Exercise Sheet 2
Due: November 8, 2019

Exercise 2.1 (Model Checking, 3+3)
Consider the following Kripke model which contains two different kinds of accessibility relations. The equivalence relations 1 and 2 can be interpreted as epistemic indistinguishability relations for the knowledge of two different agents. The relation $a$ can be interpreted as a temporal successor relation specifying the transitions resulting from the execution of an action $a$.

![Kripke Model Diagram]

(a) Check whether or not the following is true. Remember that $K_i \varphi$ is a notation for $[i] \varphi$ and $\hat{K}_i \varphi$ is a notation for $\langle i \rangle \varphi$ (and equivalent to $\neg K_1 \neg \varphi$). Write down all intermediate steps.

$$M, w_1 \models K_1 (\neg l \land \hat{K}_2 l) \land \langle a \rangle (g \land K_1 l \land K_2 l)$$

(b) Assume that proposition $g$ stands for “the garage door is open” and proposition $l$ stands for “the light in the garage is on”. Which story does the model tell us?

Exercise 2.2 (S5: Axioms and Frame Properties, 6)
A Kripke frame $F = \langle S, R \rangle$ is defined exactly like a Kripke model $\langle S, R, V \rangle$, but without the valuation $V$. The set of all models over $\langle S, R \rangle$ is the set of all models $\langle S, R, V \rangle$ where $V$ is any propositional valuation. A formula is valid in a frame $F$, if it is valid in all models over $F$. It is valid in a class of frames, if it is valid in each frame in that class. We say that an axiom defines a class of frames if the axiom is valid exactly in this class of frames. Show that

(a) the axiom $T$ defines the class of reflexive frames,

(b) the axiom $4$ defines the class of transitive frames,

(c) the axiom $5$ defines the class of Euclidean frames.