## **Principles of AI Planning**

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## Exercise Sheet 13 Due: Friday, February 7th, 2020

Send your solution to mario.kantz@gmail.com (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of two or three students. Please indicate all names on your solution.

## Exercise 13.1 (EVMDDs, 2+2 points)

An EVMDD is called *reduced* if it contains no two isomorphic subgraphs and if it contains no nodes where all outgoing edges lead to the same successor node and carry the same weight. It is *canonical* if for each node, the minimal outgoing edge weight is zero.

- (a) Let  $c_1 = xy + y$  for variables x, y with  $\mathcal{D}_x = \mathcal{D}_y = \{0, 1\}$ . Draw the canonical reduced ordered EVMDDs for  $c_1$  for both possible variable orders (x, y and y, x). Compare their sizes.
- (b) Let  $c_2 = x \cdot (2 + y + z) u^2 + 7$  for variables x, y, z, u with  $\mathcal{D}_x = \mathcal{D}_y = \mathcal{D}_z = \{0, 1\}$  and  $\mathcal{D}_u = \{0, 1, 2\}$ . Draw the canonical reduced ordered EVMDD for  $c_2$  and variable order x, y, z, u.

Exercise 13.2 (EVMDD sizes and variable orders, 2 points)

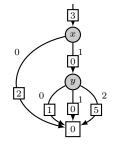
Let  $v_0, \ldots, v_{2n-1}$  be variables with domains  $\mathcal{D}_{v_i} = \{0, \ldots, k-1\}$  for all  $i = 0, \ldots, 2n-1$ , let  $\pi : \{0, \ldots, 2n-1\} \rightarrow \{0, \ldots, 2n-1\}$  be a permutation of the variables, let  $\kappa_j \in \mathbb{N}, j = 0, \ldots, n-1$ , be natural numbers, and let  $c = \sum_{j=0}^{n-1} \kappa_j v_{\pi(2j)} v_{\pi(2j+1)}$  be an arithmetic function over  $v_0, \ldots, v_{2n-1}$ . Intuitively, c is a weighted sum of products of two variables each, such that no variable occurs in more than one product subterm. Show that there exists a variable order for  $v_0, \ldots, v_{2n-1}$  such that there exists an EVMDD with that order that represents the function c and that has a size (number of edges) in the order of  $n \cdot k^2$ .

*Hint:* Consider the example  $c = 2v_0v_3 + 6v_1v_5 + 4v_2v_4$ . How should the variables be ordered to minimize the size of the EVMDD?

**Exercise 13.3** (Evaluating states with EVMDDs, 1 point)

Consider a cost function represented by the EVMDD on the right.

Let s be a state with s(x) = 1 and s(y) = 2. To which value does the EVMDD evaluate for state s?



Exercise 13.4 (EVMDD-based action compilation, 2+1 points)

Consider again the EVMDD from Exercise 13.3. Assume it encodes the cost  $c_{o_1}$  of operator  $o_1 = \langle z = 1 \land u = 1, x := 0 \rangle$ .

(a) Give the EVMDD-based action compilation of  $o_1$  using this EVMDD.

(b) Let  $\Pi = \langle V, I, O, \gamma, (c_o)_{o \in O} \rangle$  with  $V = \{x, y, z, u\}$ ,  $\mathcal{D}_x = \mathcal{D}_z = \mathcal{D}_u = \{0, 1\}$  and  $\mathcal{D}_y = \{0, 1, 2\}$ , initial state I with I(x) = I(y) = I(z) = I(u) = 1, operators  $O = \{o_1, o_2\}$  with  $o_1$  as above and  $o_2 = \langle x = 0, z := 0 \rangle$  with cost function  $c_{o_2} = 1$  and goal formula  $\gamma = (z = 0)$ . Give an optimal plan  $\pi$  for  $\Pi$  and an optimal plan  $\pi'$  for the EVMDD-based action compilation of  $\Pi$  and their respective costs.