Principles of AI Planning
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Exercise Sheet 11
Due: Friday, January 24th, 2020

Send your solution to mario.kantz@gmail.com (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of two or three students. Please indicate all names on your solution.

Exercise 11.1 (Strong stubborn sets, 1+3 points)
Consider the SAS\(^+\) planning task II with variables \(V = \{\text{pos}, \text{left}, \text{right}, \text{hat}\}\), \(D_{\text{pos}} = \{\text{home, uni}\}\) and \(D_{\text{left}} = D_{\text{right}} = D_{\text{hat}} = \{t, f\}\). The initial state is \(I = \{\text{pos} \mapsto \text{home}, \text{left} \mapsto f, \text{right} \mapsto f, \text{hat} \mapsto f\}\) and the goal specification is \(\gamma = \{\text{pos} \mapsto \text{uni}\}\). There are four operators in \(O\), namely

\begin{align*}
\text{wear-left-shoe} &= (\text{pos} = \text{home} \land \text{left} = f, \text{left} := t), \\
\text{wear-right-shoe} &= (\text{pos} = \text{home} \land \text{right} = f, \text{right} := t), \\
\text{wear-hat} &= (\text{pos} = \text{home} \land \text{hat} = f, \text{hat} := t), \\
\text{go-to-university} &= (\text{pos} = \text{home} \land \text{left} = t \land \text{right} = t, \text{pos} := \text{uni}).
\end{align*}

(a) Draw the breadth-first search graph (with duplicate detection) for planning task II without any form of partial-order reduction.

(b) Draw the breadth-first search graph (with duplicate detection) for planning task II using strong stubborn set pruning. For each expansion of a node for a state \(s\), specify in detail how \(T_s\) (and thus \(T_{\text{app}(s)}\)) are computed, i.e., explain how the initial disjunctive action landmark is chosen and how operators are iteratively added to \(T_s\) as part of necessary enabling sets or interfering operators, respectively. Break ties in favor of \text{wear-left-shoe} over \text{wear-right-shoe}.

How many node expansion do you save with strong stubborn sets compared to plain breadth-first search? What about the lengths of the resulting solutions?

You may abbreviate the operator names, state descriptions etc. appropriately.

Exercise 11.2 (Weak vs. strong stubborn sets, 6 points)
Show that weak stubborn sets admit exponentially more pruning than strong stubborn sets.

Hint: Consider the family of planning tasks \((\Pi_n)_{n \in \mathbb{N}}, \text{where } \Pi_n = \langle V_n, I_n, O_n, \gamma \rangle\) is the planning task with the following components:

- \(V_n = \{a, x, y, b_1, \ldots, b_n\}\) with variable domains \(D_a = D_x = D_y = \{0, 1\}\) and \(D_{b_i} = \{0, 1, 2\}\) for all \(i \in \{1, \ldots, n\}\)
- \(O_n = \{o, o', o_d, o_{d'}, o_1, \ldots, o_n, \overline{o_1}, \ldots, \overline{o_n}\}\)
- \(\text{pre}(o) = \{a \mapsto 0\}, \text{eff}(o) = \{x \mapsto 1\}\)
- \(\text{pre}(o') = \{a \mapsto 0\}, \text{eff}(o') = \{y \mapsto 1\}\)
- \(\text{pre}(o_d) = \{a \mapsto 0\}, \text{eff}(o_d) = \{a \mapsto 1, b_1 \mapsto 1, \ldots, b_n \mapsto 1\}\)
- \(\text{pre}(o_{d'}) = \{a \mapsto 1\}, \text{eff}(o_{d'}) = \{a \mapsto 0, b_1 \mapsto 1, \ldots, b_n \mapsto 1\}\)
- \(\text{pre}(o_i) = \{b_i \mapsto 1\}, \text{eff}(o_i) = \{b_i \mapsto 2\}\) for \(1 \leq i \leq n\)
- \(\text{pre}(\overline{o_i}) = \{b_i \mapsto 2\}, \text{eff}(\overline{o_i}) = \{b_i \mapsto 1\}\) for \(1 \leq i \leq n\)
- \(I_n = \{a \mapsto 0, x \mapsto 0, y \mapsto 0, b_1 \mapsto 0, \ldots, b_n \mapsto 0\}\)
- \(\gamma = \{x \mapsto 1, y \mapsto 1\}\)