Principles of AI Planning

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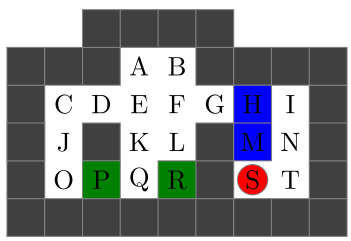
Exercise Sheet 9

Due: Friday, January 10th, 2020

Send your solution to `mario.kantz@gmail.com` (PDF only) or submit a hardcopy before the lecture. The exercise sheets may and should be worked on and handed in in groups of two or three students. Please indicate all names on your solution.

Exercise 9.1 (Additive patterns and canonical heuristic, 2+1+2 points)

Consider the Sokoban problem given by the picture below. The red circle denotes the agent's position, the blue squares are boxes, and the green grid cells are the target positions of the boxes (it is irrelevant which box ends up in which target position). The letters only denote the names of the grid cells.



We will model this problem in finite-domain representation using the variables $\mathit{position}_p$, $\mathit{position}_{s_1}$, $\mathit{position}_{s_2}$, $\mathit{at\text{-}goal}_{s_1}$, $\mathit{at\text{-}goal}_{s_2}$, $\mathit{content}_A$, $\mathit{content}_B, \dots, \mathit{content}_T$ with the following domains:

- $\mathcal{D}_{\mathit{position}_p} = \mathcal{D}_{\mathit{position}_{s_1}} = \mathcal{D}_{\mathit{position}_{s_2}} = \{A, B, \dots, T\}$
- $\mathcal{D}_{\mathit{at\text{-}goal}_{s_1}} = \mathcal{D}_{\mathit{at\text{-}goal}_{s_2}} = \{\text{true}, \text{false}\}$
- $\mathcal{D}_{\mathit{content}_A} = \dots = \mathcal{D}_{\mathit{content}_T} = \{\text{nothing}, p, s_1, s_2\}$

The initial state is given as

- $\mathit{position}_p = S, \mathit{position}_{s_1} = M, \mathit{position}_{s_2} = H, \mathit{at\text{-}goal}_{s_1} = \mathit{at\text{-}goal}_{s_2} = \text{false}$
- $\mathit{content}_H = s_2, \mathit{content}_M = s_1, \mathit{content}_S = p$
- $\mathit{content}_X = \text{nothing for } X \in \{A, \dots, T\} \setminus \{H, M, S\}$

and the goal formula is $\mathit{at\text{-}goal}_{s_1} = \text{true} \wedge \mathit{at\text{-}goal}_{s_2} = \text{true}$. The set of available operators contains the obvious FDR formalizations of all *move*- and *push*-actions that are usually available in Sokoban.

Consider the pattern collection $\mathcal{C}$ with the following patterns:

$$
\begin{aligned}
P_1 &= \{\mathit{at\text{-}goal}_{s_2}\}\\
P_2 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{position}_{s_1}\}\\
P_3 &= \{\mathit{at\text{-}goal}_{s_2}, \mathit{position}_{s_2}\}\\
P_4 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{position}_{s_1}, \mathit{position}_p\}\\
P_5 &= \{\mathit{position}_{s_1}, \mathit{position}_p\}\\
P_6 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_H\}\\
P_7 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_G\}\\
P_8 &= \{\mathit{at\text{-}goal}_{s_2}, \mathit{content}_D\}\\
P_9 &= \{\mathit{content}_A, \mathit{content}_E\}\\
P_{10} &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_Q\}
\end{aligned}
$$

(a) Specify the compatibility graph of $\mathcal{C}$ and determine its maximal cliques.

(b) Determine the canonical heuristic $h^{\mathcal{C}}$ and simplify it as much as possible.

(c) Not all patterns in $\mathcal{C}$ are reasonable. Which can obviously be omitted, and why? What would the canonical heuristic look like if we omitted those patterns before even constructing the compatibility graph?

Exercise 9.2 (Orthogonality and pairwise orthogonality, 5 points)

Recall: We call abstractions mappings $\alpha_1, \ldots, \alpha_n$ over the same transition system $\mathcal{T}$ *orthogonal* if for all transitions $\langle s, \ell, t \rangle$ of $\mathcal{T}$, we have $\alpha_i(s) \neq \alpha_i(t)$ for at most one $i \in \{1, \ldots, n\}$. Moreover, we say that $\alpha_1, \ldots, \alpha_n$ are *pairwise orthogonal* if for all $j, k \in \{1, \ldots, n\}$ with $j \neq k$, mappings α_j and α_k are orthogonal.

Prove the following: $\alpha_1, \ldots, \alpha_n$ are orthogonal if and only if they are pairwise orthogonal.