Multi-Agent Systems

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Exercise Sheet 4

Due: November 22, 2019

Exercise 4.1 (Public Announcement Logic, 3+3+3+3)

We want to model and solve a simple logics puzzle using Public Announcement Logic (PAL).[1] Consider the following situation with two agents. Both have a number written on their forehead, which can only be seen by the other agent. It is also common knowledge between the agents that both numbers have to be consecutive integers between 0 and 9. Furthermore, let us assume that Agent 1 sees the number 4 on Agent 2's forehead. Now consider the following sequence of truthful announcements:

Agent 1: *"I don't know my number!"*
Agent 2: *"I don't know my number!"*
Agent 1: *"I don't know my number!"*
Agent 2: *"I don't know my number!"*
Agent 1: *"I know my number!"*

(a) Model the initial situation (prior to the announcements) as epistemic model M. Identify the worlds that Agent 1 considers possible, given the problem description (but not the dialogue).

(b) Model the announcements from the dialogue as epistemic formulas $\alpha_1, \ldots, \alpha_5$.

(c) Compute all the models M_{α_1}, $M_{\alpha_1\alpha_2}$, $\ldots$, $M_{\alpha_1\ldots\alpha_5}$.

(d) What is Agent 1's number? Will Agent 2 know his number in the end? Justify your answers!

Exercise 4.2 (Public Announcement Logic II, Bonus: 2+2+2)

Consider the following scenario:

> Three logicians walk into a bar. The bartender asks the group: "Do **all of you** want a beer?" Without consulting each other, the logicians reply with the following sequence of (public) announcements:
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> Logician 1: "I don't know."
> Logician 2: "I don't know."
> Logician 3: "Yes!"

(a) Model the initial situation (prior to the announcements) as epistemic model M. Assume that initially it is commonly known that each agent knows whether he himself wants a beer or not, but not whether the other agents do.

(b) Model the announcements from the dialogue as epistemic formulas $\alpha_1, \alpha_2, \alpha_3$. Note that when an agent says "I don't know", he means "I don't know whether **all of us** want a beer".

(c) Compute all the updated models M_{α_1}, $M_{\alpha_1\alpha_2}$, and $M_{\alpha_1\alpha_2\alpha_3}$ and show that the announcements have been in fact truthful.

[1] Different versions of this puzzle and their origins are discussed in the book *One Hundred Prisoners and a Light Bulb* by Hans van Ditmarsch and Barteld Kooi, Springer, 2015.