

Multi-Agent Systems

B. Nebel, R. Bergdoll, T. Engesser
Winter Semester 2019/20

University of Freiburg
Department of Computer Science

Exercise Sheet 14

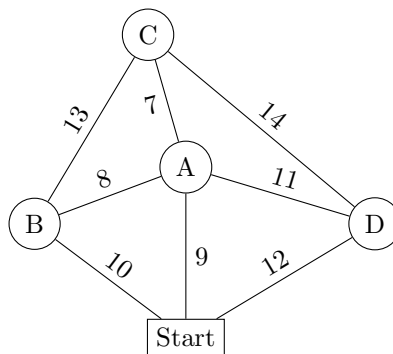
Due: February 14, 2020, 14:00

Exercise 14.1 (The Shapley Value, 3+2)

- Prove that the Shapley value satisfies the axioms *Symmetry*, *Dummy player*, and *Additivity*.
- Prove that the Shapley value is *feasible* and *efficient*.

Exercise 14.2 (Taxi Cost Sharing, 2+2+2+2+2)

Consider a scenario for sharing taxis and taxi fares that is slightly more complex than the one presented in the lecture. There are four agents who each want to be transported from a common starting point (a taxi stand) to their individual destinations (A , B , C , and D). The costs a taxi charges for a route can be calculated by adding up the costs for the single route segments, which are given as edge labels in the following graph:



E.g., the route Start-A-C has a cost of $9 + 7 = 16$. Since taxis take always the shortest and cheapest route, the taxi fare for a coalition is always the cost of the cheapest route starting from “Start” and containing all the destinations of the agents in the coalition.

- Model the problem as cooperative game (N, v) .
- Calculate the Shapley value $\Psi(N, v)$.
- Find the socially optimal coalition structure CS^* for (N, v) .
- Calculate the Shapley value $\Psi(N', v)$ for each coalition $N' \in CS^*$.
- Which of the Shapley values $\Psi(X, v)$ calculated in (b) and (d) are in the core of their respective games (X, v) ? Prove your answer!