Principles of AI Planning

18. Strong nondeterministic planning

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel and Robert Mattmüller

February 12th, 2020

Strong planning

In this chapter, we will consider the simplest case of nondeterministic planning by restricting attention to strong plans.

Concepts

Strong plans

Recall the definition of strong plans:

Definition (strong plan)

Let S be the set of states of a planning task Π. Then a strong plan for Π is a function $\pi : S_\pi \to O$ for some subset $S_\pi \subseteq S$ such that

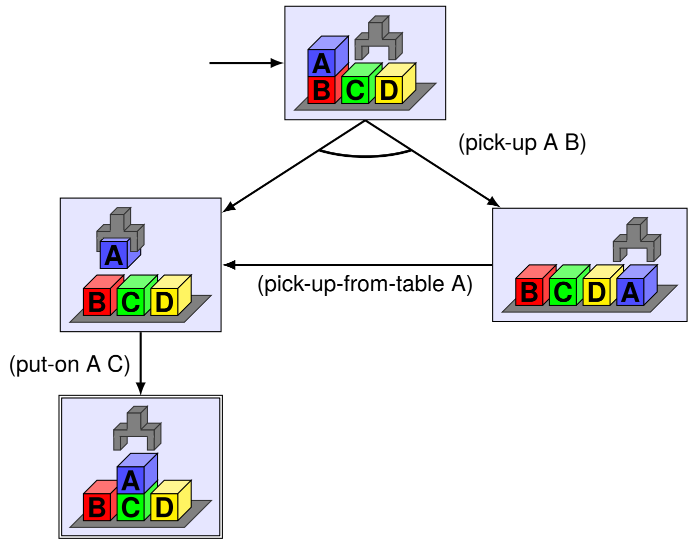
- $\pi(s)$ is applicable in s for all $s \in S_\pi$,
- $S_\pi(s') \cap S_\star \neq \emptyset$ for all $s' \in S_\pi(s_0)$ (π is proper), and
- there is no state $s' \in S_\pi(s_0)$ such that s' is reachable from s' following π in a strictly positive number of steps (π is acyclic).

Strong plans

Execution of a strong plan

1. Determine the current state s.
2. If s is a goal state then terminate.
3. Execute action $\pi(s)$.
4. Repeat from first step.

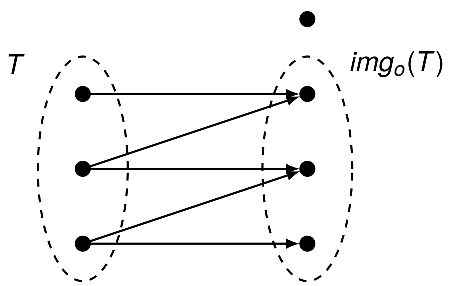
Strong plans



Images

Image

The image of a set T of states with respect to an operator o is the set of those states that can be reached by executing o in a state in T.



Images

Definition (image of a state)

$img_o(s) = \{s' \in S | s \xrightarrow{o} s'\} = app_o(s)$

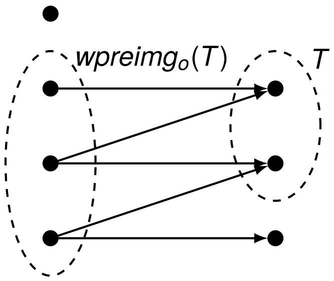
Definition (image of a set of states)

$img_o(T) = \bigcup_{s \in T} img_o(s)$

Weak preimages

Weak preimage

The weak preimage of a set T of states with respect to an operator o is the set of those states from which a state in T can be reached by executing o.



Concepts
Strong plans
Images
Weak preimages
Strong preimages
Algorithms
Summary

Weak preimages

Definition (weak preimage of a state)

$wpreimg_o(s') = \{s \in S | s \xrightarrow{o} s'\}$

Definition (weak preimage of a set of states)

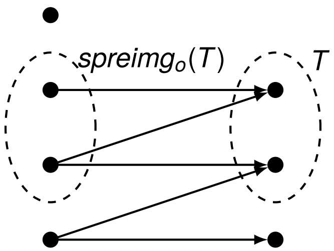
$wpreimg_o(T) = \bigcup_{s \in T} wpreimg_o(s)$.

Concepts
Strong plans
Images
Weak preimages
Strong preimages
Algorithms
Summary

Strong preimages

Strong preimage

The strong preimage of a set T of states with respect to an operator o is the set of those states from which a state in T is always reached when executing o.



Concepts
Strong plans
Images
Weak preimages
Strong preimages
Algorithms
Summary

Strong preimages

Definition (strong preimage of a set of states)

$spreimg_o(T) = \{s \in S \mid \exists s' \in T : s \xrightarrow{o} s' \wedge img_o(s) \subseteq T\}$

Concepts
Strong plans
Images
Weak preimages
Strong preimages
Algorithms
Summary

Algorithms

Algorithms for strong planning

1. Dynamic programming (backward)
   Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.
   1. Zero actions needed for goal states.
   2. If states with i actions to goals are known, states with $\leq i+1$ actions to goals can be easily identified.

   Automatic reuse of plan suffixes already found.
2. Heuristic search (forward)
   Strong planning can be viewed as AND/OR graph search.

   OR nodes: Choice between operators
   AND nodes: Choice between effects

   Heuristic AND/OR search algorithms:
   AO*, Proof Number Search, ...

Dynamic programming

Planning by dynamic programming

If for all successors of state s with respect to operator o a plan exists, assign operator o to s.

- Base case $i = 0$: In goal states there is nothing to do.
- Inductive case $i \geq 1$: If $\pi(s)$ is still undefined and there is $o \in O$ such that for all $s' \in img_o(s)$, the state s' is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

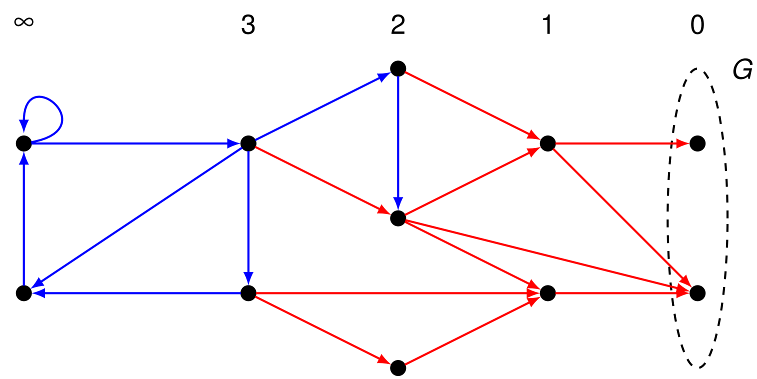
Backward distances

If s is assigned a value on iteration $i \geq 1$, then the backward distance of s is i. The dynamic programming algorithm essentially computes the backward distances of states.

Backward distances

Example

distance to G



Backward distances

Definition (backward distance sets)

Let G be a set of states and O a set of operators.
The backward distance sets D_i^{bwd} for G and O consist of those states for which there is a guarantee of reaching a state in G with at most i operator applications using operators in O:

$$D_0^{bwd} := G$$
$$D_i^{bwd} := D_{i-1}^{bwd} \cup \bigcup_{o \in O} spreimg_o(D_{i-1}^{bwd}) \text{ for all } i \geq 1$$

Backward distances

Definition (backward distance)

Let G be a set of states and O a set of operators, and let $D_0^{bwd}, D_1^{bwd}, \ldots$ be the backward distance sets for G and O.
Then the backward distance of a state s for G and O is

$$\delta_G^{bwd}(s) = \min\{i \in \mathbb{N} \mid s \in D_i^{bwd}\}$$

(where $\min \emptyset = \infty$).

Strong plans based on distances

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states $S_\star$.

Extraction of a strong plan from distance sets

1. Let $S' \subseteq S$ be those states having a finite backward distance for $G = S_\star$ and O.
2. Let $s \in S'$ be a state with distance $i = \delta_G^{bwd}(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $img_o(s) \subseteq D_{i-1}^{bwd}$. Hence o decreases the backward distance by at least one.

Then π is a strong plan for $\mathscr{T}$ iff $I \in S'$.

Question: What is the worst-case runtime of the algorithm?
Question: What is the best-case runtime of the algorithm if most states have a finite backward distance?

Making the algorithm a logic-based algorithm

- An algorithm that represents the states explicitly stops being feasible at about 10^8 or 10^9 states.
- For planning with bigger transition systems structural properties of the transition system have to be taken advantage of.
- As before, representing state sets as propositional formulae (or BDDs) often allows taking advantage of the structural properties: a formula (or BDD) that represents a set of states or a transition relation that has certain regularities may be very small in comparison to the set or relation.
- In the following, we will present an algorithm using a boolean-formula representation (without going into the details of how to implement it using BDDs).

Making the algorithm a logic-based algorithm

Remark: The following algorithm assumes a propositional representation of the state space as opposed to a finite-domain representation. We have already seen how to translate an FDR encoding into a propositional encoding in Chapter 9 (cf. definition of the "induced propositional planning task").

Therefore, for the rest of the present section, we will assume without loss of generality that all $v \in V$ are propositional variables with domain $\mathscr{D}_v = \{0,1\}$.

Breadth-first search with progression and state sets (deterministic case)

Progression breadth-first search

def bfs-progression(V, I, O, γ):
    goal := *formula-to-set*(γ)
    reached := $\{I\}$
    loop:
        if $\mathit{reached} \cap \mathit{goal} \neq \emptyset$:
            return solution found
        $\mathit{new\text{-}reached} := \mathit{reached} \cup \bigcup_{o \in O} \mathit{img}_o(\mathit{reached})$
        if *new-reached* = *reached*:
            return no solution exists
        reached := *new-reached*

$\rightsquigarrow$ This can easily be transformed into a regression algorithm.

Breadth-first search with regression and state sets (deterministic case)

Regression breadth-first search

def bfs-regression(V, I, O, γ):
    init := I
    reached := *formula-to-set*(γ)
    loop:
        if $\mathit{init} \in \mathit{reached}$:
            return solution found
        $\mathit{new\text{-}reached} := \mathit{reached} \cup \bigcup_{o \in O} \mathit{wpreimg}_o(\mathit{reached})$
        if *new-reached* = *reached*:
            return no solution exists
        reached := *new-reached*

- This algorithm is very similar to the dynamic programming algorithm for the nondeterministic case!

Breadth-first search with regression and state sets (strong nondeterministic case)

Regression breadth-first search

def bfs-regression(V, I, O, γ):
    init := I
    reached := *formula-to-set*(γ)
    loop:
        if $\mathit{init} \in \mathit{reached}$:
            return solution found
        $\mathit{new\text{-}reached} := \mathit{reached} \cup \bigcup_{o \in O} \mathit{spreimg}_o(\mathit{reached})$
        if *new-reached* = *reached*:
            return no solution exists
        reached := *new-reached*

Remark: Do you recognize the assignments $D_0^{bwd} := G$ and $D_i^{bwd} := D_{i-1}^{bwd} \cup \bigcup_{o \in O} \mathit{spreimg}_o(D_{i-1}^{bwd})$ for $i \geq 1$?

Breadth-first search with regression and state sets (strong nondeterministic case, symbolic)

Regression breadth-first search

def bfs-regression(V, I, O, γ):
    init := I
    reached := γ
    loop:
        if *init* $\models$ *reached*:
            return solution found
        new-reached := *reached* $\vee$
                $\bigvee_{o \in O}$ *spreimgsymb*$_o$(*reached*)
        if *new-reached* $\equiv$ *reached*:
            return no solution exists
        reached := *new-reached*

- How do we define *spreimgsymb* with logic (or BDD) operations?

Symbolic strong preimage computation

Let φ be a logic formula and $[\![\varphi]\!] = \{s \in S \mid s \models \varphi\}$.

We want: a symbolic preimage operation *spreimgsymb* such that if $\psi = \mathit{spreimgsymb}_o(\varphi)$, then $[\![\psi]\!] = \{s \in S \mid s \models \psi\} = \mathit{spreimg}_o([\![\varphi]\!])$.

In other words, we want the following diagram to commute:

$$
\begin{tikzcd}
\psi \arrow[d, "{[\![\cdot]\!]}"'] & \varphi \arrow[l, "\mathit{spreimgsymb}_o(\cdot)"'] \arrow[d, "{[\![\cdot]\!]}"] \\
{[\![\psi]\!]} & {[\![\varphi]\!]} \arrow[l, "\mathit{spreimg}_o(\cdot)"']
\end{tikzcd}
$$

Transition formula for nondeterministic operators

Let V be the set of state variables and $V' := \{v' \mid v \in V\}$ a set of primed copies of the variables in V. Intuition:

- Variables in V describe the current state s.
- Variables in V' describe the next state s'.

We would like to define a formula $\tau_V(o)$ that describes the transitions labeled with o between states s (over V) and s' (over V') in terms of V and V'.

Transition formula for nondeterministic operators

The formula $\tau_V(o)$ must express

- the conditions for applicability of o,
- how o changes state variables, and
- which state variables o does not change.

A significant difficulty lies in the third requirement because different variables may be affected depending on nondeterministic choices.

Transition formula for nondeterministic operators

$\tau_V(o)$ for deterministic operators $o = \langle \chi, e\rangle$

$$\tau_V(o) = \chi \wedge \bigwedge_{v\in V} ((EPC_v(e) \vee (v \wedge \neg EPC_{\neg v}(e))) \leftrightarrow v') \wedge \bigwedge_{v\in V} \neg(EPC_v(e) \wedge EPC_{\neg v}(e))$$

Assume that $e = \bigwedge_{a\in A} a \wedge \bigwedge_{d\in D} \neg d$ for $A = \{a_1,\dots,a_k\}$ and $D = \{d_1,\dots,d_l\}$ with $A \cap D = \emptyset$. Then this becomes simpler.

$\tau_V(o)$ for STRIPS operators $o = \langle \chi, \bigwedge_{a\in A} a \wedge \bigwedge_{d\in D} \neg d\rangle$

$$\tau_V(o) = \chi \wedge \bigwedge_{a\in A} a' \wedge \bigwedge_{d\in D} \neg d' \wedge \bigwedge_{v\in V\setminus(A\cup D)} (v \leftrightarrow v')$$

Transition formula for nondeterministic operators

For nondeterministic operators $o = \langle \chi, \{e_1,\dots,e_n\}\rangle$ with corresponding add and delete lists A_i and D_i of e_i such that $A_i \cap D_i = \emptyset$, $i = 1,\dots,n$, we get:

$\tau_V(o)$ for nondeterministic operators $o = \langle \chi, \{e_1,\dots,e_n\}\rangle$

$$\tau_V(o) = \chi \wedge \bigvee_{i=1}^{n} \left(\bigwedge_{a\in A_i} a' \wedge \bigwedge_{d\in D_i} \neg d' \wedge \bigwedge_{v\in V\setminus(A_i\cup D_i)} (v \leftrightarrow v') \right)$$

Example

Let $V = \{a,b\}$, $V' = \{a',b'\}$, and $o = \langle \neg a, \{a, a \wedge \neg b\}\rangle$. Then

$$\tau_V(o) = \neg a \wedge \Big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \Big).$$

Computing strong preimages

Definition (substitution)

Let $\varphi, t_1,\dots,t_n$ be propositional formulas and $v_1,\dots,v_n$ atomic propositions.

We denote the formula obtained from φ by simultaneous replacement of all variables v_i by the corresponding formulas t_i, $i = 1,\dots,n$, by $\varphi[t_1,\dots,t_n/v_1,\dots,v_n]$.

Computing strong preimages

Definition (existential abstraction)

Let φ be a propositional formula and $v_1,\dots,v_n$ be atomic propositions. Then the existential abstraction of φ wrt. $v_1,\dots,v_n$ is recursively defined as follows:

$$\exists v.\varphi := \varphi[\top/v] \vee \varphi[\bot/v]$$

For a set of variables $V = \{v_1,\dots,v_n\}$ we use the abbreviation

$$\exists V.\varphi := \exists v_1 \dots \exists v_n.\varphi.$$

Note: Even with intermediate formula simplifications this can lead to an exponential blowup. BDDs can be useful here.

Computing strong preimages

Strong preimages

$$
\begin{aligned}
spreimg_o(T) &= \{s \in S \mid \exists s' \in T : s \xrightarrow{o} s' \wedge img_o(s) \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad \{s' \in S \mid s \xrightarrow{o} s'\} \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad (\forall s' \in S : s \xrightarrow{o} s' \Rightarrow s' \in T)\} \\
&= \{s \in S \mid (\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&\qquad (\neg \exists s' \in S : s \xrightarrow{o} s' \wedge \neg(s' \in T))\}
\end{aligned}
$$

Computing strong preimages with boolean function operations

$$
\begin{aligned}
spreimg_o(T) = \{s \in S \mid (&\exists s' \in S : s \xrightarrow{o} s' \wedge s' \in T) \wedge \\
&(\neg \exists s' \in S : s \xrightarrow{o} s' \wedge \neg(s' \in T))\}
\end{aligned}
$$

Strong preimages with boolean functions

For formula φ characterizing set T of strongly backward-reached states:

$$
\begin{aligned}
spreimgsymb_o(\varphi) = &\left(\exists V'.(\tau_V(o) \wedge \varphi[v'_1, \dots, v'_n / v_1, \dots, v_n])\right) \wedge \\
&\left(\neg \exists V'.(\tau_V(o) \wedge \neg \varphi[v'_1, \dots, v'_n / v_1, \dots, v_n])\right)
\end{aligned}
$$

We can use this regression formula for efficient symbolic regression search. BDDs support all necessary operations (atomic propositions, $\neg$, $\wedge$, $\vee$, substitution, $\exists$, ...).

Computing strong preimages with boolean function operations

Example

Let $V = \{a, b\}$, $V' = \{a', b'\}$, and

$$
\begin{aligned}
o &= \langle \neg a, \{a, a \wedge \neg b\} \rangle, \quad \text{i.e.,} \\
\tau_V(o) &= \neg a \wedge \Big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \Big).
\end{aligned}
$$

Moreover, let $\varphi = a$. Then

$$
\begin{aligned}
spreimgsymb_o(\varphi) = &\exists a' \exists b'. \Big(\neg a \wedge \big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \big) \wedge a' \Big) \wedge \\
&\neg \exists a' \exists b'. \Big(\neg a \wedge \big((a' \wedge (b \leftrightarrow b')) \vee (a' \wedge \neg b') \big) \wedge \neg a' \Big) \\
\equiv &\neg a
\end{aligned}
$$

Computing strong preimages with boolean function operations

Theorem

The previous definition of the symbolic preimage operator makes the following diagram commute:

$$
\begin{tikzcd}
\psi \arrow[d, "{[\![\cdot]\!]}"'] & \varphi \arrow[l, "spreimgsymb_o(\cdot)"'] \arrow[d, "{[\![\cdot]\!]}"] \\
{[\![\psi]\!]} & {[\![\varphi]\!]} \arrow[l, "spreimg_o(\cdot)"']
\end{tikzcd}
$$

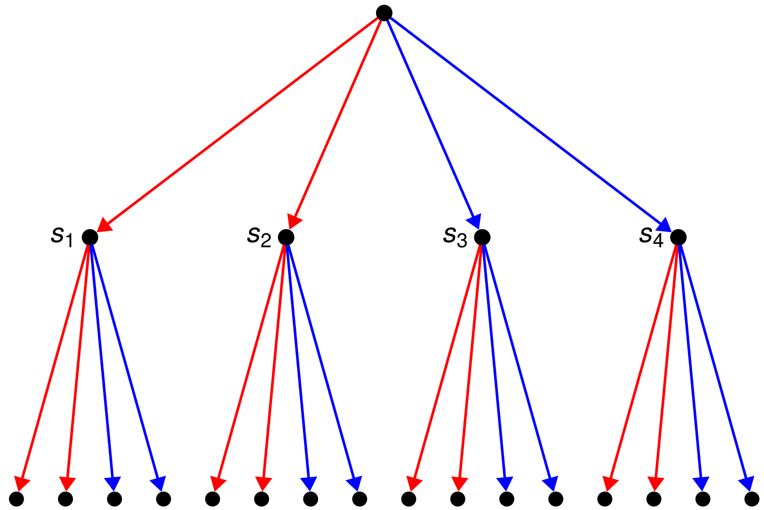
Proof.

Homework □

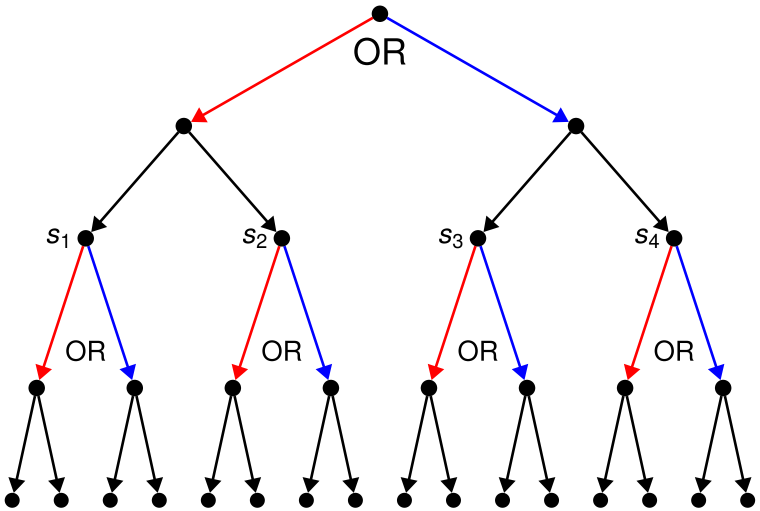
Progression Search

- We saw a generalization of regression search to strong planning.
- However, this search is uninformed (breadth-first search).
- Is there an analogue to A* search for strong planning?
- Yes: AO* search
  - Progression search (like A*)
  - Guided by a heuristic (like A*)
  - Guaranteed optimality (under certain conditions, like A*)

AND/OR search



AND/OR search



Progression Search

- We describe AO* on a graph representation without intermediate nodes, i.e., as in the first figure.
- There are different variants of AO*, depending on whether the graph that is being searched is an AND/OR tree, an AND/OR DAG, or a general, possibly cyclic, AND/OR graph.
- The graphs we want to search, $\mathcal{T}(\Pi)$, are in general cyclic.
- However, AO* becomes a bit more involved when dealing with cycles, so we only discuss AO* under the assumption of acyclicity and leave the generalization to cyclic state spaces as an exercise.

- The search is over $\mathcal{T}(\Pi)$.
- For ease of presentation, we do not distinguish between states of $\mathcal{T}(\Pi)$ and search nodes.
- Also, for ease of presentation, we do not handle the case that no strong plan exists.

UNI FREIBURG

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

Definition (solution graph)

A solution graph for a nondeterministic transition system $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ is an acyclic subgraph of $\mathcal{T}$ (viewed as a graph), $\mathcal{T}' = \langle S', L, T' \rangle$, such that

- $s_0 \in S'$,
- for each $s' \in S' \setminus S_\star$, there is exactly one label $l \in L$ s.t.
  - T' contains at least one outgoing transition from s' labeled with l,
  - T' contains all outgoing transitions from s' labeled with l (and S' contains the states reached via such transitions),
  - T' contains no outgoing transitions from s' labeled with any $\tilde{l} \neq l$, and
- every directed path in $\mathcal{T}'$ terminates at a goal state.

UNI FREIBURG

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

Conceptually, there are three graphs/transition systems:

- The induced transitions system $\mathcal{T} = \mathcal{T}(\Pi)$, which only exists as a mathematical object, but is in general not made explicit completely during AO* search,
- The current portion of $\mathcal{T}$ explicitly represented by the search algorithm, $\mathcal{T}_e$, and
- The current portion of $\mathcal{T}_e$ considered by the algorithm as the cheapest/best current partial solution graph, $\mathcal{T}_p$.

UNI FREIBURG

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

Definition (partial solution graph)

A partial solution graph for a nondeterministic transition system $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ is an acyclic subgraph of $\mathcal{T}$ (viewed as a graph), $\mathcal{T}_p = \langle S_p, L, T_p \rangle$, s.t.

- $s_0 \in S_p$,
- for each $s' \in S_p \setminus S_\star$ that is not an unexpanded leaf node in $\mathcal{T}_p$ there is exactly one label $l \in L$ such that
  - T_p contains at least one outgoing transition from s' labeled with l,
  - T_p contains all outgoing transitions from s' labeled with l (and S_p contains the states reached via such transitions),
  - T_p contains no outgoing transitions from s' labeled with any $\tilde{l} \neq l$, and
- every directed path in $\mathcal{T}_p$ terminates at a goal state or an unexpanded leaf node in $\mathcal{T}_p$.

AO* Search

Definition (cost of a partial solution graph)

Let $h : S \to \mathbb{N} \cup \{\infty\}$ be a heuristic function for the state space S of $\mathcal{T}$, and let $\mathcal{T}_p = \langle S_p, L, T_p \rangle$ be a partial solution graph. The cost labeling of $\mathcal{T}_p$ is the solution to the following system of equations over the states S_p of $\mathcal{T}_p$:

$$f(s) = \begin{cases} 0 & \text{if } s \text{ is a goal state} \\ h(s) & \text{if } s \text{ is an unexpanded non-goal} \\ 1 + \max_{s \xrightarrow{o} s'} f(s') & \text{for the unique outgoing action} \\ & o \text{ of } s \text{ in } \mathcal{T}_p\text{, otherwise.} \end{cases}$$

The cost of $\mathcal{T}_p$ is the cost labeling of its root.

AO* search keeps track of a cheapest partial solution graph by marking for each expanded state s an outgoing action o minimizing $1 + \max_{s \xrightarrow{o} s'} f(s')$.

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

AO* Search

Procedure ao-star

def ao-star($\mathcal{T}$):
- let $\mathcal{T}_e$ and $\mathcal{T}_p$ initially consist of the initial state s_0.
- **while** $\mathcal{T}_p$ has unexpanded non-goal node:
  - expand an unexpanded non-goal node s of $\mathcal{T}_p$
  - add new successor states to $\mathcal{T}_e$
  - **for all** new states s' added to $\mathcal{T}_e$:
    - $f(s') \leftarrow h(s')$ (or 0 if $s' \in S_\star$)
  - $Z \leftarrow s$ and its ancestors in $\mathcal{T}_e$ along marked actions.
  - **while** Z is not empty:
    - remove from Z a state s w/o descendant in Z.
    - $f(s) \leftarrow \min_{o \text{ applicable in } s}(1 + \max_{s \xrightarrow{o} s'} f(s'))$.
    - mark the best outgoing action for s
      (this may implicitly change $\mathcal{T}_p$).
- **return** an optimal solution graph.

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

AO* Search

Correctness (proof sketch)

- Solution graphs directly correspond to strong plans.
- Algorithm eventually terminates (finite number of possible node expansions).
- Acyclicity guarantees that extraction of $\mathcal{T}_p$ and dynamic programming back-propagation of f values always terminates.
- Marking makes sure that existing solutions are eventually marked.

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

AO* Search

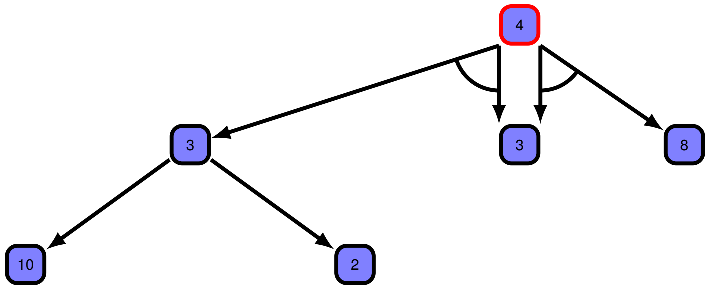
Details

- Pseudocode omits bookkeeping of solved states (can improve performance).
- Choice of unexpanded non-goal node of best partial solution graph is unspecified.
  - Correctness/optimality not affected.
  - One possibility: choose node with lowest cost estimate.
  - Alternative: expand several nodes simultaneously.
- Algorithm can be extended to deal with cycles in the AND/OR graph.

Concepts
Algorithms
Regression
Efficient implementation of regression
Progression
Summary

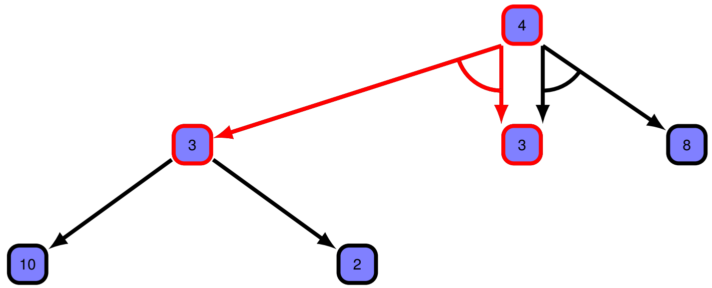
AO* Search

Example



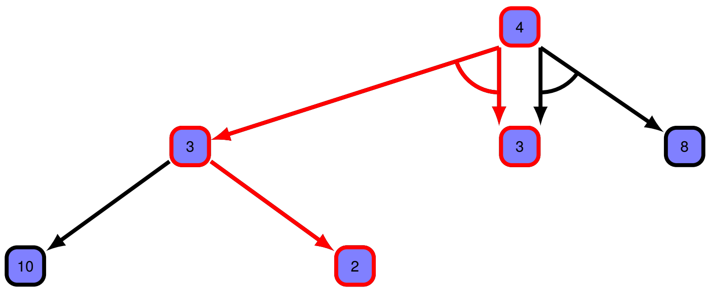
AO* Search

Example



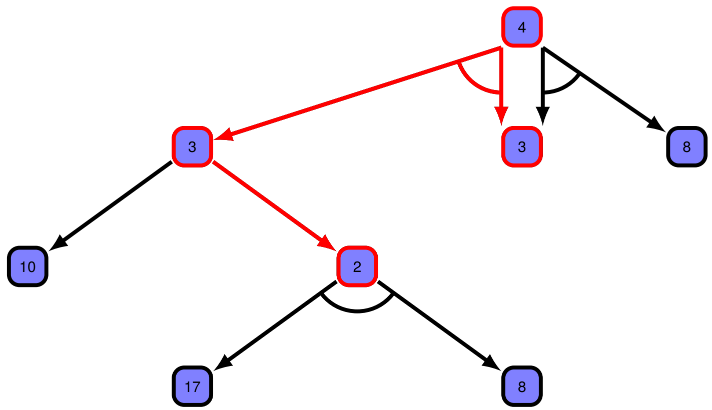
AO* Search

Example

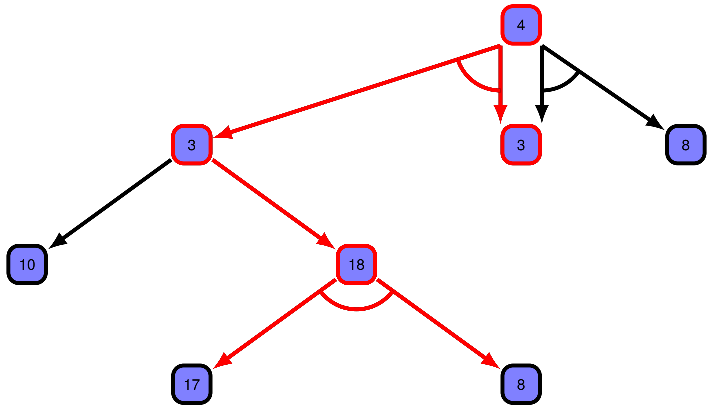


AO* Search

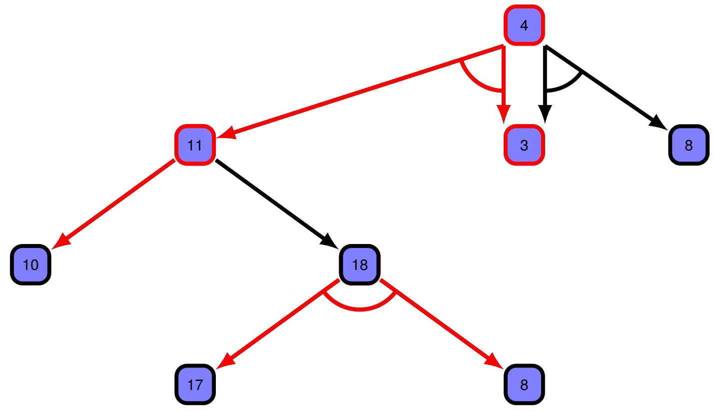
Example



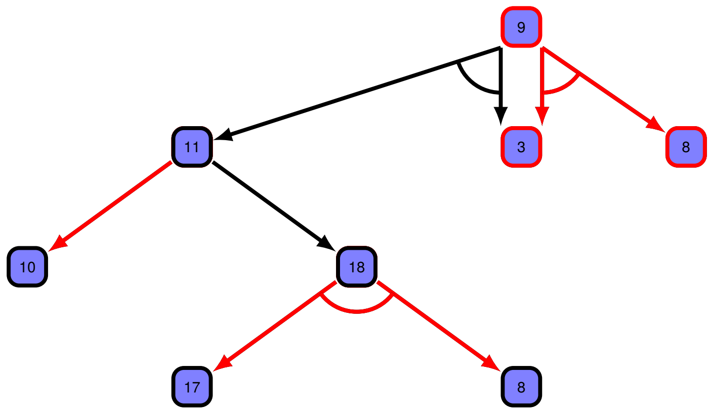
Example



Example



Example



Heuristic Evaluation Function

- Desirable: informative, domain-independent heuristic to initialize cost estimates.
- Heuristic should estimate (strong) goal distances.
- Heuristic does not necessarily have to be admissible (unless we seek optimal solutions).
- We can adapt many heurstics we already know from classical planning (details omitted).

Summary

Summary

- We have considered the special case of nondeterministic planning where
  - planning tasks are fully observable and
  - we are interested in strong plans.
- We have introduced important concepts also relevant to other variants of nondeterministic planning such as
  - images and
  - weak and strong preimages.
- We have discussed some basic classes of algorithms:
  - backward induction by dynamic programming, and
  - forward search in AND/OR graphs.