Multi-Agent Systems

Dynamic Epistemic Logic

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel, Rolf Bergdoll, and Thorsten Engesser

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Action Models

So far: Only public announcements.

Now: How to model other ways of knowledge changes, such as private announcements, sensing, or ontic (world-changing) actions that affect knowledge along the way?

Idea: Action models similar to Kripke models.

Action Models

Example

Agents *a* and *b* both don't know the value of proposition *p*. This is common knowledge among them. In fact, *p* is true. Then agent *a* receives a letter containing the value of *p* and reads it. Agent *b* observes *a* reading the letter and knows that it is about *p*, but *b* does not learn the value of *p*.

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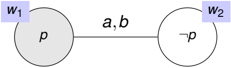
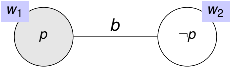
Model Before:



Model After:



Action Models

Question: How to get from Before to After?

Answer: Action models.

Remark: After, $w_1' \models$
$K_a p \wedge (\neg K_b p \wedge \neg K_b \neg p) \wedge K_b(K_a p \vee K_a \neg p) \wedge K_a(\neg K_b p \wedge \neg K_b \neg p)$
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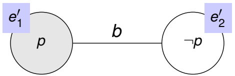
$\leadsto$ action model needs to achieve exactly that!

Action model Read:



With this action model, After = Before $\otimes$ Read, for an appropriate definition of $\otimes$.

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Product update, informally

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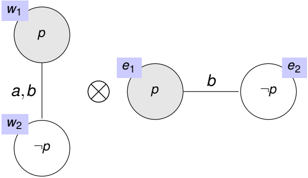

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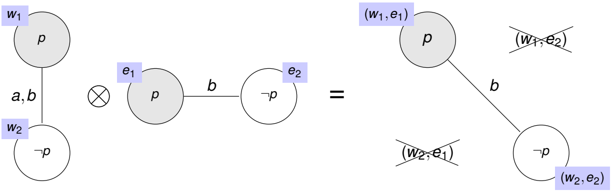

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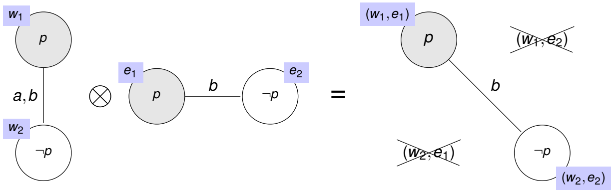


- $(w_1, e_1) \sim_b (w_2, e_2)$ because $w_1 \sim_b w_2$ and $e_1 \sim_b e_2$.
- (w_1, e_2) and (w_2, e_1) were eliminated because e_2 cannot be applied in w_1 and e_1 cannot be applied in w_2.

Action Models

Action model

Let $\mathscr{L}$ be any logical language for a set of agents $\mathscr{I}$ and a set of atoms P. Then an $S5$ action model A is a structure $(E, \sim, pre)$ such that:

- E is the domain of events,
- $\sim_a$ is an equivalence relation on E for all $a \in \mathscr{I}$, the indistinguishability relation for agent a, and
- $pre : E \to \mathscr{L}$ is the precondition function that assigns a precondition $pre(e) \in \mathscr{L}$ to all $e \in E$.

A pointed action model is such a structure (A, e) with $e \in E$.

Action Models

Example (Action model Read, formally)

Read is the action model $(\{e_1, e_2\}, \sim, pre)$ with

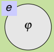
$$\sim_a = \{(e_1, e_1), (e_2, e_2)\} \qquad pre(e_1) = p$$
$$\sim_b = \{(e_1, e_1), (e_1, e_2), (e_2, e_1), (e_2, e_2)\} \qquad pre(e_2) = \neg p.$$

(and with pointed event e_1).

Remark: Public announcements are a special case of action models.

Example (Public announcements)

Action model for the public announcement of φ:



Action Models

Fix agents $\mathscr{I}$ and atomic propositions P.

Example (Skip)

Action skip (or **1**) is the pointed action model $((\{e\}, \sim, pre), e)$ with $pre(e) = \top$ and $\sim_a = \{(e,e)\}$ for all $a \in \mathscr{I}$.

Example (Crash)

Action crash (or **0**) is the pointed action model $((\{e\}, \sim, pre), e)$ with $pre(e) = \bot$ and $\sim_a = \{(e,e)\}$ for all $a \in \mathscr{I}$.

Language

Let P be a countable set of atomic propositions and $\mathscr{I}$ a finite set of agent symbols. Then the language of action model logic is the union of the formulas φ and the actions α defined by the following BNF:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid C_B\varphi \mid [\alpha]\varphi$$
$$\alpha ::= (A, e) \mid \alpha \cup \alpha$$

where $p \in P$, $a \in \mathscr{I}$, $B \subseteq \mathscr{I}$, and (A, e) is a pointed action model with a finite domain E, and

- for all events $e' \in E$, the precondition $pre(e')$ is a formula that has already been constructed in a previous step of the induction.

Action Models

Intuition:

- $[\alpha]\varphi$: After (every) application of action α, φ is true.

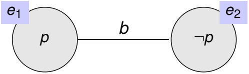
Abbreviations:

- $\langle\alpha\rangle\varphi := \neg[\alpha]\neg\varphi$
  After (some) application of action α, φ is true.
- $A := \bigcup_{e \in E}(A, e)$

Action Models

Deterministic vs. nondeterministic actions:

- $\alpha = (A, e)$: Deterministic action α with unique pointed event e. Example: $\alpha = (\text{Read}, e_1)$.
- $\alpha = \alpha_1 \cup \alpha_2$: Nondeterministic choice, i. e., either α_1 or α_2 happens. Example: $\alpha = (\text{Read}, e_1) \cup (\text{Read}, e_2) = \text{Read}$.
  - Remark 1a: $\alpha = \text{Read}$ not properly nondeterministic, since preconditions of e_1 and e_2 are mutually exclusive.
  - Remark 1b: We will see a properly nondeterministic action later (action Mayread).
  - Remark 2: If, for $\alpha = (A_1, e_1) \cup (A_2, e_2)$, we have $A_1 = A_2$, then we can depict α as a multi-pointed model, like $(\text{Read}, e_1) \cup (\text{Read}, e_2)$:



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Product update

Let $M = (W, \sim, V)$ be an epistemic (i.e., S5) model and let $A = (E, \sim, pre)$ be an action model. Then the product update $M \otimes A$ is the epistemic model $M' = (W', \sim', V')$ with:

- $W' = \{(w,e) \in W \times E \mid M, w \models pre(e)\}$,
- $(w,e) \sim'_a (t,\varepsilon)$ iff $w \sim_a t$ and $e \sim_a \varepsilon$, for $a \in \mathscr{I}$, and
- $(w,e) \in V'_p$ iff $w \in V_p$.

Example

$(\text{Before}, w_1) \otimes (\text{Read}, e_1) = (\text{After}, (w_1, e_1))$

Action Models

Semantics of formulas and actions

Let (M, w) be an epistemic state, φ be a formula and α an action model.

$$M, w \models p,\ \neg\varphi,\ \varphi \wedge \psi,\ K_a\varphi,\ C_B\varphi \ \text{ as usual}$$

$$M, w \models [\alpha]\varphi \quad \text{iff} \quad \text{for all } (M', w') : (M, w)[\![\alpha]\!](M', w') \text{ implies } (M', w') \models \varphi$$

where

- $(M, w)[\![(A, e)]\!](M', w')$ iff
  $(M, w) \models pre(e)$ and $(M', w') = (M \otimes A, (w, e))$, and
- $[\![\alpha \cup \alpha']\!] = [\![\alpha]\!] \cup [\![\alpha']\!]$.

Remarks:

- For $\alpha = (A, e)$, $[\![\alpha]\!]$ is functional, i. e., for each (M, w), there is at most one (M', w') with $(M, w)[\![(A, e)]\!](M', w')$.
- For $\alpha = \alpha_1 \cup \alpha_2$, this is no longer necessarily the case. Careful with duality between $[\alpha]$ and $\langle\alpha\rangle$, then.

Special case $\alpha = (A, e)$: Then $M, w \models [\alpha]\varphi$ iff $M, w \models pre(e)$ implies $(M \otimes A, (w, e)) \models \varphi$.

Dual $\langle\alpha\rangle$, for $\alpha = (A, e)$:

$$M, w \models \langle\alpha\rangle\varphi \quad \text{iff}$$

$$M, w \not\models [\alpha]\neg\varphi \quad \text{iff}$$

$$M, w \models pre(e) \;\text{ does not imply }\; (M \otimes A, (w, e)) \models \neg\varphi \quad \text{iff}$$

$$M, w \models pre(e) \;\text{ and }\; (M \otimes A, (w, e)) \not\models \neg\varphi \quad \text{iff}$$

$$M, w \models pre(e) \;\text{ and }\; (M \otimes A, (w, e)) \models \varphi$$

Action Models

Remark: This is very similar to the semantics of $[\varphi]\psi$ and $\langle\varphi\rangle\psi$ in public announcement logic.

For completeness, dual $\langle\alpha\rangle$, for general α:

$M, w \models \langle\alpha\rangle\varphi$ iff

$M, w \not\models [\alpha]\neg\varphi$ iff

not f. a. $(M', w') : (M, w)[\![\alpha]\!](M', w')$ implies $(M', w') \models \neg\varphi$ iff

there ex. $(M', w') : (M, w)[\![\alpha]\!](M', w')$ and $(M', w') \not\models \neg\varphi$ iff

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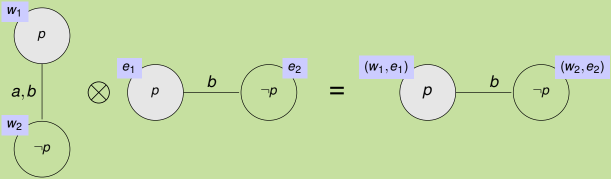
Action Models

Example

Model $(\text{Before}, w_1) \otimes (\text{Read}, e_1)$:



Then:

- $\text{Before}, w_1 \models [\text{Read}, e_1] K_a p$
- $\text{Before}, w_1 \models [\text{Read}, e_1] \neg K_b K_a p$
- $\text{Before}, w_1 \models [\text{Read}, e_1] C_{ab}(K_a p \vee K_a \neg p)$

Example

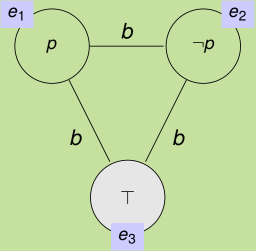
Now, a may only read the letter, but does not have to. Agent b does not know whether a will read it or not. Actually, a does not read the letter.

From b's perspective, there are three possibilities:

- a reads the letter and learns that p is true.
- a reads the letter and learns that p is false.
- a does not read the letter and learns nothing about p.

Example (ctd.)

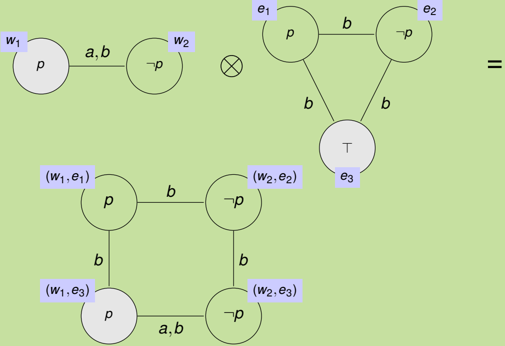
Action model (Mayread, e_3):



$$\text{Mayread} = (\text{Mayread}, e_1) \cup (\text{Mayread}, e_2) \cup (\text{Mayread}, e_3)$$

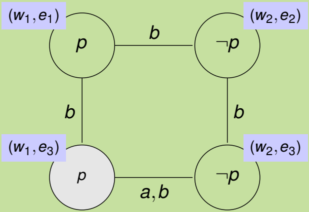
Example (ctd.)

Model $(\text{Before}, w_1) \otimes (\text{Mayread}, e_3)$:



Example (ctd.)

Model $(\text{Before}, w_1) \otimes (\text{Mayread}, e_3)$:



- $\text{Before}, w_1 \models [\text{Mayread}, e_3]\neg(K_a p \vee K_a \neg p) \wedge \hat{K}_b(K_a p \vee K_a \neg p)$
- $\text{Before} \models p \rightarrow$
  $(\langle\text{Mayread}\rangle K_a p \wedge \langle\text{Mayread}\rangle \neg K_a p \wedge \neg\langle\text{Mayread}\rangle K_a \neg p)$

Summary

- Action models allow more epistemic change than just public announcements.
- Action models similar to Kripke structures. State update by product update operator.
- Axiomatization similar to public announcement logic. Actions and (common) knowledge slightly trickier.

Literature

L. S. Moss, Dynamic Epistemic Logic, Chapter 6, In H. van Dithmarschen, J. Y. Halpern, W. van der Hoek, B. Kooi (eds.) **Handbook of Epistemic Logic**, College Publications, 2015.

Hans P. van Ditmarsch and Wiebe van der Hoek and Barteld Kooi, **Dynamic Epistemic Logic**, Springer, 2007.