

# Principles of AI Planning

## 17. Nondeterministic planning

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# Motivation

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- The world is not predictable.
- AI robotics:
  - imprecise movement of the robot
  - other robots
  - human beings, animals
  - machines (cars, trains, airplanes, lawn-mowers, ...)
  - natural phenomena (wind, water, snow, temperature, ...)
- Games: other players are outside our control.
  - To win a game (reaching a goal state) with certainty, all possible actions by the other players have to be anticipated (a **winning strategy** of a game).
  - The world is not predictable because it is unknown: we cannot **observe** everything.

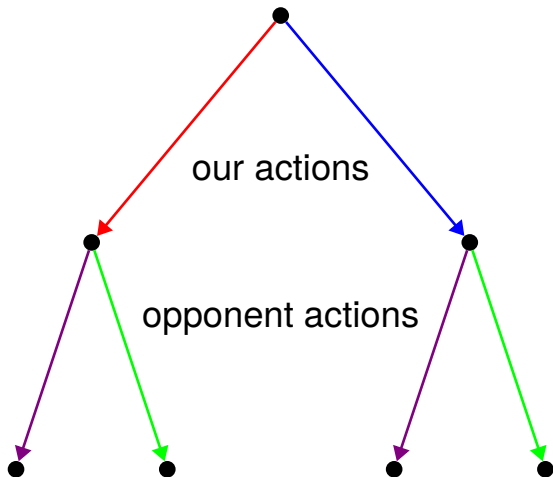
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In this lecture, we will only deal with uncertain operator outcomes, not with partial observability.

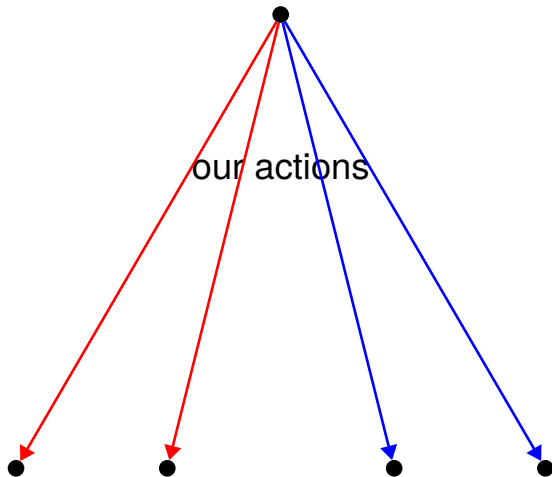
# Nondeterminism in games



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- In **deterministic planning** we have assumed that the only changes taking place in the world are those caused by us and that we can **exactly predict** the results of our actions.
- **Other agents** and processes, beyond our control, are formalized as **nondeterminism**.
- Implications:
  - 1 The future state of the world cannot be predicted.
  - 2 We cannot reliably plan ahead: no single operator sequence achieves the goals.
  - 3 In some cases it is not possible to achieve the goals with certainty no matter which outcomes the actions have, but only under certain fairness assumptions.

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# Transition systems and planning tasks

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# Transition systems with nondeterminism (cf. Chapter 2)



## Definition (transition system)

A **nondeterministic transition system** is a 5-tuple

$\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$  where

- $S$  is a finite set of **states**,
- $L$  is a finite set of (transition) **labels**,
- $T \subseteq S \times L \times S$  is the **transition relation**,
- $s_0 \in S$  is the **initial state**, and
- $S_\star \subseteq S$  is the set of **goal states**.

**Note:**  $T \subseteq S \times L \times S$  allows **nondeterministic operators** with more than one possible outcome.

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## Definition (nondeterministic operator)

Let  $V$  be a set of finite-domain state variables. A nondeterministic operator in unary nondeterminism normal form with conjunctive precondition and unconditional effects, or **nondeterministic operator** for short, is a pair  $o = \langle \chi, E \rangle$ , where

- $\chi$  is a conjunction of atoms over  $V$  (the **precondition**), and
- $E = \{e_1, \dots, e_n\}$  is a finite set of possible **effects** of  $o$ , each  $e_i$  being a conjunction of atomic finite-domain effects over  $V$ .

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## Definition (nondeterministic operator application)

Let  $o = \langle \chi, E \rangle$  be a nondeterministic operator and  $s$  a state.

Applicability of  $o$  in  $s$  is defined as in the deterministic case, i.e.,  $o$  is **applicable** in  $s$  iff  $s \models \chi$  and the change set of each effect  $e \in E$  is consistent.

If  $o$  is applicable in  $s$ , then the **application** of  $o$  in  $s$  leads to one of the states in the set  $app_o(s) := \{app_{\langle \chi, e \rangle}(s) \mid e \in E\}$  nondeterministically.

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## Example

$put\text{-}on\text{-}block(A, B) = \langle \chi, \{e_1, e_2\} \rangle$  where

- $\chi = \{handempty \mapsto false, clear\text{-}B \mapsto true, pos\text{-}A \mapsto hand\}$ ,
- $e_1 = \{handempty \mapsto true, clear\text{-}B \mapsto false, pos\text{-}A \mapsto on\text{-}B\}$ ,
- $e_2 = \{handempty \mapsto true, pos\text{-}A \mapsto table\}$ .

Applied to a state where the agent is holding block  $A$  and block  $B$  is clear, this operator leads to one of two possible successor states. Either  $A$  gets stacked on  $B$  successfully, or  $A$  is dropped to the table.

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## Definition (nondeterministic planning task)

A (fully observable) **nondeterministic planning task** is a 4-tuple  $\Pi = \langle V, I, O, \gamma \rangle$  where

- $V$  is a finite set of **finite-domain state variables**,
- $I$  is an **initial state** over  $V$ ,
- $O$  is a finite set of **nondeterministic operators** over  $V$ , and
- $\gamma$  is a conjunctions of atoms over  $V$  describing the **goal states**.

**Remark:** In the following, we will always assume that our nondeterministic planning tasks are fully observable.

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## Definition (induced transition system)

Every nondeterministic planning task  $\Pi = \langle V, I, O, \gamma \rangle$  induces a corresponding nondeterministic transition system

$$\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle:$$

- $S$  is the set of all states over  $V$ ,
- $L$  is the set of operators  $O$ ,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' \in \text{app}_o(s) \}$ ,
- $s_0 = I$ , and
- $S_\star = \{ s \in S \mid s \models \gamma \}$

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# Plans

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# What is a plan?



In nondeterministic planning, plans are more complicated objects than in the deterministic case:

The best action to take may **depend on nondeterministic effects** of previous operators.

Nondeterministic plans thus often require **branching**.  
Sometimes, they even require **looping**.

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## Example (Branching)

(Part of) a plan for winning the game **Connect Four** can be described as follows:

- Place a tile in the 4th column.
  - If opponent places a tile in the 1st, 4th or 7th column, place a tile in the 4th column.
  - If opponent places a tile in the 2nd or 5th column, place a tile in the 2nd column.
  - If opponent places a tile in the 3rd or 6th column, place a tile in the 6th column.

There is no **non-branching** plan that solves the task  
(= is guaranteed to win the game).

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## Example (Looping)

A plan for building a card house can be described as follows:

- 1 Build a wall with two cards.  
If the structure falls apart, redo from start.
- 2 Build a second wall with two cards.  
If the structure falls apart, redo from start.
- 3 Build a ceiling on top of the walls with a fifth card.  
If the structure falls apart, redo from start.
- 4 Build a wall on top of the ceiling with two cards.  
If the structure falls apart, redo from start.

There is no **non-looping** plan that solves the task (unless the planning agent is very dextrous).

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- Plans should be allowed to **branch**. Otherwise, most interesting nondeterministic planning tasks cannot be solved.
- We may or may not allow plans to **loop**.
  - Non-looping plans are preferable because they **guarantee** that the goal is reached within a bounded number of steps.
  - Where non-looping plans are not possible, looping plans may be adequate because they at least guarantee that the goal will be reached **eventually** unless nature is **unfair**.

We will now introduce the formal concepts necessary to define branching and looping plans.

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## Definition (strategy)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ .

A **strategy** for  $\Pi$  is a function  $\pi : S_\pi \rightarrow O$  for some subset  $S_\pi \subseteq S$  such that for all states  $s \in S_\pi$  the action  $\pi(s)$  is applicable in  $s$ .

The set of states reachable in  $\mathcal{T}(\Pi)$  starting in state  $s$  and following  $\pi$  is denoted by  $S_\pi(s)$ .

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## Definition (weak, proper, and acyclic strategies)

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ , and let  $\pi$  be a strategy for  $\Pi$ . Then  $\pi$  is called

- **weak** iff  $S_\pi(s_0) \cap S_* \neq \emptyset$ ,
- **proper** iff  $S_\pi(s') \cap S_* \neq \emptyset$  for all  $s' \in S_\pi(s_0)$ , and
- **acyclic** iff there is no state  $s' \in S_\pi(s_0)$  such that  $s'$  is reachable from  $s'$  following  $\pi$  in a strictly positive number of steps.

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- **Strategies** in nondeterministic planning correspond to **applicable operator sequences** in deterministic planning.
- In deterministic planning, a **plan** is an applicable operator sequence that results in a goal state.
- In nondeterministic planning, we define different notions of “resulting in a goal state”.

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## Definition

Let  $\Pi = \langle V, I, O, \gamma \rangle$  be a nondeterministic planning task with state set  $S$  and goal states  $S_*$ .

- A strategy for  $\Pi$  is called a **weak plan** for  $\Pi$  iff it is weak.
- A strategy for  $\Pi$  is called a **strong cyclic plan** for  $\Pi$  iff it is proper.
- A strong cyclic plan for  $\Pi$  is called a **strong plan** for  $\Pi$  iff it is acyclic.

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We extended the deterministic (**classical**) planning formalism:

- **operators** can be nondeterministic

**Remark:** We could also introduce nondeterminism in the initial situation by allowing more than one initial state, but this can be easily compiled into our formalism. (**How?**)

As a consequence, **plans** can contain

- **branches** and
- **loops**.

In the following chapter, we consider the **strong planning** problem and the **strong cyclic planning** problem and discuss some algorithms.

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