Multi-Agent Systems

B. Nebel, T. Engesser, R. Bergdoll
Winter Semester 2019/20

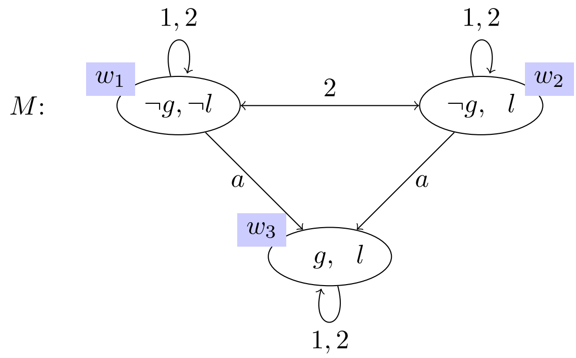
University of Freiburg
Department of Computer Science

Exercise Sheet 2

Due: November 8, 2019

Exercise 2.1 (Model Checking, 3+3)

Consider the following Kripke model which contains two different kinds of accessibility relations. The equivalence relations 1 and 2 can be interpreted as *epistemic* indistinguishability relations for the knowledge of two different agents. The relation a can be interpreted as a *temporal* successor relation specifying the transitions resulting from the execution of an action a.



(a) Check whether or not the following is true. Remember that $K_i\varphi$ is a notation for $[i]\varphi$ and $\hat{K}_i\varphi$ is a notation for $\langle i\rangle\varphi$ (and equivalent to $\neg K_i\neg\varphi$). Write down all intermediate steps.

$$M, w_1 \models K_1(\neg l \wedge \hat{K}_2 l) \wedge [a](g \wedge K_1 l \wedge K_2 l)$$

(b) Assume that proposition g stands for "the garage door is open" and proposition l stands for "the light in the garage is on". Which story does the model tell us?

Exercise 2.2 (**S5**: Axioms and Frame Properties, 6)

A Kripke frame $\mathcal{F} = \langle S, R\rangle$ is defined exactly like a Kripke model $\langle S, R, V\rangle$, but without the valuation V. The set of all models over $\langle S, R\rangle$ is the set of all models $\langle S, R, V\rangle$ where V is any propositional valuation. A formula is valid in a frame $\mathcal{F}$, if it is valid in all models over $\mathcal{F}$. It is valid in a class of frames, if it is valid in each frame in that class. We say that an axiom defines a class of frames if the axiom is valid exactly in this class of frames. Show that

(a) the axiom **T** defines the class of *reflexive* frames,

(b) the axiom **4** defines the class of *transitive* frames,

(c) the axiom **5** defines the class of *Euclidean* frames.