Principles of AI Planning

16. Planning via SAT

Albert-Ludwigs-Universität Freiburg

UNI FREIBURG

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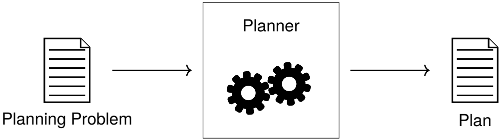
February 5th, 2020

SAT Modelling

Idea: Problem Transformation



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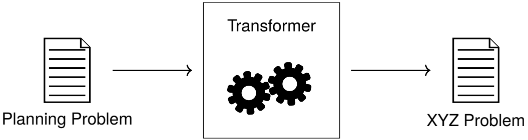

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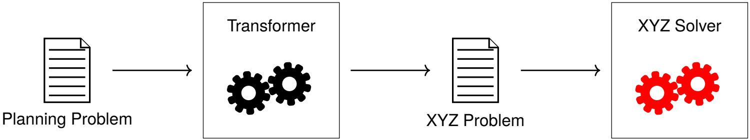

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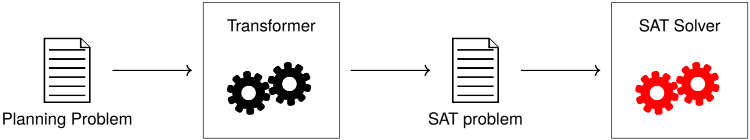

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Given a propositional formula $\mathscr{F}$, decide whether $\mathscr{F}$ has a satisfying valuation.

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CNF:

$$\mathscr{F} = \bigwedge_{C \in \mathfrak{C}} \bigvee_{\ell \in C} \ell$$

($\mathfrak{C}$ is the set of clauses; C is a clause, a set of literals.)

SAT Solvers

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- Standardised input format DIMACS:

```
p cnf 5 3
1 -5 4 0
-1 5 3 4 0
-3 -4 0
```

$\equiv$ CNF with 5 vars and 3 clauses:

$$(v_1 \vee \neg v_5 \vee v_4) \wedge$$
$$(\neg v_1 \vee v_5 \vee v_3 \vee v_4) \wedge$$
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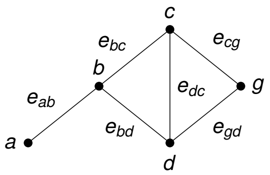
Colouring

Definition

Given a graph $G = (V, E)$ and a number k.
Is there an assignment of k colours to the vertices of G, such that all adjacent vertices have different colours?



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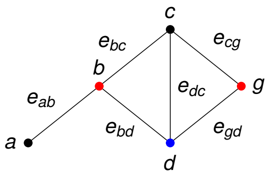

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Every node has at most one colour

$$\bigwedge_{i=1}^{k} \left[\texttt{colour}_v^i \rightarrow \bigwedge_{j=1, i \neq j}^{k} \neg\texttt{colour}_v^j \right] \qquad \forall v \in V$$

Theoretical Background

Computational Complexity

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$\mathbb{PSPACE}$ with $\mathbb{NP}$ calculus?

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  - Practical limit: usually smaller (sometimes polynomially bounded).
- Start with a small k and increase until a solution is found.

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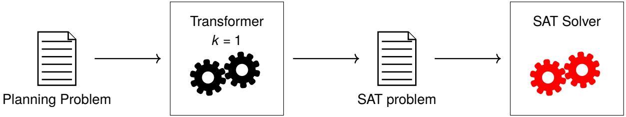

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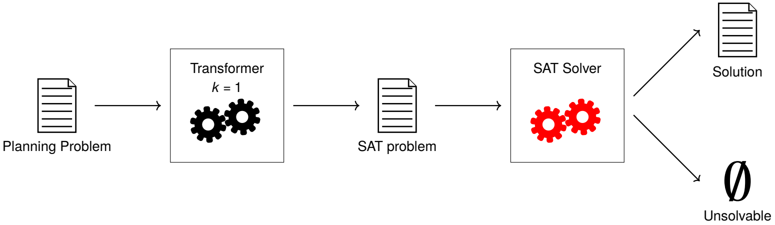

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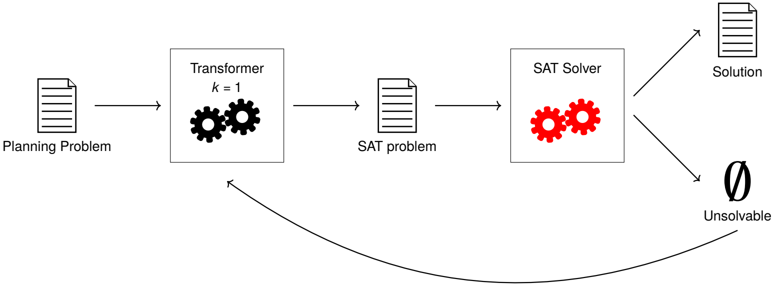

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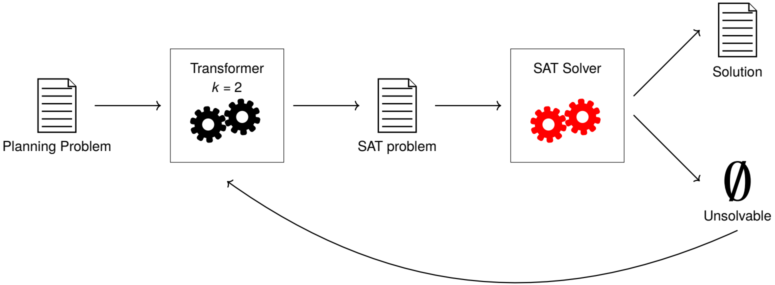

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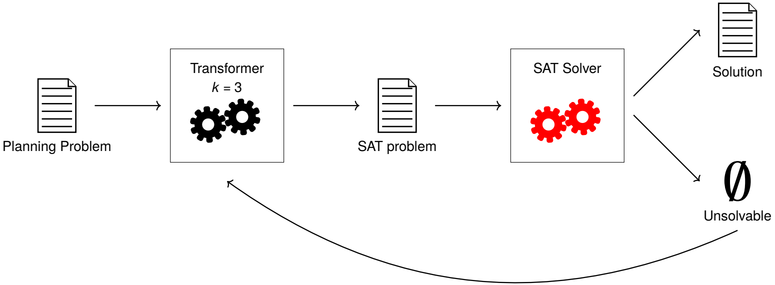

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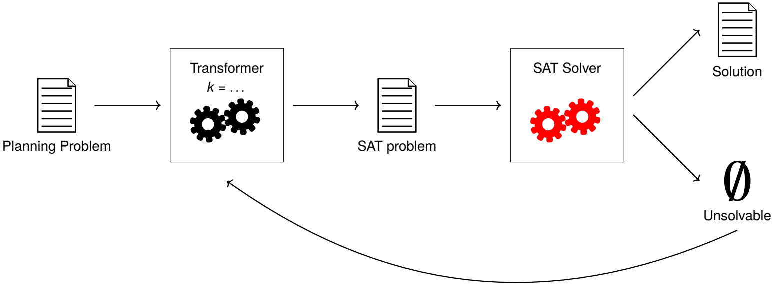

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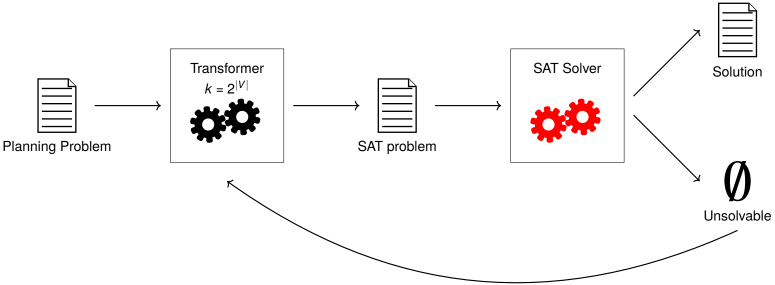
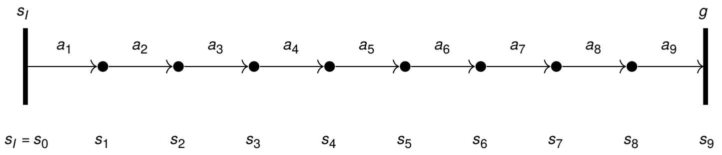


Sequential Classical Planning in SAT

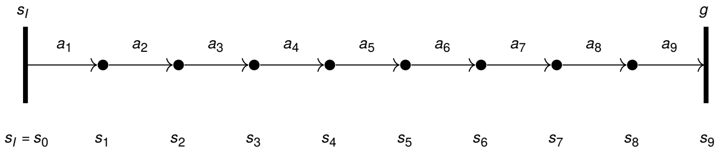
Classical Planning via SAT [Kautz&Selman'92]



A plan is just a sequence of state transitions.

- "Mechanics" is identical in all timesteps.
- Just model one timestep and copy'n'paste.
- Edge constraints!

Decision Variables



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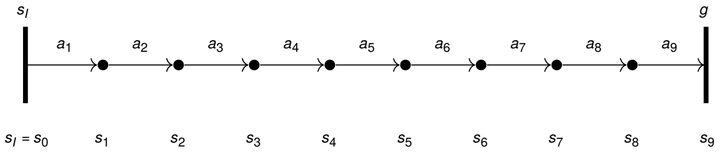

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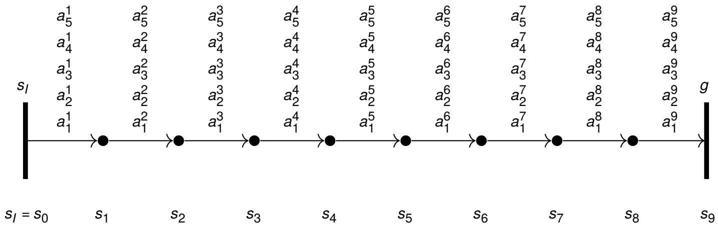

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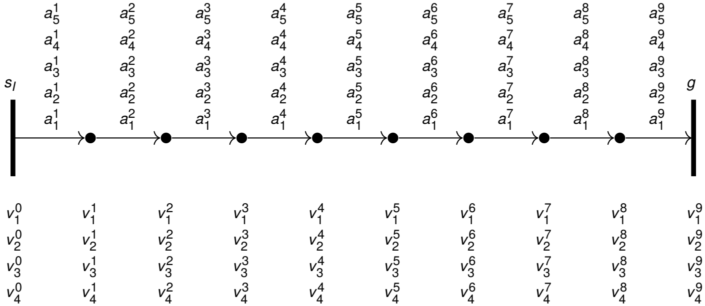

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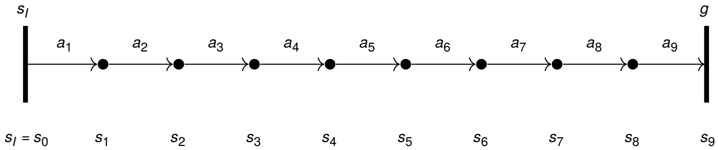
1. a_i^t – Action i is executed at time t.
2. v_i^t – State variable i is true at time t.

Overall Formula



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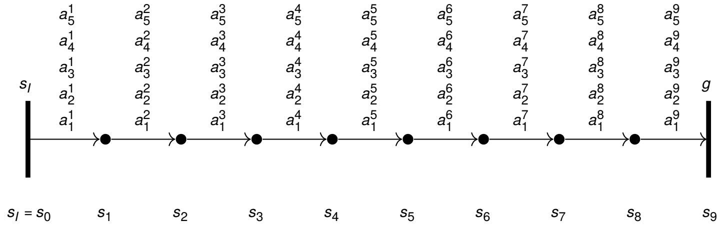

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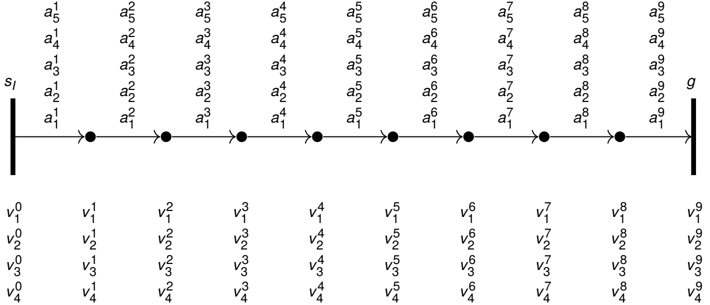

Constraints to check:

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- s_I and g must be respected.

$$\mathscr{F} = \bigwedge_{t=0}^{k-1} \tau(t) \wedge \bigwedge_{v_i \in s_I} v_i^0 \wedge \bigwedge_{v_i \in V \setminus s_I} \neg v_i^0 \wedge \bigwedge_{v_i \in g} v_i^k \quad \text{here: } k = 9$$

Classical Planning via SAT

$$s \xrightarrow{a} s'$$

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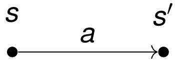

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$$F_2 = \left[\bigwedge_{a \in A} a^{t+1} \rightarrow \bigwedge_{v \in add(a)} v^{t+1}\right] \quad \wedge$$

$$\left[\bigwedge_{a \in A} a^{t+1} \rightarrow \bigwedge_{v \in del(a)} \neg v^{t+1}\right]$$

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$$F_3 = \bigwedge_{v \in V} \left[(\neg v^t \wedge v^{t+1}) \rightarrow \bigvee_{a \in A \text{ with } v \in add(a)} a^{t+1} \right] \wedge$$

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$$F_4 = \text{at-most-one}(\{a^t \mid a \in A\})$$

At-most-one

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Naive encoding:

$$\bigwedge_{x_1 \in X} \bigwedge_{x_2 \in X \setminus \{x_1\}} \underbrace{\neg x_1 \vee \neg x_2}_{\substack{(x_1 \Rightarrow \neg x_2) \wedge \\ (x_2 \Rightarrow \neg x_1)}}$$

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Sequential encoding:

$$\bigwedge_{i=1}^{|X|-1} \underbrace{\neg x_i \vee f_i}_{x_i \Rightarrow f_i} \qquad \bigwedge_{i=2}^{|X|-1} \underbrace{\neg f_{i-1} \vee f_i}_{f_{i-1} \Rightarrow f_i}$$

$$\bigwedge_{i=1}^{|X|} \underbrace{\neg x_i \vee \neg f_{i-1}}_{\substack{(x_i \Rightarrow \neg f_{i-1}) \wedge \\ (f_{i-1} \Rightarrow \neg x_i)}}$$

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n_i – bit i (0-index) of a $\lceil \log(|X|) \rceil$-digit binary number if one

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Binary encoding:

$$\neg x_i \lor n_j \qquad \text{if } \frac{i}{2^j} \mod 2 = 1$$
$$\neg x_i \lor \neg n_j \qquad \text{if } \frac{i}{2^j} \mod 2 = 0$$

Different AMO Implementations[1]

encoding	#clauses	#new variables
binomial	n^2	0
binary	$n \log n$	$\log n$
sequential	$3n$	n
commander	$\frac{7}{2}n$	$\frac{n}{2}$
product	$2(n + n^{\frac{1}{m+1}})$	$2n^{\frac{1}{2}}$

where n is the number of atoms, i.e., $|X|$

[1] Frisch and Giannaros; SAT Encodings of the At-Most-k Constraint – Some Old, Some New, Some Fast, Some Slow; 2010

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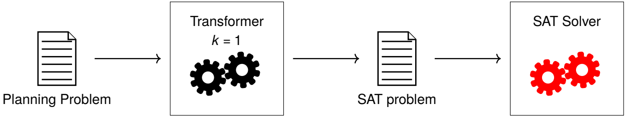

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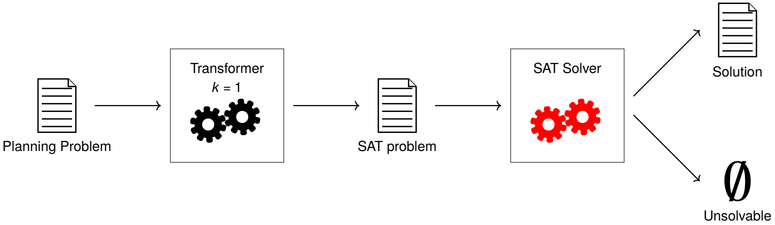

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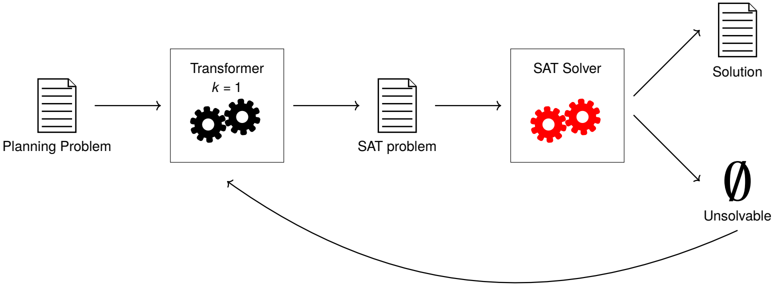

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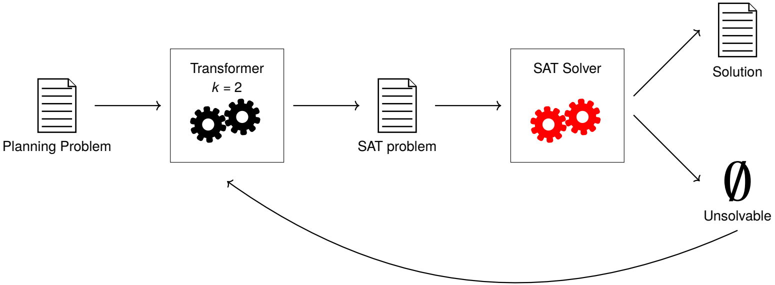

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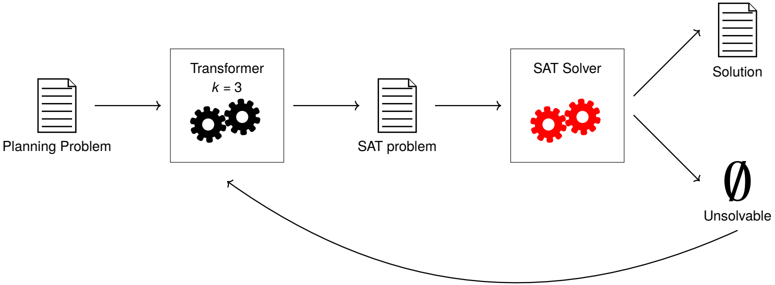

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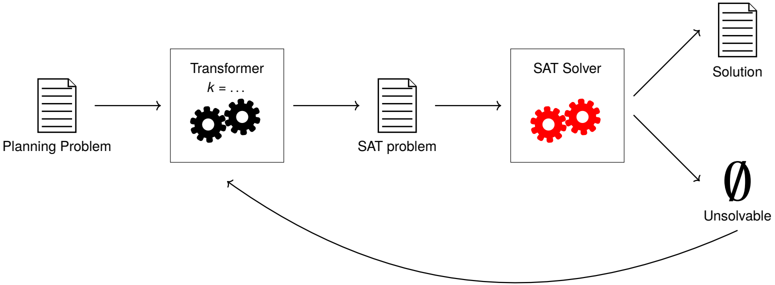

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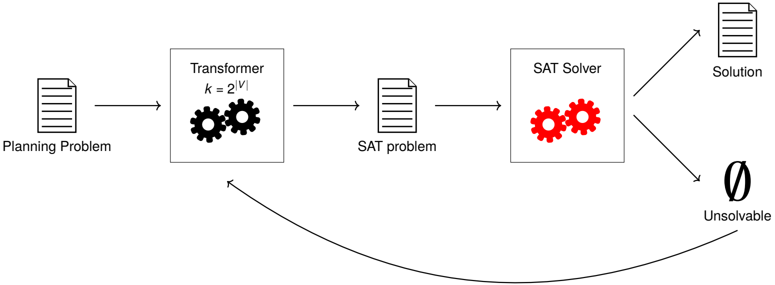

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Invariants

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Definition (Invariant)

An invariant $\mathscr{I}$ is a formula over the state variables such that for all states s reachable from s_I it holds $s \models \mathscr{I}$.

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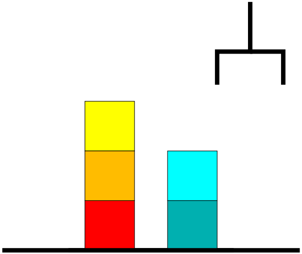

Predicates:

- *on*(x,y) – x lies directly on y.
- *free*(x) – x has no block above it.

Actions:

- *pickup*(x) – pick up x, if it is free.
- *putdown*(x,y) – put x on y, if y is free (*table* is always free).

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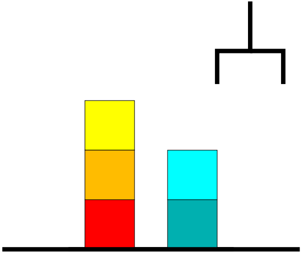

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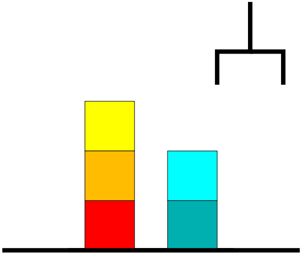

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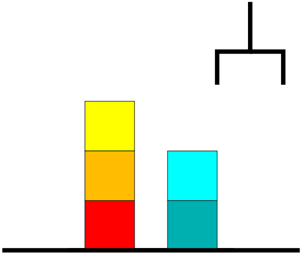

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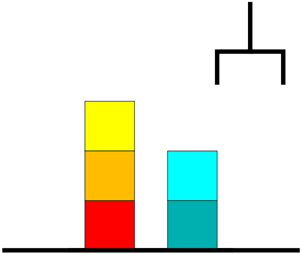

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- Compute an approximation of all invariants of a fixed form.
- Restrict to binary-or invariants:

$$\ell_1 \vee \ell_2$$

Computing Invariants [Rintanen'98]

Note: Here we consider some action $a = (pre, add, del)$ and denote with $\mathit{eff} = add(a) \cup \{\neg v \mid v \in del(a)\}$ its effects (as a literal set).

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$F_{\langle pre,eff\rangle}(\mathscr{I})$ is a *filter* for invariants, returning those that hold after the execution of an action $a = \langle pre, eff\rangle$

$$F_{\langle pre,eff\rangle}(\mathscr{I}) = \begin{cases} \mathscr{I} & \text{if } \mathscr{I} \cup pre \models \bot \text{ and otherwise:} \\ \{\ell_1 \vee \ell_2 \in \mathscr{I} \mid & \big(\neg \ell_1 \notin eff \text{ or } \ell_2 \in U_{\langle pre,eff\rangle}(\mathscr{I})\big) \text{ and} \\ & \big(\neg \ell_2 \notin eff \text{ or } \ell_1 \in U_{\langle pre,eff\rangle}(\mathscr{I})\big)\} \end{cases}$$

Computing Invariants [Rintanen'98], cont'd

Call $R_A(\mathscr{I}) := F_{a_1}(F_{a_2}(\cdots F_{a_n}(\mathscr{I})\cdots))$ with initial invariant
$I_{init} = \{v \vee \ell \mid v \in s_I, \ell \in V \cup \neg V\} \cup \{\neg v \vee \ell \mid v \notin s_I, \ell \in V \cup \neg V\}$
and arbitrary linearization of action set A, $a_1, \ldots, a_n$,
until $\mathscr{I}$ does not change anymore.

R stands for "*reduce* invariant set".

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∀-step

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Further implications?
The resulting state must always be the same!
Thus we forbid two actions a_1, a_2 with $del(a_1) \cap add(a_2) \neq \emptyset$ to be executed in parallel.
(Otherwise the resulting state would not be unique.)

∃-step

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(Re-)Consider the following (single) planning problem:

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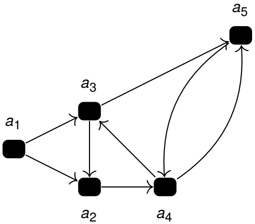

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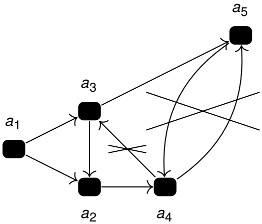

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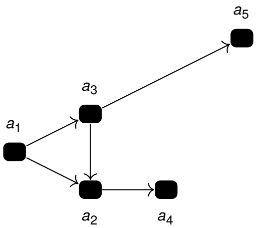

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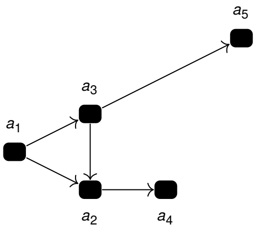

a_5, a_4, a_2, a_3, a_1

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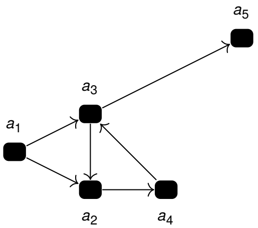

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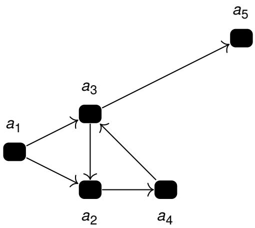

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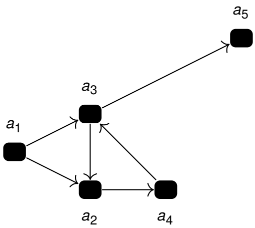
- If executed in parallel, we will always execute actions in **this** order.



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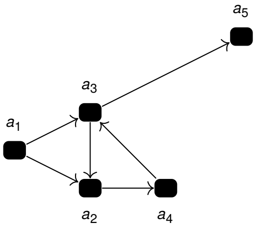

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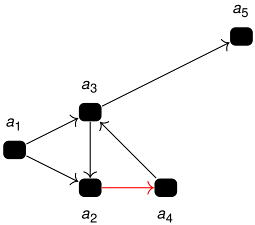

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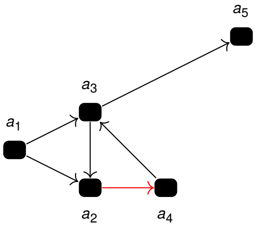

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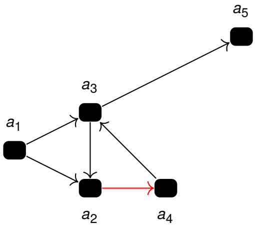

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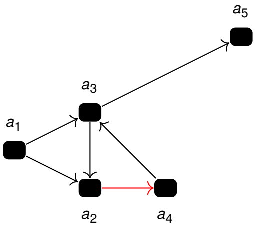
6. Enforced via *chaines*.



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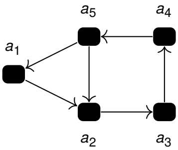
Chains

We are given an SCC and an ordering of its vertices.



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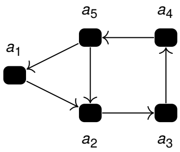

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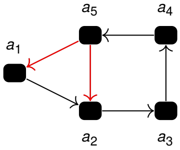

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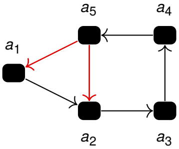

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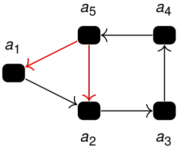

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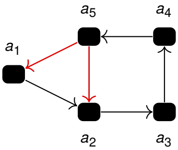

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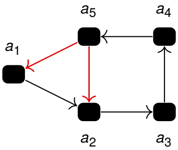

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$$\{\mathtt{f}_v^i \rightarrow \neg\pi^i \mid \pi^i \in R_v\}$$

Further Improvements

Improvements for classical planning:

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- Relaxed ∃-step [Wehrle&Rintanen'07].
- Parallel SAT search [Rintanen'04] [Rintanen,Heljanko,Niemelä'06].
- Specialised heuristics for SAT solvers [Rintanen'10a] [Rintanen'10b].
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Extensions to non-classical planning:

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→ `https://users.aalto.fi/~rintanj1/satplan.html`