# Principles of AI Planning 

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## Exercise Sheet 13

## Due: Friday, February 7th, 2020

Send your solution to mario.kantz@gmail.com (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of two or three students. Please indicate all names on your solution.

Exercise 13.1 (EVMDDs, $2+2$ points)
An EVMDD is called reduced if it contains no two isomorphic subgraphs and if it contains no nodes where all outgoing edges lead to the same successor node and carry the same weight. It is canonical if for each node, the minimal outgoing edge weight is zero.
(a) Let $c_{1}=x y+y$ for variables $x, y$ with $\mathcal{D}_{x}=\mathcal{D}_{y}=\{0,1\}$. Draw the canonical reduced ordered EVMDDs for $c_{1}$ for both possible variable orders ( $x, y$ and $y, x$ ). Compare their sizes.
(b) Let $c_{2}=x \cdot(2+y+z)-u^{2}+7$ for variables $x, y, z, u$ with $\mathcal{D}_{x}=\mathcal{D}_{y}=\mathcal{D}_{z}=\{0,1\}$ and $\mathcal{D}_{u}=\{0,1,2\}$. Draw the canonical reduced ordered EVMDD for $c_{2}$ and variable order $x, y, z, u$.

Exercise 13.2 (EVMDD sizes and variable orders, 2 points)
Let $v_{0}, \ldots, v_{2 n-1}$ be variables with domains $\mathcal{D}_{v_{i}}=\{0, \ldots, k-1\}$ for all $i=0, \ldots, 2 n-1$, let $\pi$ : $\{0, \ldots, 2 n-1\} \rightarrow\{0, \ldots, 2 n-1\}$ be a permutation of the variables, let $\kappa_{j} \in \mathbb{N}, j=0, \ldots, n-1$, be natural numbers, and let $c=\sum_{j=0}^{n-1} \kappa_{j} v_{\pi(2 j)} v_{\pi(2 j+1)}$ be an arithmetic function over $v_{0}, \ldots, v_{2 n-1}$. Intuitively, $c$ is a weighted sum of products of two variables each, such that no variable occurs in more than one product subterm. Show that there exists a variable order for $v_{0}, \ldots, v_{2 n-1}$ such that there exists an EVMDD with that order that represents the function $c$ and that has a size (number of edges) in the order of $n \cdot k^{2}$.

Hint: Consider the example $c=2 v_{0} v_{3}+6 v_{1} v_{5}+4 v_{2} v_{4}$. How should the variables be ordered to minimize the size of the EVMDD?

Exercise 13.3 (Evaluating states with EVMDDs, 1 point)

Consider a cost function represented by the EVMDD on the right.
Let $s$ be a state with $s(x)=1$ and $s(y)=2$. To which value does the EVMDD evaluate for state $s$ ?


Exercise 13.4 (EVMDD-based action compilation, $2+1$ points)
Consider again the EVMDD from Exercise 13.3. Assume it encodes the cost $c_{o_{1}}$ of operator $o_{1}=\langle z=1 \wedge u=1, x:=0\rangle$.
(a) Give the EVMDD-based action compilation of $o_{1}$ using this EVMDD.
(b) Let $\Pi=\left\langle V, I, O, \gamma,\left(c_{o}\right)_{o \in O}\right\rangle$ with $V=\{x, y, z, u\}, \mathcal{D}_{x}=\mathcal{D}_{z}=\mathcal{D}_{u}=\{0,1\}$ and $\mathcal{D}_{y}=$ $\{0,1,2\}$, initial state $I$ with $I(x)=I(y)=I(z)=I(u)=1$, operators $O=\left\{o_{1}, o_{2}\right\}$ with $o_{1}$ as above and $o_{2}=\langle x=0, z:=0\rangle$ with cost function $c_{o_{2}}=1$ and goal formula $\gamma=(z=0)$. Give an optimal plan $\pi$ for $\Pi$ and an optimal plan $\pi^{\prime}$ for the EVMDD-based action compilation of $\Pi$ and their respective costs.

