

Principles of AI Planning

9. Interlude: Finite-domain representation

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December 6th, 2019



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Summary

- When we as humans reason about planning tasks, we implicitly make use of “obvious” properties of these tasks.
 - **Example:** we are never in two places at the same time
- We can express this as a logical formula φ that is **true in all reachable states**.
 - **Example:** $\varphi = \neg(at\text{-}uni \wedge at\text{-}home)$
- Such formulae are called **invariants** of the task.

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How does an **automated** planner come up with invariants?

- Theoretically, testing if an arbitrary formula φ is an invariant is **as hard as planning** itself.
- Still, many practical invariant synthesis algorithms exist.
- To remain efficient (= polynomial-time), these algorithms only compute a **subset** of all useful invariants.
- Empirically, they tend to at least find the “obvious” invariants of a planning task.

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Most algorithms for generating invariants are based on a **generate-test-repair** paradigm:

- **Generate:** Suggest some invariant candidates, e. g., by enumerating all possible formulas φ of a certain size.
- **Test:** Try to prove that φ is indeed an invariant. Usually done **inductively**:
 - 1 Test that **initial state** satisfies φ .
 - 2 Test that if φ is true in the current state, it remains true after applying a single operator.
- **Repair:** If invariant test fails, replace candidate φ by a **weaker** formula, ideally exploiting **why** the proof failed.

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We discussed invariant synthesis in detail in previous courses on AI planning, but this year we will focus on other aspects of planning.

Literature on invariant synthesis:

- DISCOPLAN (Gerevini & Schubert, 1998)
- TIM (Fox & Long, 1998)
- Edelkamp & Helmert's algorithm (1999)
- Rintanen's algorithm (2000)
- Bonet & Geffner's algorithm (2001)
- Helmert's algorithm (2009)

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Invariants have many uses in planning:

- **Regression search:**
Prune states that violate (are inconsistent with) invariants.
- **Planning as satisfiability:**
Add invariants to a SAT encoding of a planning task to get tighter constraints.
- **Reformulation:**
Derive a more compact state space representation (i. e., with lower percentage of unreachable states).

We now briefly discuss the last point, since it leads to **planning tasks in finite-domain representation**, which are very important for the next chapters.

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Planning tasks in finite-domain representation

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Invariants that take the form of **binary clauses** are called **mutexes** because they state that certain variable assignments cannot be simultaneously true and are hence **mutually exclusive**.

Example (Blocksworld)

The invariant $\neg A\text{-on-}B \vee \neg A\text{-on-}C$ states that $A\text{-on-}B$ and $A\text{-on-}C$ are mutex.

Often, a larger **set of literals** is mutually exclusive because every pair of them forms a mutex.

Example (Blocksworld)

Every pair in $\{B\text{-on-}A, C\text{-on-}A, D\text{-on-}A, A\text{-clear}\}$ is mutex.

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Encoding mutex groups as finite-domain variables



Let $L = \{l_1, \dots, l_n\}$ be mutually exclusive literals over n different variables $A_L = \{a_1, \dots, a_n\}$.

Then the planning task can be rephrased using a single **finite-domain** (i.e., non-binary) state variable v_L with $n + 1$ possible values in place of the n variables in A_L :

- n of the possible values represent situations in which **exactly one** of the literals in L is true.
- The remaining value represents situations in which **none of the literals** in L is true.
 - **Note:** If we can prove that one of the literals in L has to be true in each state, this additional value can be omitted.

In many cases, the reduction in the number of variables can dramatically improve performance of a planning algorithm.

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Definition (finite-domain state variable)

A **finite-domain state variable** is a symbol v with an associated **finite domain**, i. e., a non-empty finite set.

We write \mathcal{D}_v for the domain of v .

Example

$v = \textit{above-a}$, $\mathcal{D}_{\textit{above-a}} = \{\mathbf{b}, \mathbf{c}, \mathbf{d}, \textit{nothing}\}$

This state variable encodes the same information as the propositional variables $\textit{B-on-A}$, $\textit{C-on-A}$, $\textit{D-on-A}$ and $\textit{A-clear}$.

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Definition (finite-domain state)

Let V be a finite set of finite-domain state variables.

A **state** over V is an assignment $s : V \rightarrow \bigcup_{v \in V} \mathcal{D}_v$ such that $s(v) \in \mathcal{D}_v$ for all $v \in V$.

Example

$$s = \{ \textit{above-a} \mapsto \textit{nothing}, \textit{above-b} \mapsto \textit{a}, \textit{above-c} \mapsto \textit{b}, \\ \textit{below-a} \mapsto \textit{b}, \textit{below-b} \mapsto \textit{c}, \textit{below-c} \mapsto \textit{table} \}$$

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Definition (finite-domain formulae)

Logical formulae over finite-domain state variables V are defined as in the propositional case, except that instead of atomic formulae of the form $a \in A$, there are atomic formulae of the form $v = d$, where $v \in V$ and $d \in \mathcal{D}_v$.

Example

The formula $(above-a = nothing) \vee \neg(below-b = c)$ corresponds to the formula $A-clear \vee \neg B-on-C$.

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Definition (finite-domain effects)

Effects over finite-domain state variables V are defined as in the propositional case, except that instead of atomic effects of the form a and $\neg a$ with $a \in A$, there are atomic effects of the form $v := d$, where $v \in V$ and $d \in \mathcal{D}_v$.

Example

The effect

$(below-a := table) \wedge ((above-b = a) \triangleright (above-b := nothing))$

corresponds to the effect

$A-on-T \wedge \neg A-on-B \wedge \neg A-on-C \wedge \neg A-on-D \wedge (A-on-B \triangleright (B-clear \wedge \neg A-on-B \wedge \neg C-on-B \wedge \neg D-on-B))$.

\rightsquigarrow definition of **finite-domain operators** follows from this

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Definition (planning task in finite-domain representation)

A **deterministic planning task in finite-domain representation** or **FDR planning task** is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of **finite-domain state variables**,
- I is an **initial state** over V ,
- O is a finite set of **finite-domain operators** over V , and
- γ is a formula over V describing the **goal states**.

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Definition (induced propositional planning task)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an FDR planning task.

The **induced propositional planning task** Π' is the (regular) planning task $\Pi' = \langle A', I', O', \gamma' \rangle$, where

- $A' = \{(v, d) \mid v \in V, d \in \mathcal{D}_v\}$
- $I'((v, d)) = 1$ iff $I(v) = d$
- O' and γ' are obtained from O and γ by replacing
 - each atomic formula $v = d$ with the proposition (v, d) ,
 - each atomic effect $v := d$ with the effect $(v, d) \wedge \bigwedge_{d' \in \mathcal{D}_v \setminus \{d\}} \neg(v, d')$.
- \rightsquigarrow can define operator semantics, plans, relaxed planning graphs, ... for Π in terms of its induced propositional planning task

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Definition (SAS⁺ planning task)

An FDR planning task $\Pi = \langle V, I, O, \gamma \rangle$ is called an **SAS⁺ planning task** iff there are no conditional effects in O and all operator preconditions in O and the goal formula γ are conjunctions of atoms.

- analogue of STRIPS planning tasks for finite-domain representations
- induced propositional planning task of a SAS⁺ planning task is STRIPS
- FDR tasks obtained by invariant-based reformulation of STRIPS planning task are SAS⁺

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Summary



- **Invariants** are common properties of all reachable states, expressed as logical formulas.
- A number of algorithms for **computing invariants** exist.
- These algorithms will not find **all useful invariants** (which is too hard), but try to find some useful subset within reasonable (polynomial) time.
- **Mutexes** are invariants that express that certain pairs of state variable assignments are mutually exclusive.
- Groups of mutexes can be used for **problem reformulation**, transforming a planning task into **finite-domain representation (FDR)**.
- Many planning algorithms are more efficient when working on these FDR tasks (rather than the original tasks) because they contain **fewer unreachable states**.

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