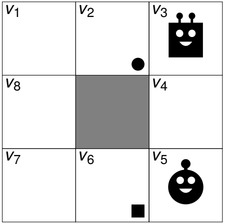

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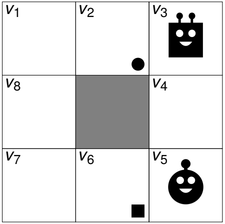

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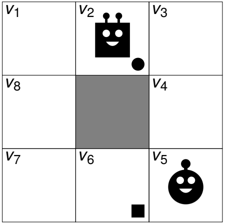

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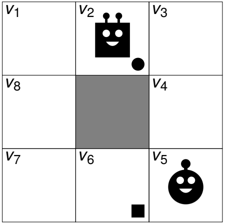

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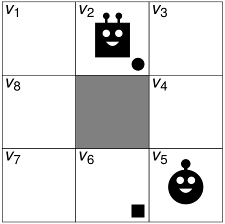

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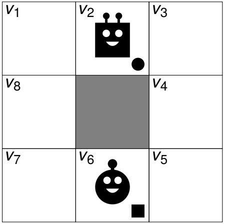

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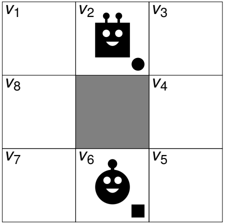

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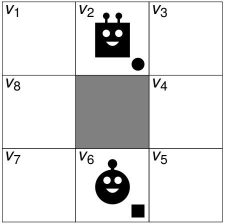

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Optimally eager agents

- Eager agents avoid *deadlocks*, however they are *hyper-active*.
- They might even move away from their destination!
- So, let force them to be smart: They should generate only optimal plans . . . and among those optimal plans they should also be eager.
- In our previous example: After the square agent moved right, the circle agent will choose to move left!

→ Does it always work out?

Optimally eager agents are always successful

Theorem

Optimally eager agents are always successful on all solvable DMAPF instances.

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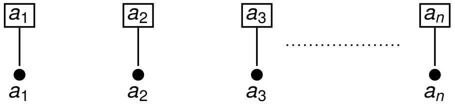

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- Conservative replanning is not helpful in this context, because the executed actions might not be a prefix of a valid plan!

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→ Models multi-robot interactions without communication

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→ Since MAPF/DU is a special case of epistemic planning (initial state uncertainty which is monotonically decreasing), we can use concepts and results from this area.

MAPF/DU representation & state space

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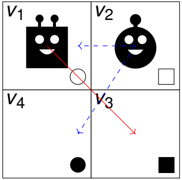
- A *MAPF/DU instance* is given by $\langle A, G, s_0, \alpha_* \rangle$, where $s_0 = \langle \alpha_0, \beta_0 \rangle$.

MAPF/DU: Implicitly coordinated branching plans



- Square agent S wants to go to v_3 and knows that circle agent C wants to go to v_1 or v_4.

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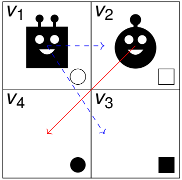

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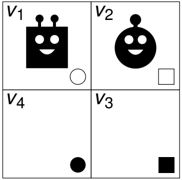

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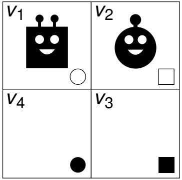

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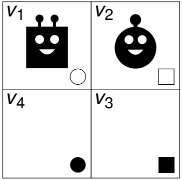

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- S needs *shifting its perspective* in order to plan for all possible destinations of C (*branching on destinations*).
- Planning for C, S must *forget* about its own destination.

Branching plans: Building blocks

Branching plans consist of:

- *Movement actions*: $(\langle agent \rangle, \langle sourcenode \rangle, \langle targetnode \rangle)$, i.e., a movement of an agent
- *Success announcement*: $(\langle agent \rangle, \mathscr{S})$, after that all agents know that the agent has reached its destination and it cannot move anymore
- *Perspective shift*: $[\langle agent \rangle : \ldots]$, i.e., from here on we assume to plan with the knowledge of agent $\langle agent \rangle$. This can be unconditional or conditional on $\langle agent \rangle$'s destinations.
- *Branch on all destinations*: $(?\langle dest_1 \rangle \{\ldots\}, \ldots, ?\langle dest_n \rangle \{\ldots\})$, where all destinations of the current agent have to be listed. For each case we try to find a successful plan to reach the goal state.

Semantics of branching plans

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- A perspective shift from i to j with subsequent branching on destinations transforms the subjective state $s^i = \langle \alpha, \beta, i, v_i \rangle$ to a set of subjective states $s^{j_k} = \langle \alpha, \beta, j, v_{j_k} \rangle$ with all $v_{j_k} \in \beta(j)$.

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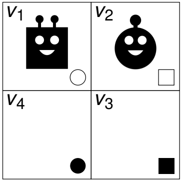
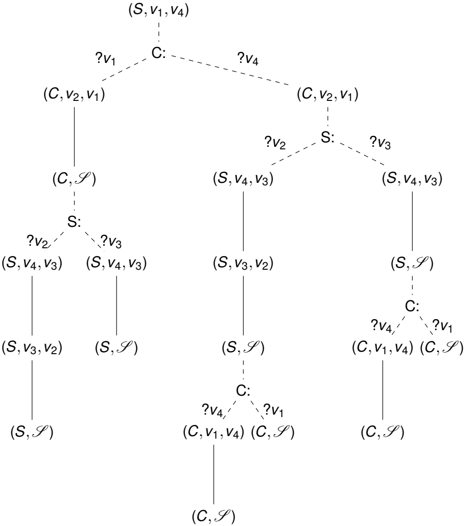
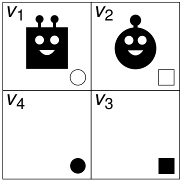
- Note: After a perspective shift to j, only j can move and announce success!

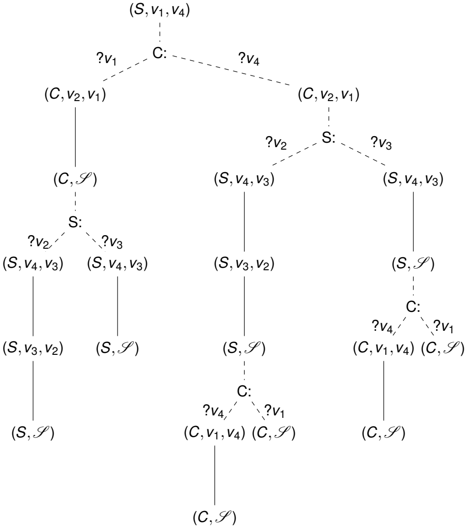
Branching plan: Example





Branching plan: Subjective execution example





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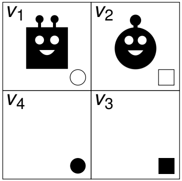

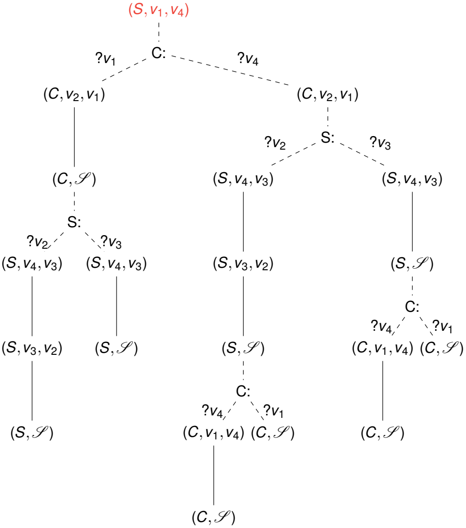

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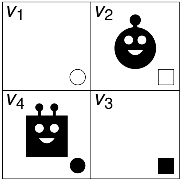

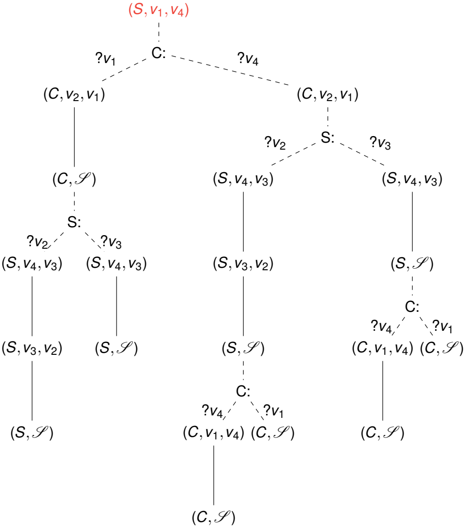

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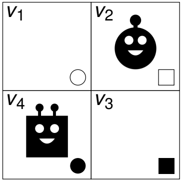

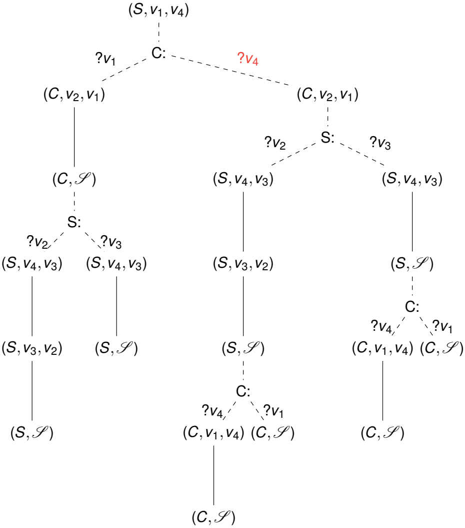

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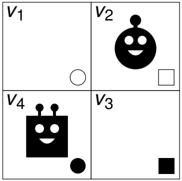

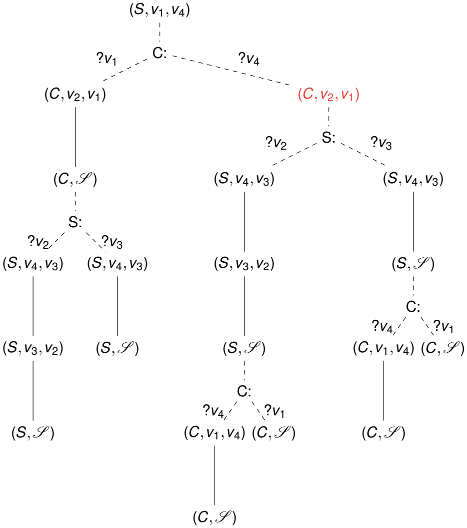

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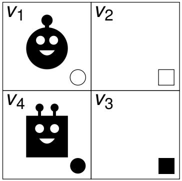

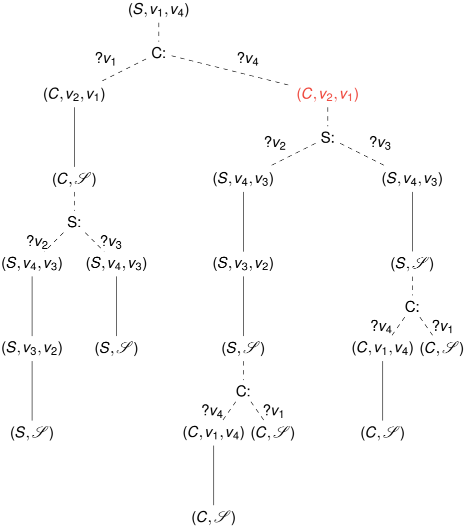

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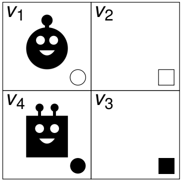

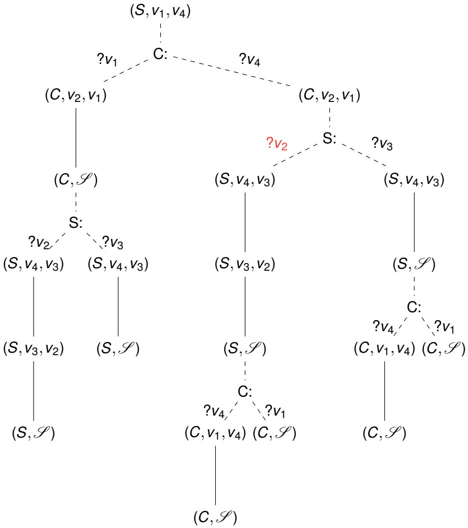

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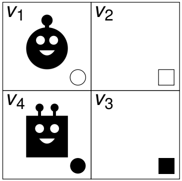

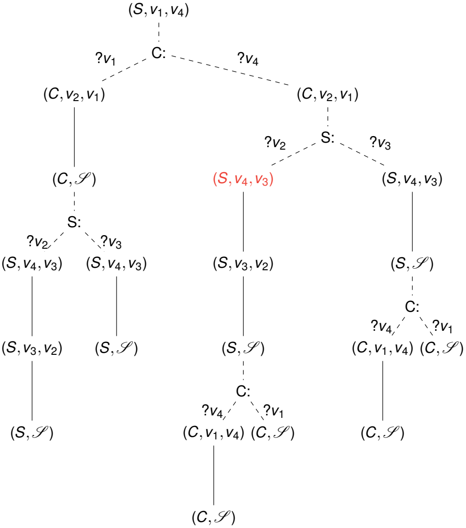

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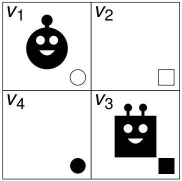

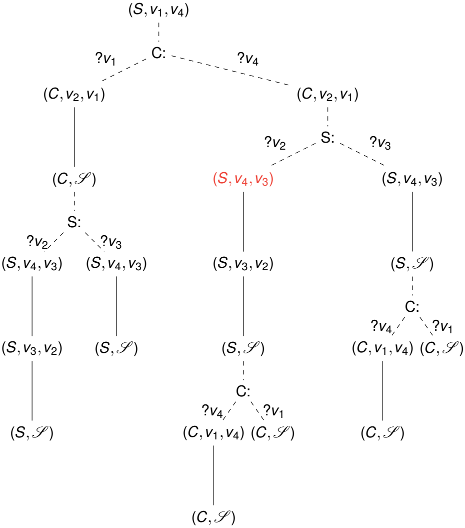

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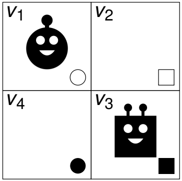

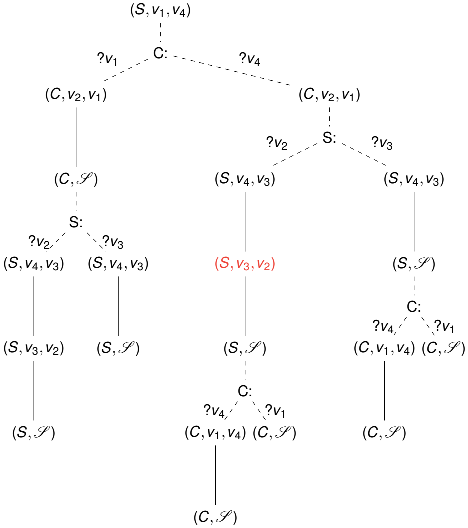

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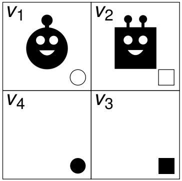

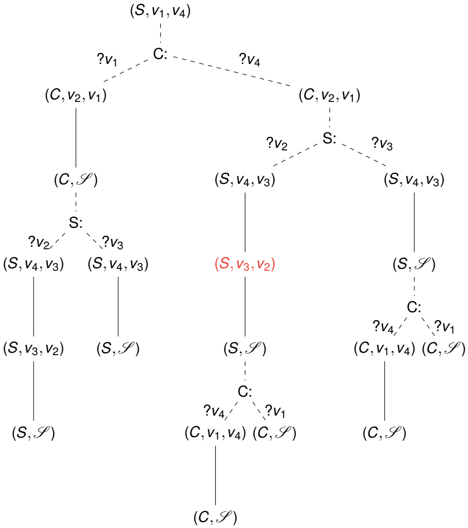

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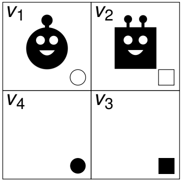

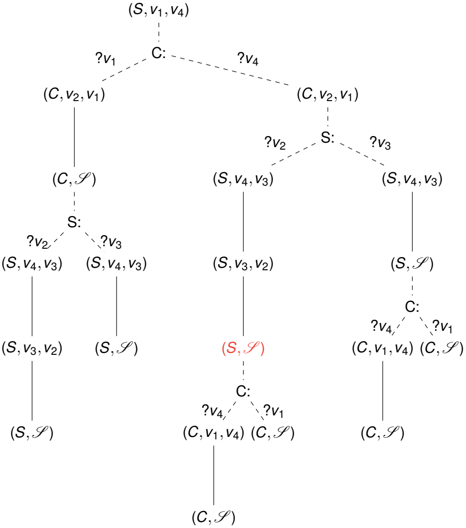

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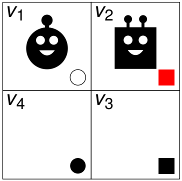

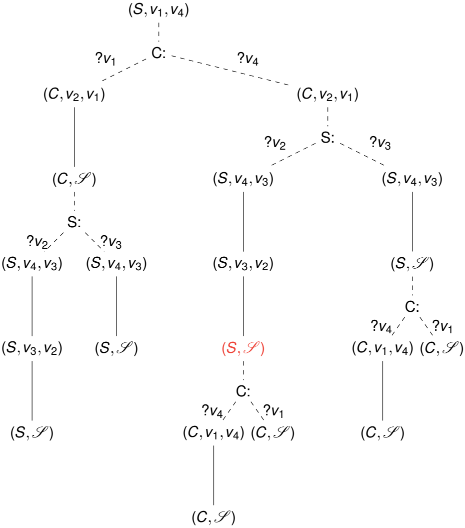

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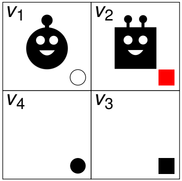

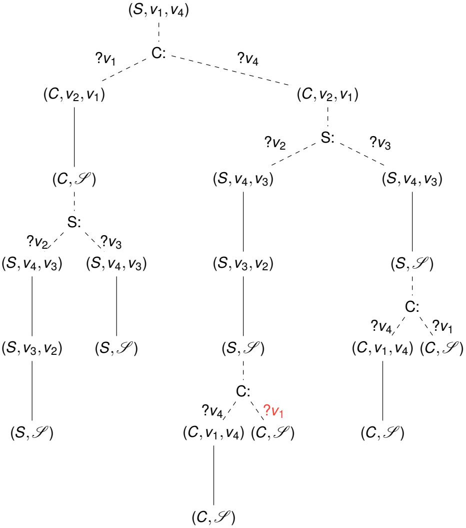

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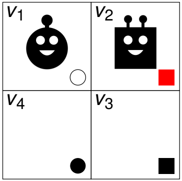

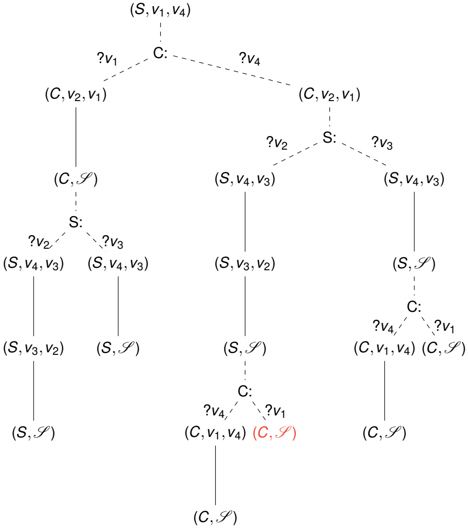

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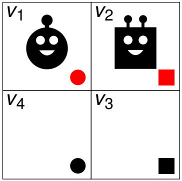

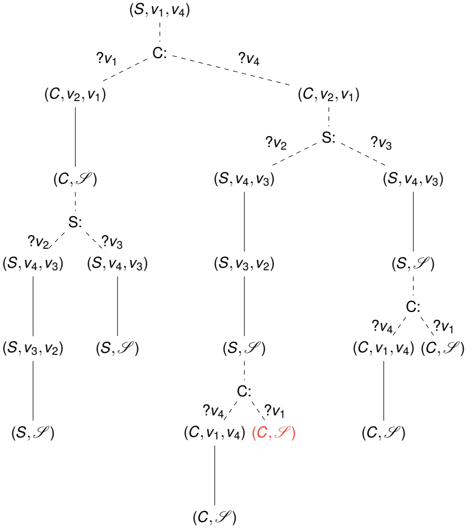

Strong plans

Similar to the notion of strong plans in non-deterministic single-agent planning, we define *i-strong plans* for an agent *i* to be:

- *cycle-free*, i.e., not visiting the same objective state twice;
- *always successful*, i.e. always ending up in a state such that all agents have announced success;
- *covering*, i.e., for all combinations of possible destinations of agents different from *i*, success can be reached.

Subjectively and objectively strong plans

- A plan is called *subjectively strong* if it is i-strong for some agent i.
- A plan is called *objectively strong* if it is i-strong for each agent i.
- An instance is *objectively* or *subjectively solvable* if there exists an objectively or subjectively strong plan, respectively.

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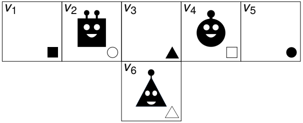

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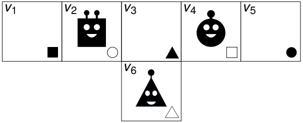

→ There does not exist a T-strong plan, but an S- and a C-strong plan.

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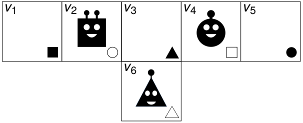
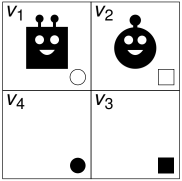


→ There does not exist a T-strong plan, but an S- and a C-strong plan.

- Difference between subjective and objective solvability concerns only the first acting agent!

Structure of strong plans: Stepping stones

- A *stepping stone* for agent *i* is a state in which *i* can move to each of its possible destinations, announcing success, and afterwards, for each possible destination, there exists an *i*-strong plan to solve the resulting states.



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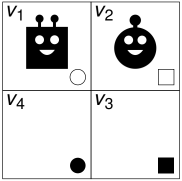
- S can create a stepping stone for C by moving from v_1 via v_4 to v_3.



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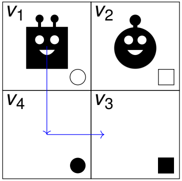

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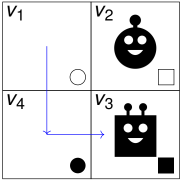

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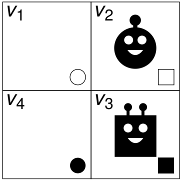
- *C* can now move to v_1 or v_4 and announce success.



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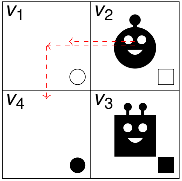

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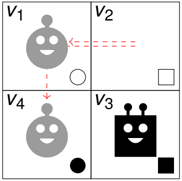

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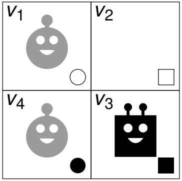

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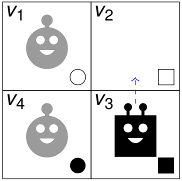

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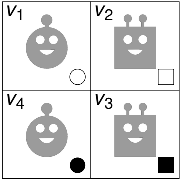

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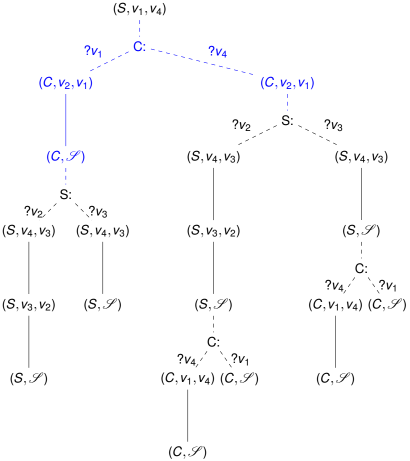

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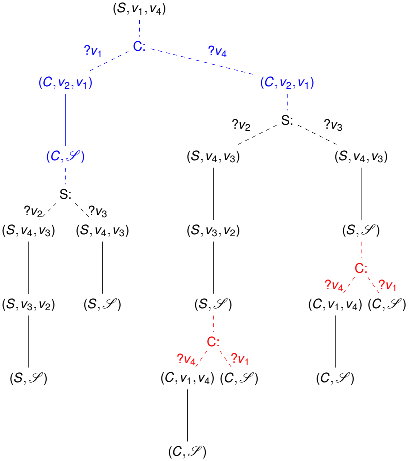

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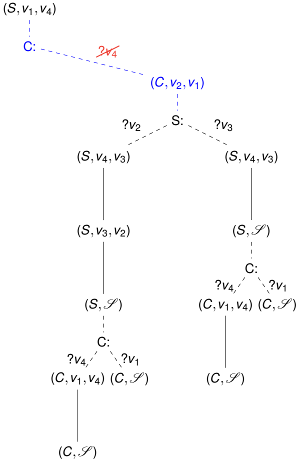

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Direct consequence of the stepping stone theorem and the maximal number of movements in the MAPF problem. □

Joint execution and execution guarantees

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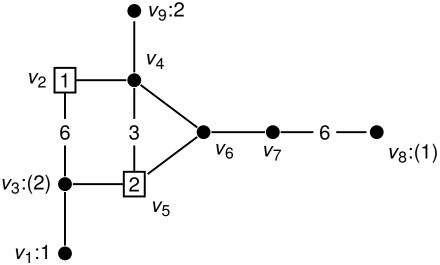
- We do not have uniform knowledge … and indeed, execution cycles cannot be excluded.

A counter example



A number on an edge means that there are as many nodes on a line.

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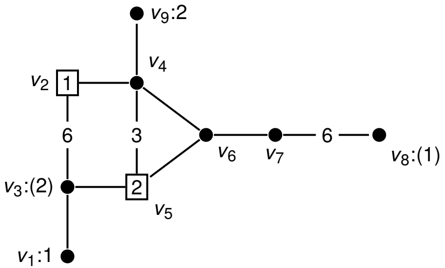

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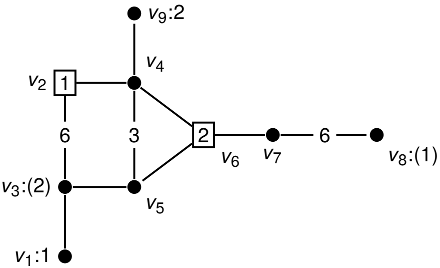

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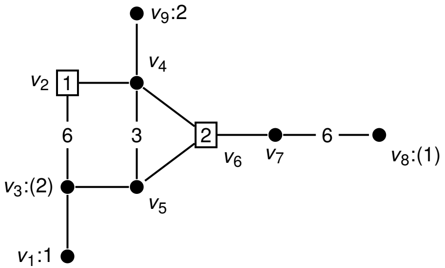

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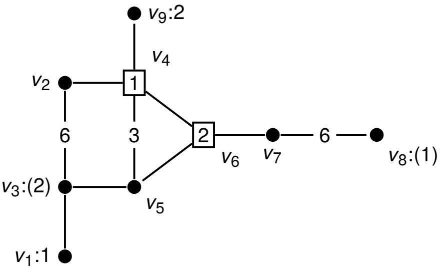

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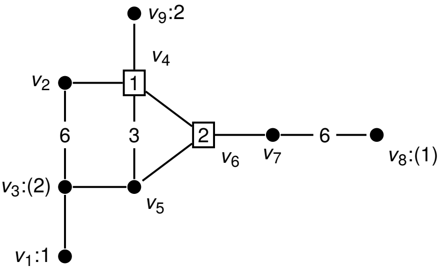

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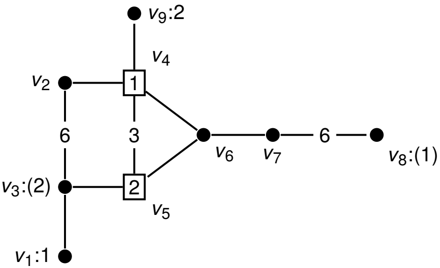

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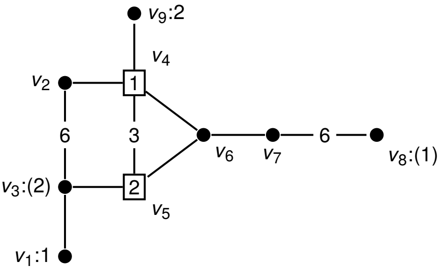

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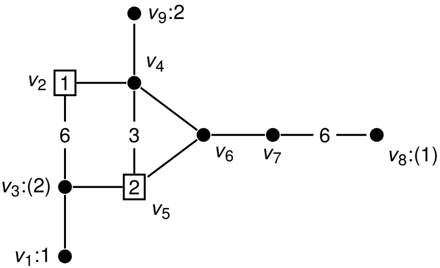

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Conservatism

- Perhaps conservatism can help!
- Similarly to DMAPF, conservative replanning means that the already executed actions are used as a prefix in the plan to be generated.
- Differently from DMAPF, we assume that after a success announcement, the initial state is modified so that the *real destination* of the agent is known in the initial state.
- Otherwise we could not solve instances that are only subjectively solvable.

Conservative, optimally eager agents

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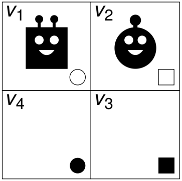
Proof idea.

After the second agent starts to act, all agents have an identical perspective and for this reason produce objectively strong plans with the same execution costs, which can be shown to be bounded polynomially using the stepping stone theorem. □

Conservative replanning example

- Assume S moves first to v_4.



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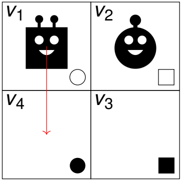

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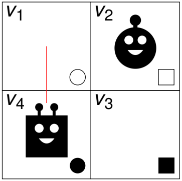

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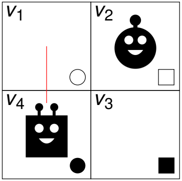

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- Assume C re-plans. From now on, in replanning from the beginning, it has to do a perspective shift to S, because it now has to extend the partial plan starting with (S, v_4, v_1), i.e., it has to create an objectively strong plan.

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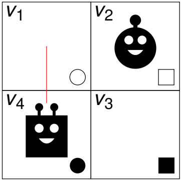

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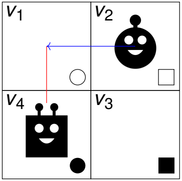

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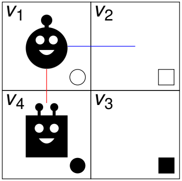

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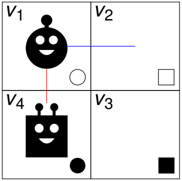

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- Assume that C moves now to v_1.
- From now on, also S has to make a perspective shift to C, effectively “forgetting” its own destination, i.e., it also has to create a objectively strong plan.

Computational Complexity: Algorithms and Turing machines

- We use Turing machines as formal models of algorithms
- This is justified, because:
  - we assume that Turing machines can compute all computable functions
  - the resource requirements (in term of time and memory) of a Turing machine are only polynomially worse than other models
- The regular type of Turing machine is the deterministic one: DTM (or simply TM)
- Often, however, we use the notion of nondeterministic TMs: NDTM

Computational Complexity: Complexity classes P and NP

Problems are categorized into complexity classes according to the requirements of computational resources:

- The class of problems decidable on deterministic Turing machines in polynomial time: P
  - Problems in P are assumed to be efficiently solvable (although this might not be true if the exponent is very large)
  - In practice, a reasonable definition
- The class of problems decidable on non-deterministic Turing machines in polynomial time, i.e., having a poly. length accepting computation for all positive instances: NP
- More classes are definable using other resource bounds on time and memory

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  - show that some hard problem can be reduced to the problem at hand

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- Given languages L_1 and L_2, L_1 can be polynomially reduced to L_2, written $L_1 \leq_p L_2$, if there exists a polynomial time-computable function f such that

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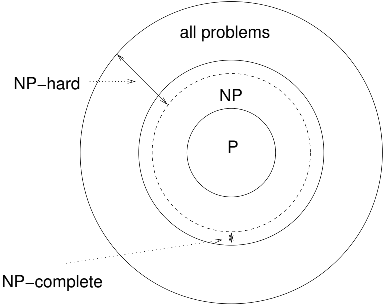

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  - What if we want to decide UNSAT, the complementary problem?
  - It seems necessary to check all possible truth-assignments!
- Define co-C = $\{L \subseteq \Sigma^* : \Sigma^* \setminus L \in C\}$ (provided Σ is our alphabet)
- co-NP = $\{L \subseteq \Sigma^* : \Sigma^* \setminus L \in \mathrm{NP}\}$
- Examples: UNSAT, TAUT $\in$ co-NP!
- *Note:* P is closed under complement, in particular,

$$\mathrm{P} \subseteq \mathrm{NP} \cap \text{co-NP}$$

Computational Complexity: PSPACE

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Some facts about PSPACE:

- PSPACE is closed under complements (… as all other deterministic classes)
- PSPACE is identical to NPSPACE (because non-deterministic Turing machines can be simulated on deterministic TMs using only quadratic space: Savitch's Theorem)
- NP$\subseteq$PSPACE (because in polynomial time one can "visit" only polynomial space, i.e., NP$\subseteq$NPSPACE)

Computational Complexity: PSPACE-completeness

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A decision problem (or language) is PSPACE-complete if it is in PSPACE and all other problems in PSPACE can be polynomially reduced to it.

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Intuitively, PSPACE-complete problems are the “hardest” problems in PSPACE (similar to NP-completeness). They appear to be “harder” than NP-complete problems from a practical point of view.

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An example for a PSPACE-complete problem is the NDFA equivalence problem:

Instance: Two non-deterministic finite state automata A_1 and A_2.

Question: Are the languages accepted by A_1 and A_2 identical?

Computational complexity of MAPF/DU bounded plan existence

Theorem

Deciding whether there exists an eager MAPF/DU i-strong or objectively strong plan with execution cost k or less is PSPACE-complete.

Proof sketch.

Since plans have polynomial depth, all execution traces can be generated non-deterministically and tested using only polynomial space, i.e., PSPACE-membership.

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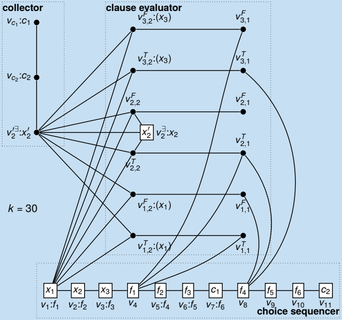
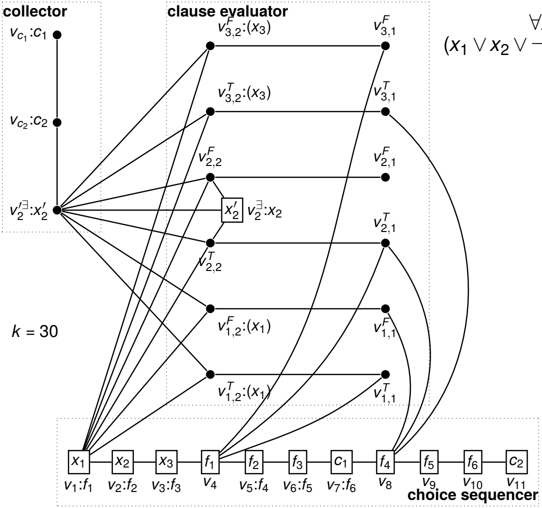


The reduction enlarged



$$\forall x_1 \exists x_2 \forall x_3 :$$
$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

Complexity with a fixed number of agents

These results probably imply that the technique could not be used online.

For a fixed number of agents, however, the bounded planning problem is polynomial.

Theorem

For a fixed number c of agents, deciding whether there exists a MAPF/DU i-strong or objectively strong plan with execution cost of k or less can be done in time $O(n^{c^2+c})$.

That means, for two agents, it takes “only” $O(n^6)$ time – but in practice it should be faster.

An algorithm for generating an objective MAPF/DU plan for two agents

1. Determine in the *state space of all node assignments* the distance to the initial state using Dijkstra: $O(|V|^4)$ time.

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2. For each of the $O(|V|^2)$ configurations check, whether it is a *potential stepping stone* for one agent, i.e., whether all potential destinations of this agent are reachable using Dijkstra on the modified graph, where the other agent blocks the way: $O(|V|^4)$ time.

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4. Consider all stepping stones and minimize over the maximum plan depth. Among the minimal plans select those that are eager for the planning agent.

Summary

- DMAPF generalizes the MAPF problem by dropping the assumption that plans are generated centrally and then communicated.
- MAPF/DU generalizes the MAPF problem further by dropping the assumptions that destinations are common knowledge.
- A solution concept for this setting are *i*-strong branching plans corresponding to implicitly coordinated policies in the area of epistemic planning.
- The backbone of such plans are stepping stones.
- Joint execution can be guaranteed to be successful and polynomially bounded if all agents are conservative and optimally eager.
- While bounded plan existence in general is PSPACE-complete, it is polynomial for a fixed number of

Outlook

- → Do the results still hold for planar graphs?
- Is MAPF/DU plan existence also PSPACE-complete?
- How would more general forms of describing the common knowledge about destinations affect the results?
- → Overlap of destinations or general Boolean combinations
- Can we get similar results for other execution semantics?
- → Concurrent executions of actions
- Can we be more aggressive in expectations about possible destinations?
- → Use forward induction, i.e., assume that actions in the past were rational.
- Are other forms of implicit coordination possible?
- → More communication? Coordination in competitive scenarios?

Literature

B. Nebel, T. Bolander, T. Engesser and R. Mattmüller.
Implicitly Coordinated Multi-Agent Path Finding under Destination Uncertainty.
Journal of Artificial Intelligence research 64: 497-527, 2019.

T. Bolander, T. Engesser, R. Mattmüller and B. Nebel.
Better Eager Than Lazy? How Agent Types Impact the Successfulness of Implicit Coordination.
In *Proceedings of the Sixteenth Conference on Principles of Knowledge Representation and Reasoning (KR-18)*, pages 445-453. 2018.

T. Engesser, T. Bolander, R. Mattmüller, and B. Nebel.
Cooperative epistemic multi-agent planning for implicit coordination.
In *Proceedings of the Ninth Workshop on Methods for Modalities (M4MICLA-17)*, pages 75–90, 2017.