# Principles of AI Planning

## 12. Planning as search: potential heuristics

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# Motivation

# Motivation: declarative heuristics

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Motivation

Potential Heuristics

Summary

## Previous chapters:

### "Procedural" method for obtaining a heuristic

Solve an easier version of the problem.

We have studied two common simplification methods: relaxation and abstraction.

## This chapter:

### "Declarative" method for obtaining a heuristic

- Declaratively describe the information we want the heuristic estimator to exploit.
- Let a computer find a heuristic that fits the declarative description.

# Motivation: potential heuristics



## Example (potential heuristic in chess)

Evaluation function for chess position $s$ (from White's perspective; the higher, the better):

$$h(s) = 9 \cdot (\text{♕} - \text{♛}) + 5 \cdot (\text{♖} - \text{♜}) + 3 \cdot (\text{♗} - \text{♝}) + 3 \cdot (\text{♘} - \text{♞}) + 1 \cdot (\text{♙} - \text{♟})$$

where ♕, ♛, ♖, ♜, … is the number of white and black queens, rooks, etc. still on the board.

Question: Can we derive a similar heuristic for planning?

Answer: Yes! (Even declaratively!)

# Potential Heuristics

## Potential heuristics: idea

Heuristic design as an optimization problem:

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

$$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$.
- Find potentials for which $h$ is admissible and well-informed.

Motivation:

- declarative approach to heuristic design
- heuristic very fast to compute if features are

# Potential heuristics

## Definition (feature)

A (state) feature for a planning task is a numerical function defined on the states of the task: $f : S \to \mathbb{R}$.

Atomic features test if some atom is true in a state.

## Definition (atomic feature)

Let $v = d$ be an atom of an FDR planning task.
Then the atomic feature $f_{v=d}$ is defined as:

$$f_{v=d}(s) = \begin{cases} 1 & \text{if } s \models v = d \\ 0 & \text{otherwise} \end{cases}$$

$\leadsto$ atomic features $\approx$ facts

## Definition (potential heuristic)

A potential heuristic for a set of features $\mathscr{F} = \{f_1, \ldots, f_n\}$ is a heuristic function $h$ defined as a linear combination of the features:

$$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

with weights (potentials) $w_i \in \mathbb{R}$.

- We only consider atomic potential heuristics, which are based on the set of all atomic features.
- Example for a task with state variables $v_1$ and $v_2$ and $\mathscr{D}_{v_1} = \mathscr{D}_{v_2} = \{d_1, d_2, d_3\}$:

$$h(s) = 3f_{v_1=d_1} + {}^1\!/{}_2 f_{v_1=d_2} - 2f_{v_1=d_3} + {}^5\!/{}_2 f_{v_2=d_1}$$

# How to set the weights?



We want to find good atomic potential heuristics:

- admissible
- consistent
- well-informed

Question: How to achieve this?

Answer: Linear programming.

Goal: solve a system of linear inequalities over $n$ real-valued variables while optimizing some linear objective function.

## Example (Production domain)

Two sorts of items with time requirements and profit per item.

| | Cutting | Assembly | Postproc. | Profit per item |
|---|---|---|---|---|
| ($x$) sort 1 | 25 | 60 | 68 | 30 |
| ($y$) sort 2 | 75 | 60 | 34 | 40 |
| per day | $\leq 450$ | $\leq 480$ | $\leq 476$ | maximize! |

Aim: Find numbers of pieces $x$ of sort 1 and $y$ of sort 2 produced per day such that resource constraints are met and objective function is maximized.

## Example (ctd., formalization)

$$\text{maximize } z = 30x + 40y \text{ subject to:} \quad (1)$$

$$x \geq 0, \; y \geq 0 \quad (2)$$

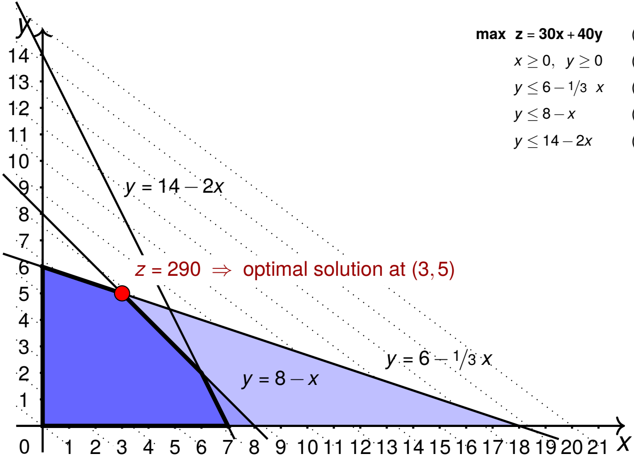
$$25x + 75y \leq 450 \quad (3)$$

$$60x + 60y \leq 480 \quad (4)$$

$$68x + 34y \leq 476 \quad (5)$$

- Line (1): Objective function
- Inequalities (2)–(5): Admissible solutions

## Example (ctd., visualization)




$$\mathbf{max}\ \ \mathbf{z = 30x + 40y} \quad (1)$$

$$x \geq 0, \quad y \geq 0 \quad (2)$$

$$y \leq 6 - 1/3\ x \quad (3)$$

$$y \leq 8 - x \quad (4)$$

$$y \leq 14 - 2x \quad (5)$$

## Definition (Linear program)

A linear program (LP) over variables $x_1, \dots, x_n$ consists of

- $m$ linear constraints of the form

$$\sum_{i=1}^{n} a_{ji} x_i \leq b_j$$

  with $a_{ji} \in \mathbb{R}$ for all $j = 1, \dots, m$ and $i = 1, \dots, n$, and
- a linear objective function to be maximized ($x_i \geq 0$):

$$\sum_{i=1}^{n} c_i x_i$$

  with $c_i \in \mathbb{R}$ for all $i = 1, \dots, n$.

# Linear Programming

Solution of an LP:
assignment of values to the $x_i$ satisfying the constraints and maximizing the objective function.

Solution algorithms:

- Usually: simplex algorithm (worst-case exponential).
- There are also polynomial-time algorithms.

# Transition normal form



Standard description of LP-based derivation of potentials assumes transition normal form.

Assumption (for the rest of the chapter): only SAS⁺ tasks.

Notation: variables occurring in conditions and effects.

## Definition ($vars(\varphi)$, $vars(e)$)

For a logical formula $\varphi$ over finite-domain variables $\mathcal{V}$, $vars(\varphi)$ denotes the set of finite-domain variables occurring in $\varphi$.

For an effect $e$ over finite-domain variables $\mathcal{V}$, $vars(e)$ denotes the set of finite-domain variables occurring in $e$.

# Transition normal form



## Definition (transition normal form)

An SAS$^+$ planning task $\Pi = \langle \mathscr{V}, I, O, \gamma \rangle$ is in transition normal form (TNF) if

- for all $o \in O$, $vars(pre(o)) = vars(eff(o))$, and
- $vars(\gamma) = \mathscr{V}$.

In words, an operator in TNF must mention the same variables in the precondition and effect, and a goal in TNF must mention all variables (= specify exactly one goal state).

# Converting operators to TNF: violations



There are two ways in which an operator $o$ can violate TNF:

- There exists a variable $v \in vars(pre(o)) \setminus vars(eff(o))$.
- There exists a variable $v \in vars(eff(o)) \setminus vars(pre(o))$.

The first case is easy to address: if $v = d$ is a precondition with no effect on $v$, just add the effect $v := d$.

## Example (TNF: adding effects)

Let $o = \langle x = 0 \wedge y = 0, y := 1 \rangle$.

Fix: rewrite $o = \langle x = 0 \wedge y = 0, x := 0 \wedge y := 1 \rangle$.

# Converting operators to TNF: violations

The second case is more difficult: if we have the effect $v := d$ but no precondition on $v$, how can we add a precondition on $v$ without changing the meaning of the operator (and without introducing exponentially many new operators)?

## Example (TNF: adding precondition)

Let $o = \langle \top, y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$ with $\mathcal{D}_{y_i} = \{0, 1\}$ for all $i$.

One possible fix: rewrite $o$ as set of operators

$$o_{00\ldots0} = \langle y_1 = 0 \wedge y_2 = 0 \wedge \cdots \wedge y_n = 0,\ y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$$
$$o_{00\ldots1} = \langle y_1 = 0 \wedge y_2 = 0 \wedge \cdots \wedge y_n = 1,\ y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$$
$$\vdots$$
$$o_{11\ldots1} = \langle y_1 = 1 \wedge y_2 = 1 \wedge \cdots \wedge y_n = 1,\ y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$$

Problem: $2^n$ new operators (exponentially many!)

# Converting operators to TNF: violations

The second case is more difficult: if we have the effect $v := d$ but no precondition on $v$, how can we add a precondition on $v$ without changing the meaning of the operator (and without introducing exponentially many new operators)?

## Example (TNF: adding precondition (ctd.))

Let $o = \langle \top, y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$ with $\mathscr{D}_{y_i} = \{0, 1\}$ for all $i$.

Better fix: rewrite $o = \langle y_1 = \mathit{don't\_care} \wedge y_2 = \mathit{don't\_care} \wedge \cdots \wedge y_n = \mathit{don't\_care},\ y_1 := 1 \wedge \cdots \wedge y_n := 1 \rangle$ and make sure that every variable can take its *don't_care* value for free.

# Converting Operators to TNF

## Formally:

1. For every variable $v$, add a new auxiliary value $u$ to its domain.
2. For every variable $v$ and value $d \in \mathscr{D}_v \setminus \{u\}$, add a new operator to change the value of $v$ from $d$ to $u$ at no cost: $\langle v = d, v := u \rangle$.
3. For all operators $o$ and all variables $v \in vars(eff(o)) \setminus vars(pre(o))$, add the precondition $v = u$ to $pre(o)$.

## Properties:

- Transformation can be computed in linear time.
- Due to the auxiliary values, there are new states and transitions in the induced transition system, but all path costs between original states remain the same.

# Converting Goals to TNF



- The auxiliary value idea can also be used to convert the goal $\gamma$ to TNF.
- For every variable $v \notin \mathit{vars}(\gamma)$, add the condition $v = u$ to $\gamma$.

With these ideas, every SAS⁺ planning task can be converted into transition normal form in linear time.

# Producers and consumers





Assume that $\Pi = \langle \mathscr{V}, I, O, \gamma \rangle$ is in TNF.

## Definition (producers and consumers)

Fact $v = d$ is produced by operator $o \in O$
if $v = d$ is an effect of $o$, but not a precondition of $o$.

Fact $v = d$ is consumed by operator $o \in O$
if $v = d$ is a precondition of $o$, but not an effect of $o$.

# Admissible and consistent potential heuristics

Assume feature set $\mathscr{F} = \{f_{v=d} \mid v \in \mathscr{V}, d \in \mathscr{D}_v\}$ and corresponding potentials $\mathscr{W} = \{w_{v=d} \mid v \in \mathscr{V}, d \in \mathscr{D}_v\}$.

Constraints on potentials characterize (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

## Goal-awareness constraint

$$\sum_{\text{goal fact } v=d} w_{v=d} = 0$$

## Example (Goal-awareness constraint)

$\mathscr{V} = \{x, y\}$, $\mathscr{D}_x = \mathscr{D}_y = \{0, 1, u\}$, $\gamma = (x = 1 \wedge y = u)$.

Goal-awareness constraint: $w_{x=1} + w_{y=u} = 0$.

## Theorem

*For a task in TNF, a potential heuristic with feature set $\mathcal{F} = \{f_{v=d} \mid v \in \mathcal{V}, d \in \mathcal{D}_v\}$ and corresponding potentials $\mathcal{W} = \{w_{v=d} \mid v \in \mathcal{V}, d \in \mathcal{D}_v\}$ that satisfy the goal-awareness constraint is goal-aware.*

## Proof.

See blackboard. □

## Consistency constraints (for all operators $o \in O$)

$$\sum_{\text{fact } v=d \text{ consumed by } o} w_{v=d} - \sum_{\text{fact } v=d \text{ produced by } o} w_{v=d} \leq cost(o)$$

### Example (Consistency constraint)

$\mathscr{V} = \{x, y\}$, $\mathscr{D}_x = \mathscr{D}_y = \{0, 1, u\}$,
$o = \langle x = 0 \land y = 0, x := 0 \land y := 1 \rangle$ with $cost(o) = 1$.

Then $o$ consumes $y = 0$ and produces $y = 1$.

Consistency constraint for o: $w_{y=0} - w_{y=1} \leq 1$.

# Admissible and consistent potential heuristics



## Theorem

*For a task in TNF, a potential heuristic with feature set $\mathscr{F} = \{f_{v=d} \mid v \in \mathscr{V}, d \in \mathscr{D}_v\}$ and corresponding potentials $\mathscr{W} = \{w_{v=d} \mid v \in \mathscr{V}, d \in \mathscr{D}_v\}$ that satisfy the consistency constraints for all operators o is consistent.*

## Proof.

Homework exercise. □

# Admissible and consistent potential heuristics



Remarks:

- all linear constraints $\rightsquigarrow$ LP
- goal-aware and consistent $\rightsquigarrow$ admissible and consistent

# Well-informed potential heuristics



How to find a well-informed potential heuristic?
$\rightsquigarrow$ encode quality metric in the objective function and use LP solver to find a heuristic maximizing it

Examples:

- maximize heuristic value of a given state (e.g., initial state)
- maximize average heuristic value of all states (including unreachable ones)
- maximize average heuristic value of some sample states

# Well-informed potential heuristics





LP encoding for maximizing heuristic value of initial state while guaranteeing goal-awareness and consistency:

$$\text{maximize} \quad \sum_{\text{fact } v=d \text{ satisfied in } s_0} w_{v=d} \quad \text{subject to:}$$

goal constraint

consistency constraint for $o$ for all $o$

# Remarks

- Further constraints can be added to the LP to obtain stronger heuristics.
- The hard work is done by the LP solver.

# Summary

# Summary



- Declarative method for obtaining a heuristic
- Potential heuristics are linear combinations of features.
- Needed: features and weights (potentials)
- Features: facts (for us; can be generalized)
- Potentials: computed by solving an LP, given constraints that encode goal-awareness and consistency, and an objective function to maximize heuristic value.
- Necessary prerequisite: without loss of generality, task is in transition normal form (same variables in preconditions and effects, all variables mentioned in the goal).

# Credit



Slides heavily based on those by Gabriele Röger and Thomas Keller (Uni Basel).