

# Principles of AI Planning

## 8. Planning as search: relaxation heuristics

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# Parallel plans

## Parallel plans

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## Relaxed planning graphs

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Summary

Why does the greedy algorithm compute low-quality plans?

- It may apply many operators which are not **goal-directed**.

How can this problem be fixed?

- **Reaching the goal** of a relaxed planning task is most easily achieved with **forward search**.
- Analyzing **relevance** of an operator for achieving a goal (or subgoal) is most easily achieved with **backward search**.

**Idea:** Use a **forward-backward** algorithm that first finds a path to the goal greedily, then prunes it to a relevant subplan.

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How to decide which operators to apply in forward direction?

- We **avoid** such a decision by applying all applicable operators **simultaneously**.

## Definition (plan step)

A **plan step** is a set of operators  $\omega = \{\langle \chi_1, e_1 \rangle, \dots, \langle \chi_n, e_n \rangle\}$ .

In the **special case of all operators of  $\omega$  being relaxed**, we further define:

- Plan step  $\omega$  is **applicable** in state  $s$  iff  $s \models \chi_i$  for all  $i \in \{1, \dots, n\}$ .
- The **result** of applying  $\omega$  to  $s$ , in symbols  $app_\omega(s)$ , is defined as the state  $s'$  with  $on(s') = on(s) \cup \bigcup_{i=1}^n [e_i]_s$ .

**general** semantics for plan steps  $\rightsquigarrow$  much later

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In all cases,  $s = \{a \mapsto 0, b \mapsto 0, c \mapsto 1, d \mapsto 0\}$ .

- $\omega = \{\langle c, a \rangle, \langle \top, b \rangle\}$
- $\omega = \{\langle c, a \rangle, \langle c, a \triangleright b \rangle\}$
- $\omega = \{\langle c, a \wedge b \rangle, \langle a, b \triangleright d \rangle\}$
- $\omega = \{\langle c, a \wedge (b \triangleright d) \rangle, \langle c, b \wedge (a \triangleright d) \rangle\}$

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Applying a relaxed plan step to a state is related to applying the operators in the step to a state in sequence.

## Definition (serialization)

A **serialization** of plan step  $\omega = \{o_1^+, \dots, o_n^+\}$  is a sequence  $o_{\pi(1)}^+, \dots, o_{\pi(n)}^+$  where  $\pi$  is a permutation of  $\{1, \dots, n\}$ .

## Lemma (conservativeness of plan step semantics)

*If  $\omega$  is a plan step applicable in a state  $s$  of a relaxed planning task, then each serialization  $o_1, \dots, o_n$  of  $\omega$  is applicable in  $s$  and  $app_{o_1, \dots, o_n}(s)$  dominates  $app_{\omega}(s)$ .*

- Does equality hold for all/some serialization(s)?
- What if there are no conditional effects?
- What if we allowed general (unrelaxed) planning tasks?

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## Definition (parallel plan)

A **parallel plan** for a relaxed planning task  $\langle A, I, O^+, \gamma \rangle$  is a sequence of plan steps  $\omega_1, \dots, \omega_n$  of operators in  $O^+$  with:

- $s_0 := I$
- For  $i = 1, \dots, n$ , step  $\omega_i$  is applicable in  $s_{i-1}$  and  $s_i := \text{app}_{\omega_i}(s_{i-1})$ .
- $s_n \models \gamma$

**Remark:** By ordering the operators within each single step arbitrarily, we obtain a (regular, non-parallel) plan.

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**Idea:** In the forward phase of the heuristic computation,

- 1 apply plan step with **all operators applicable initially**,
- 2 apply plan step with **all operators applicable then**,
- 3 and so on.

## Definition (forward state/plan step/set)

Let  $\Pi^+ = \langle A, I, O^+, \gamma \rangle$  be a relaxed planning task.

The  **$n$ -th forward state**, in symbols  $s_n^F$  ( $n \in \mathbb{N}_0$ ),  
the  **$n$ -th forward plan step**, in symbols  $\omega_n^F$  ( $n \in \mathbb{N}_1$ ), and  
the  **$n$ -th forward set**, in symbols  $S_n^F$  ( $n \in \mathbb{N}_0$ ), are defined as:

- $s_0^F := I$
- $\omega_n^F := \{o \in O^+ \mid o \text{ applicable in } s_{n-1}^F\}$  for all  $n \in \mathbb{N}_1$
- $s_n^F := \text{app}_{\omega_n^F}(s_{n-1}^F)$  for all  $n \in \mathbb{N}_1$
- $S_n^F := \text{on}(s_n^F)$  for all  $n \in \mathbb{N}_0$

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## Definition (parallel forward distance)

The **parallel forward distance** of a relaxed planning task  $\langle A, I, O^+, \gamma \rangle$  is the lowest number  $n \in \mathbb{N}_0$  such that  $s_n^F \models \gamma$ , or  $\infty$  if no forward state satisfies  $\gamma$ .

**Remark:** The parallel forward distance can be computed in polynomial time. (How?)

## Definition (max heuristic $h_{\max}$ )

Let  $\Pi = \langle A, I, O, \gamma \rangle$  be a planning task in positive normal form, and let  $s$  be a state of  $\Pi$ .

The **max heuristic** estimate for  $s$ ,  $h_{\max}(s)$ , is the parallel forward distance of the relaxed planning task  $\langle A, s, O^+, \gamma \rangle$ .

**Remark:**  $h_{\max}$  is safe, goal-aware, admissible and consistent. (Why?)

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- We have seen how systematic computation of forward states leads to an admissible heuristic estimate.
- However, this estimate is **very coarse**.
- To improve it, we need to include **backward propagation** of information.

For this purpose, we use so-called **relaxed planning graphs**.

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## Definition (AND/OR dag)

An **AND/OR dag**  $\langle V, A, type \rangle$  is a directed acyclic graph  $\langle V, A \rangle$  with a label function  $type : V \rightarrow \{\wedge, \vee\}$  partitioning nodes into **AND nodes** ( $type(v) = \wedge$ ) and **OR nodes** ( $type(v) = \vee$ ).

**Note:** AND nodes drawn as squares, OR nodes as circles.

## Definition (truth values in AND/OR dags)

Let  $G = \langle V, A, type \rangle$  be an AND/OR dag, and let  $u \in V$  be a node with successor set  $\{v_1, \dots, v_k\} \subseteq V$ .

The (truth) **value** of  $u$ ,  $val(u)$ , is inductively defined as:

- If  $type(u) = \wedge$ , then  $val(u) = val(v_1) \wedge \dots \wedge val(v_k)$ .
- If  $type(u) = \vee$ , then  $val(u) = val(v_1) \vee \dots \vee val(v_k)$ .

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Let  $\Pi^+$  be a relaxed planning task, and let  $k \in \mathbb{N}_0$ .

The **relaxed planning graph** of  $\Pi^+$  for depth  $k$ , in symbols  $RPG_k(\Pi^+)$ , is an AND/OR dag that encodes

- **which propositions** can be made true in  $k$  plan steps, and
- **how** they can be made true.

Its construction is a bit involved, so we present it in stages.

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As a running example, consider the relaxed planning task  $\langle A, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$  with

$$A = \{a, b, c, d, e, f, g, h\}$$

$$I = \{a \mapsto 1, b \mapsto 0, c \mapsto 1, d \mapsto 1, \\ e \mapsto 0, f \mapsto 0, g \mapsto 0, h \mapsto 0\}$$

$$o_1 = \langle b \vee (c \wedge d), b \wedge ((a \wedge b) \triangleright e) \rangle$$

$$o_2 = \langle \top, f \rangle$$

$$o_3 = \langle f, g \rangle$$

$$o_4 = \langle f, h \rangle$$

$$\gamma = e \wedge (g \wedge h)$$

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$$I = \{a \mapsto 1, b \mapsto 0, c \mapsto 1, d \mapsto 1, e \mapsto 0, f \mapsto 0, g \mapsto 0, h \mapsto 0\}$$
$$o_1 = \langle b \vee (c \wedge d), b \wedge ((a \wedge b) \triangleright e) \rangle$$
$$o_2 = \langle \top, f \rangle, \quad o_3 = \langle f, g \rangle, \quad o_4 = \langle f, h \rangle$$

$$S_0^F = \{a, c, d\}$$

$$\omega_1^F = \{o_1, o_2\}$$

$$S_1^F = \{a, b, c, d, f\}$$

$$\omega_2^F = \{o_1, o_2, o_3, o_4\}$$

$$S_2^F = \{a, b, c, d, e, f, g, h\}$$

$$\omega_3^F = \omega_2^F$$

$$S_3^F = S_2^F \text{ etc.}$$

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A relaxed planning graph consists of four kinds of components:

- **Proposition nodes** represent the truth value of propositions after applying a certain number of plan steps.
- **Idle arcs** represent the fact that state variables, once true, remain true.
- **Operator subgraphs** represent the possibility and effect of applying a given operator in a given plan step.
- The **goal subgraph** represents the truth value of the goal condition after  $k$  plan steps.

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Let  $\Pi^+ = \langle A, I, O^+, \gamma \rangle$  be a relaxed planning task, let  $k \in \mathbb{N}_0$ .

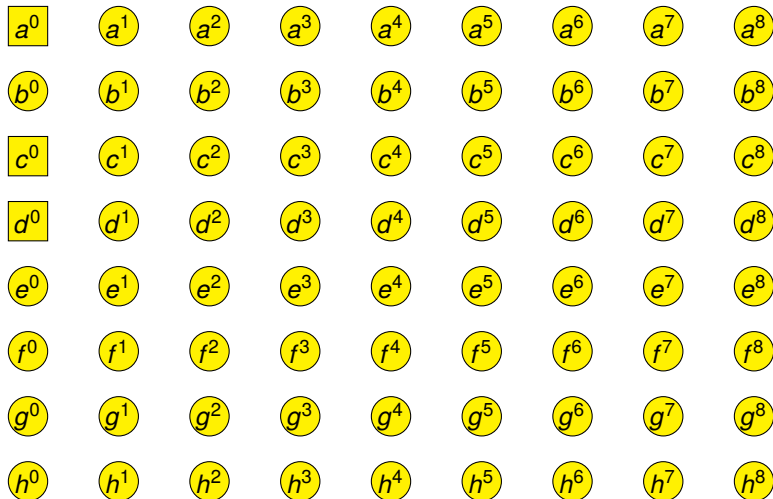
For each  $i \in \{0, \dots, k\}$ ,  $RPG_k(\Pi^+)$  contains one **proposition layer** which consists of:

- a **proposition node**  $a^i$  for each state variable  $a \in A$ .

Node  $a^i$  is an AND node if  $i = 0$  and  $I \models a$ .

Otherwise, it is an OR node.

# Relaxed planning graph: proposition layers



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For each proposition node  $a^i$  with  $i \in \{1, \dots, k\}$ ,  $RPG_k(\Pi^+)$  contains an arc from  $a^i$  to  $a^{i-1}$  (**idle arcs**).

**Intuition:** If a state variable is true in step  $i$ , one of the possible reasons is that it **was already previously true**.

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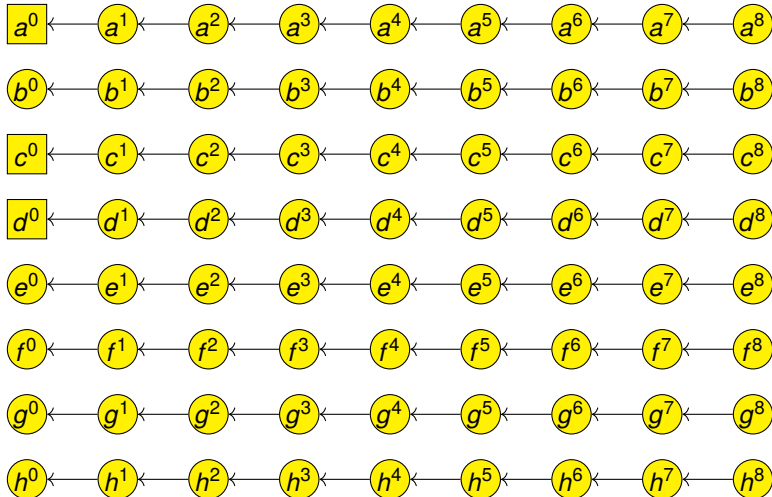
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# Relaxed planning graph: idle arcs



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For each  $i \in \{1, \dots, k\}$  and each operator  $o^+ = \langle \chi, e^+ \rangle \in O^+$ ,  $RPG_k(\Pi^+)$  contains a subgraph called an **operator subgraph** with the following parts:

- one **formula node**  $n_\varphi^i$  for each formula  $\varphi$  which is a subformula of  $\chi$  or of some effect condition in  $e^+$ :
  - If  $\varphi = a$  for some atom  $a$ ,  $n_\varphi^i$  is the proposition node  $a^{i-1}$ .
  - If  $\varphi = \top$ ,  $n_\varphi^i$  is a new AND node without outgoing arcs.
  - If  $\varphi = \perp$ ,  $n_\varphi^i$  is a new OR node without outgoing arcs.
  - If  $\varphi = (\varphi' \wedge \varphi'')$ ,  $n_\varphi^i$  is a new AND node with outgoing arcs to  $n_{\varphi'}^i$  and  $n_{\varphi''}^i$ .
  - If  $\varphi = (\varphi' \vee \varphi'')$ ,  $n_\varphi^i$  is a new OR node with outgoing arcs to  $n_{\varphi'}^i$  and  $n_{\varphi''}^i$ .

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For each  $i \in \{1, \dots, k\}$  and each operator  $o^+ = \langle \chi, e^+ \rangle \in O^+$ ,  $RPG_k(\Pi^+)$  contains a subgraph called an **operator subgraph** with the following parts:

- for each conditional effect  $(\chi' \triangleright a)$  in  $e^+$ , an **effect node**  $o^i_{\chi'}$  (an AND node) with outgoing arcs to the precondition formula node  $n^i_{\chi}$  and effect condition formula node  $n^i_{\chi'}$ , and incoming arc from proposition node  $a^i$ 
  - unconditional effects  $a$  (effects which are not part of a conditional effect) are treated the same, except that there is no arc to an effect condition formula node
  - effects with identical condition (including groups of unconditional effects) share the same effect node
  - the effect node for unconditional effects is denoted by  $o^i$

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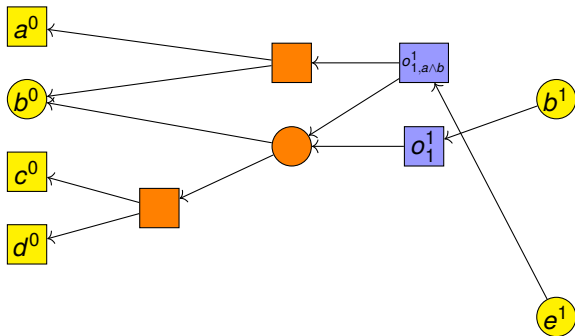
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# Relaxed planning graph: operator subgraphs



Operator subgraph for  $o_1 = \langle b \vee (c \wedge d), b \wedge ((a \wedge b) \triangleright e) \rangle$   
for layer  $i = 1$ .



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$RPG_k(\Pi^+)$  contains a subgraph called a **goal subgraph** with the following parts:

- one **formula node**  $n_\varphi^k$  for each formula  $\varphi$  which is a subformula of  $\gamma$ :
  - If  $\varphi = a$  for some atom  $a$ ,  $n_\varphi^k$  is the proposition node  $a^i$ .
  - If  $\varphi = \top$ ,  $n_\varphi^k$  is a new AND node without outgoing arcs.
  - If  $\varphi = \perp$ ,  $n_\varphi^k$  is a new OR node without outgoing arcs.
  - If  $\varphi = (\varphi' \wedge \varphi'')$ ,  $n_\varphi^k$  is a new AND node with outgoing arcs to  $n_{\varphi'}^k$  and  $n_{\varphi''}^k$ .
  - If  $\varphi = (\varphi' \vee \varphi'')$ ,  $n_\varphi^k$  is a new OR node with outgoing arcs to  $n_{\varphi'}^k$  and  $n_{\varphi''}^k$ .

The node  $n_\gamma^k$  is called the **goal node**.

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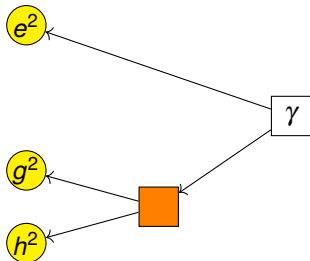
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Goal subgraph for  $\gamma = e \wedge (g \wedge h)$  and depth  $k = 2$ :



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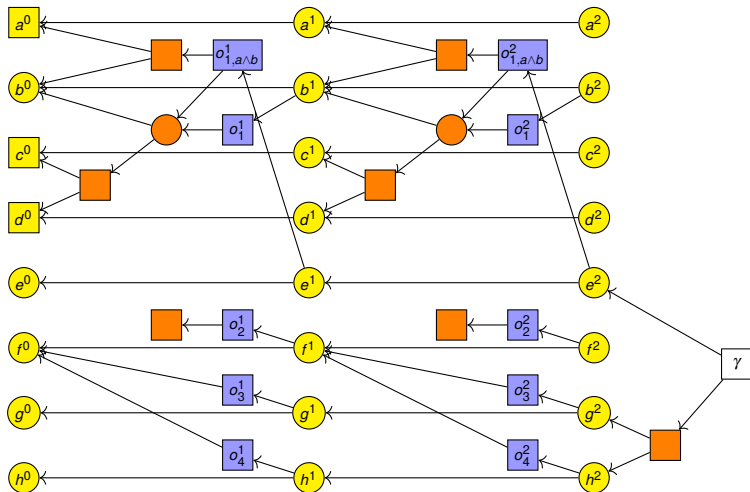
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# Relaxed planning graph: complete (depth 2)



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## Theorem (relaxed planning graph truth values)

Let  $\Pi^+ = \langle A, I, O^+, \gamma \rangle$  be a relaxed planning task.

Then the truth values of the nodes of its depth- $k$  relaxed planning graph  $RPG_k(\Pi^+)$  relate to the forward sets and forward plan steps of  $\Pi^+$  as follows:

- **Proposition nodes:**

For all  $a \in A$  and  $i \in \{0, \dots, k\}$ ,  $val(a^i) = 1$  iff  $a \in S_i^F$ .

- **(Unconditional) effect nodes:**

For all  $o \in O^+$  and  $i \in \{1, \dots, k\}$ ,  $val(o^i) = 1$  iff  $o \in \omega_i^F$ .

- **Goal nodes:**

$val(n_\gamma^k) = 1$  iff the parallel forward distance of  $\Pi^+$  is at most  $k$ .

(We omit the straight-forward proof.)

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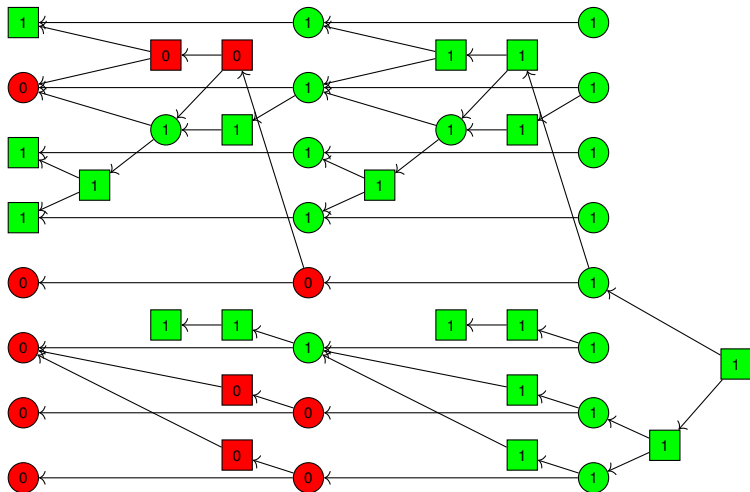
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# Computing the node truth values



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**Remark:** Relaxed planning graphs have historically been defined for STRIPS tasks only. In this case, we can simplify:

- **Only one effect node per operator:** STRIPS does not have conditional effects.
  - Because each operator has only one effect node, effect nodes are called **operator nodes** in relaxed planning graphs for STRIPS.
- **No goal nodes:** The test whether all goals are reached is done by the algorithm that evaluates the AND/OR dag.
- **No formula nodes:** Operator nodes are directly connected to their preconditions.

↪ Relaxed planning graphs for STRIPS are **layered** digraphs and only have **proposition and operator nodes**.

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$h_{add}$

$h_{sa}$

Incremental  
computation

$h_{FF}$

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So far, relaxed planning graphs offer us a way to compute parallel forward distances:

## Parallel forward distances from relaxed planning graphs

**def** *parallel-forward-distance*( $\Pi^+$ ):

Let  $A$  be the set of state variables of  $\Pi^+$ .

**for**  $k \in \{0, 1, 2, \dots\}$ :

$rpg := RPG_k(\Pi^+)$

Evaluate truth values for  $rpg$ .

**if** goal node of  $rpg$  has value 1:

**return**  $k$

**else if**  $k = |A|$ :

**return**  $\infty$

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- The relaxed planning graph for depth  $k \geq 1$  can be built **incrementally** from the one for depth  $k - 1$ :
  - Add new layer  $k$ .
  - Move goal subgraph from layer  $k - 1$  to layer  $k$ .
- Similarly, all truth values up to layer  $k - 1$  can be reused.
- Thus, overall computation with maximal depth  $m$  requires time  $O(\|RPG_m(\Pi^+)\|) = O((m + 1) \cdot \|\Pi^+\|)$ .
- This is not a very efficient way of computing parallel forward distances (and wouldn't be used in practice).
- However, it allows computing **additional information** for the relaxed planning graph nodes along the way, which can be used for heuristic estimates.

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## Computing heuristics from relaxed planning graphs

**def** *generic-rpg-heuristic*( $\langle A, I, O, \gamma \rangle, s$ ):

$\Pi^+ := \langle A, s, O^+, \gamma \rangle$

**for**  $k \in \{0, 1, 2, \dots\}$ :

$rpg := RPG_k(\Pi^+)$

Evaluate truth values for *rpg*.

**if** goal node of *rpg* has value 1:

    Annotate true nodes of *rpg*.

**if** termination criterion is true:

**return** heuristic value from annotations

**else if**  $k = |A|$ :

**return**  $\infty$

$\rightsquigarrow$  **generic template** for heuristic functions

$\rightsquigarrow$  to get **concrete heuristic**: fill in **highlighted parts**

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Many planning heuristics fit the generic template:

- **additive heuristic**  $h_{\text{add}}$  (Bonet, Loerincs & Geffner, 1997)
- **max heuristic**  $h_{\text{max}}$  (Bonet & Geffner, 1999)
- **FF heuristic**  $h_{\text{FF}}$  (Hoffmann & Nebel, 2001)
- **cost-sharing heuristic**  $h_{\text{CS}}$  (Mirkis & Domshlak, 2007)
  - not covered in this course
- **set-additive heuristic**  $h_{\text{sa}}$  (Keyder & Geffner, 2008)

## Remarks:

- For all these heuristics, equivalent definitions that don't refer to relaxed planning graphs are possible.
- Historically, such equivalent definitions have mostly been used for  $h_{\text{max}}$ ,  $h_{\text{add}}$  and  $h_{\text{sa}}$ .
- For those heuristics, the most efficient implementations do not use relaxed planning graphs explicitly.

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- The simplest relaxed planning graph heuristics are **forward cost heuristics**.
- Examples:  $h_{\max}$ ,  $h_{\text{add}}$
- Here, node annotations are **cost values** (natural numbers).
- The cost of a node estimates how expensive (in terms of required operators) it is to make this node true.

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## Forward cost heuristics

### Computing annotations:

- Propagate cost values bottom-up using a **combination rules for OR nodes and for AND nodes**.
- At **effect nodes**, **add 1** after applying combination rule.

### Termination criterion:

- **stability**: terminate if cost for proposition node  $a^k$  equals cost for  $a^{k-1}$  for all true propositions  $a$  in layer  $k$  (and true propositions in layers  $k$  and  $k - 1$  are the same)

### Heuristic value:

- The heuristic value is the cost of the goal node.
- Different forward cost heuristics only differ in their choice of combination rules.

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# The max heuristic $h_{\max}$ (again)



## Forward cost heuristics: max heuristic $h_{\max}$

### Combination rule for AND nodes:

- $cost(u) = \max(\{cost(v_1), \dots, cost(v_k)\})$   
(with  $\max(\emptyset) := 0$ )

### Combination rule for OR nodes:

- $cost(u) = \min(\{cost(v_1), \dots, cost(v_k)\})$

In both cases,  $\{v_1, \dots, v_k\}$  is the set of true successors of  $u$ .

### Intuition:

- **AND rule:** If we have to achieve several conditions, estimate this by the **most expensive** cost.
- **OR rule:** If we have a choice how to achieve a condition, pick the **cheapest** possibility.

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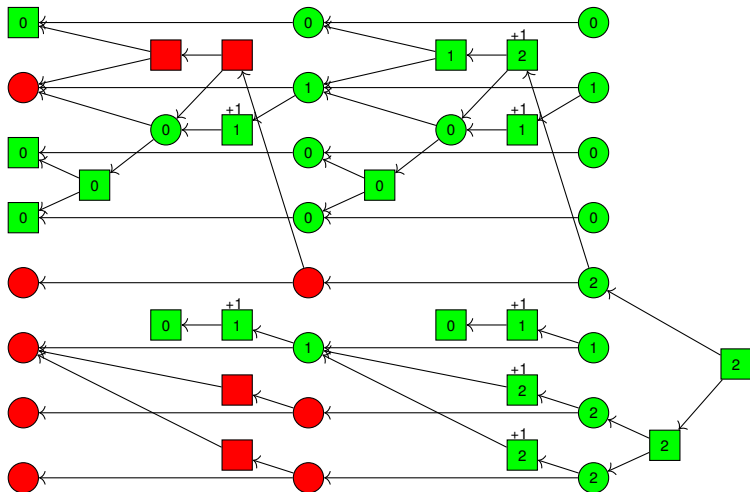
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# Running example: $h_{\max}$



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- The definition of  $h_{\max}$  as a forward cost heuristic is equivalent to our earlier definition in this chapter.
- Unlike the earlier definition, it generalizes to an extension where every operator has an associated non-negative **cost** (rather than all operators having cost 1).
- In the case without costs (and only then), it is easy to prove that the goal node has the same cost in all graphs  $RPG_k(\Pi^+)$  where it is true. (Namely, the cost is equal to the lowest value of  $k$  for which the goal node is true.)
- We can thus terminate the computation as soon as the goal becomes true, without waiting for stability.
- The same is **not true** for other forward-propagating heuristics ( $h_{\text{add}}$ ,  $h_{\text{CS}}$ ,  $h_{\text{sa}}$ ).

Parallel plans

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Generic template

$h_{\max}$

$h_{\text{add}}$

$h_{\text{sa}}$

Incremental computation

$h_{\text{FF}}$

Comparison & practice

Summary

Forward cost heuristics: additive heuristic  $h_{\text{add}}$

Combination rule for AND nodes:

- $cost(u) = cost(v_1) + \dots + cost(v_k)$   
(with  $\sum(\emptyset) := 0$ )

Combination rule for OR nodes:

- $cost(u) = \min(\{cost(v_1), \dots, cost(v_k)\})$

In both cases,  $\{v_1, \dots, v_k\}$  is the set of true successors of  $u$ .

Intuition:

- **AND rule:** If we have to achieve several conditions, estimate this by the cost of achieving **each in isolation**.
- **OR rule:** If we have a choice how to achieve a condition, pick the **cheapest** possibility.

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Generic template

$h_{\text{max}}$

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Incremental computation

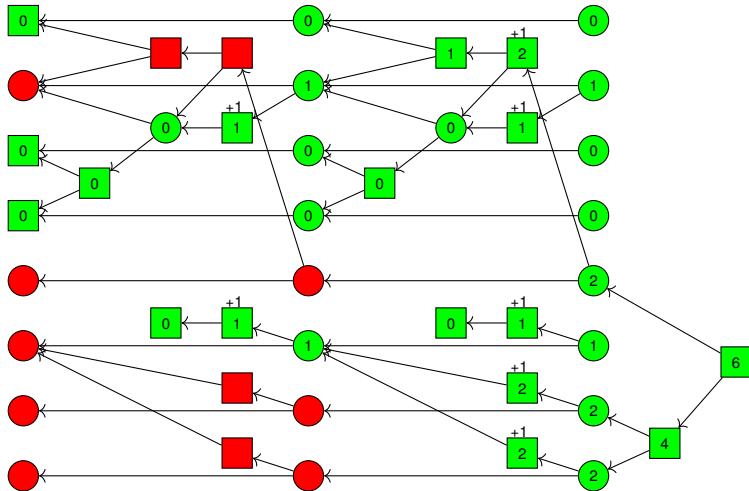
$h_{\text{FF}}$

Comparison & practice

Summary



# Running example: $h_{\text{add}}$



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- It is important to test for stability in computing  $h_{\text{add}}$ !  
(The reason for this is that, unlike  $h_{\text{max}}$ , cost values of true propositions can **decrease** from layer to layer.)
- Stability is achieved after layer  $|A|$  in the worst case.
- $h_{\text{add}}$  is **safe** and **goal-aware**.
- Unlike  $h_{\text{max}}$ ,  $h_{\text{add}}$  is a **very informative** heuristic in many planning domains.
- The price for this is that it is **not admissible** (and hence also **not consistent**), so not suitable for optimal planning.
- In fact, it **almost always** overestimates the  $h^+$  value because it does not take **positive interactions** into account.

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- We now discuss a refinement of the additive heuristic called the **set-additive heuristic**  $h_{sa}$ .
- The set-additive heuristic addresses the problem that  $h_{add}$  does not take positive interactions into account.
- Like  $h_{max}$  and  $h_{add}$ ,  $h_{sa}$  is calculated through **forward propagation** of node annotations.
- However, the node annotations are not cost values, but **sets of operators** (kind of).
- The idea is that by taking **set unions** instead of **adding costs**, operators needed only once are **counted only once**.

**Disclaimer:** There are some quite subtle differences between the  $h_{sa}$  heuristic as we describe it here and the “real” heuristic of Keyder & Geffner. We do not want to discuss this in detail, but please note that such differences exist.

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Summary

# Operators needed several times



- The original  $h_{sa}$  heuristic as described in the literature is defined for STRIPS tasks and propagates **sets of operators**.
- This is fine because in relaxed STRIPS tasks, each operator **need only be applied once**.
- The same is **not true in general**: in our running example, operator  $o_1$  must be applied twice in the relaxed plan.
- In general, it only makes sense to apply an operator again in a relaxed planning task if a **previously unsatisfied effect condition** has been made true.
- For this reason, we keep track of **operator/effect condition pairs** rather than just plain operators.

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## The set-additive heuristic $h_{sa}$

### Computing annotations:

- Annotations are **sets of operator/effect condition pairs**, computed bottom-up.

### Combination rule for AND nodes:

- $ann(u) = ann(v_1) \cup \dots \cup ann(v_k)$  (with  $\cup(\emptyset) := \emptyset$ )

### Combination rule for OR nodes:

- $ann(u) = ann(v_i)$  for some  $v_i$  minimizing  $|ann(v_i)|$

In case of several minimizers, use any tie-breaking rule.

In both cases,  $\{v_1, \dots, v_k\}$  is the set of true successors of  $u$ . At **effect nodes**, **add the corresponding operator/effect condition pair** to the set after applying combination rule.

...

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Summary

# Set-additive heuristic: fitting the template (ctd.)



## The set-additive heuristic $h_{sa}$ (ctd.)

### Computing annotations:

- ... (Effect nodes for unconditional effects are represented just by the operator, without a condition.)

### Termination criterion:

- **stability**: terminate if set for proposition node  $a^k$  has same cardinality as for  $a^{k-1}$  for all true propositions  $a$  in layer  $k$  (and true propositions in layers  $k$  and  $k - 1$  are the same)

### Heuristic value:

- The heuristic value is the **set cardinality** of the goal node annotation.

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$h_{max}$

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$h_{sa}$

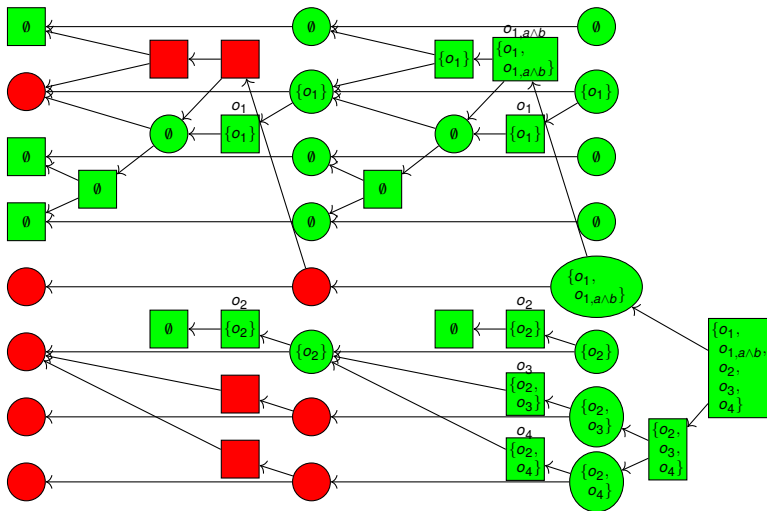
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Summary

# Running example: $h_{sa}$



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- The same remarks for stability as for  $h_{add}$  apply.
- Like  $h_{add}$ ,  $h_{sa}$  is **safe** and **goal-aware**, but neither **admissible** nor **consistent**.
- $h_{sa}$  is generally **better informed** than  $h_{add}$ , but significantly more expensive to compute.
- The  $h_{sa}$  value depends on the tie-breaking rule used, so  $h_{sa}$  is **not well-defined** without specifying the tie-breaking rule.
- The operators contained in the goal node annotation, suitably ordered, define a **relaxed plan** for the task.
  - Operators mentioned several times in the annotation must be added as many times in the relaxed plan.

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# Incremental computation of forward heuristics



One nice property of forward-propagating heuristics is that they allow **incremental computation**:

- when evaluating several states in sequence which only differ in a few state variables, can
  - **start computation from previous results** and
  - keep track only of **what needs to be recomputed**
- typical use case: **depth-first** style searches (e. g., IDA\*)
- rarely exploited in practice

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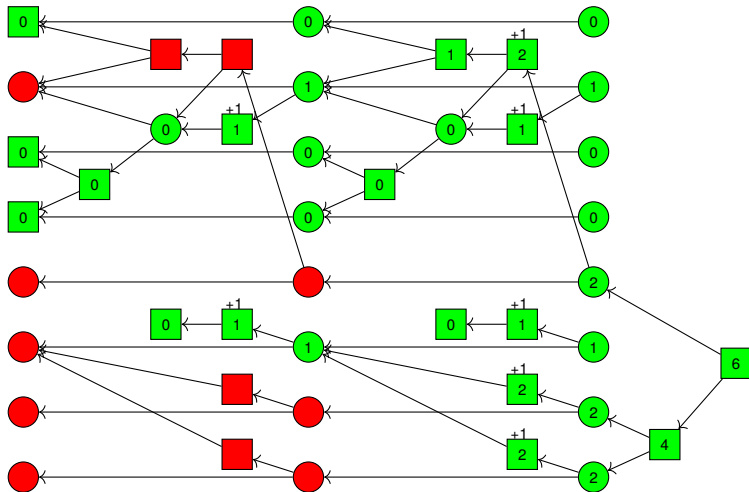
Comparison & practice

Summary

# Incremental computation example: $h_{\text{add}}$



Result for  $\{a \mapsto 1, b \mapsto 0, c \mapsto 1, d \mapsto 1, e \mapsto 0, f \mapsto 0, g \mapsto 0, h \mapsto 0\}$



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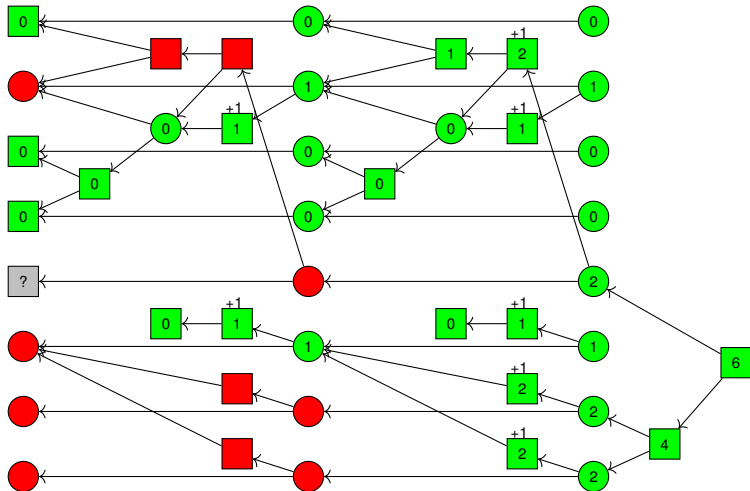
Comparison & practice

Summary

# Incremental computation example: $h_{add}$



Change value of  $e$  to 1.



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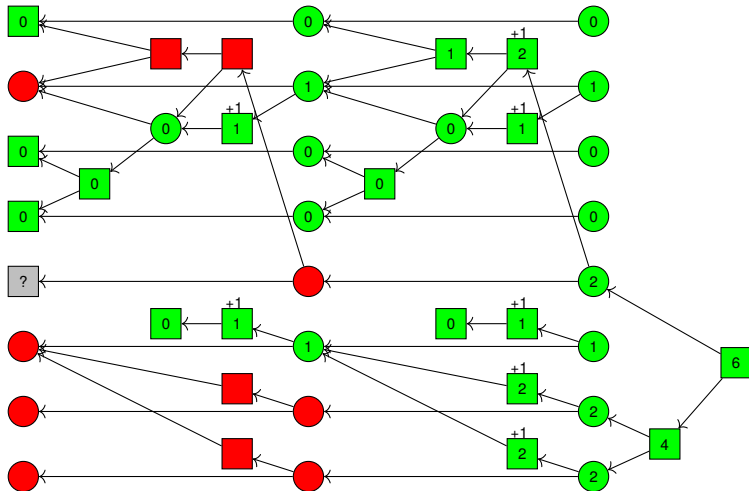
Comparison & practice

Summary

# Incremental computation example: $h_{\text{add}}$



Recompute outdated values.



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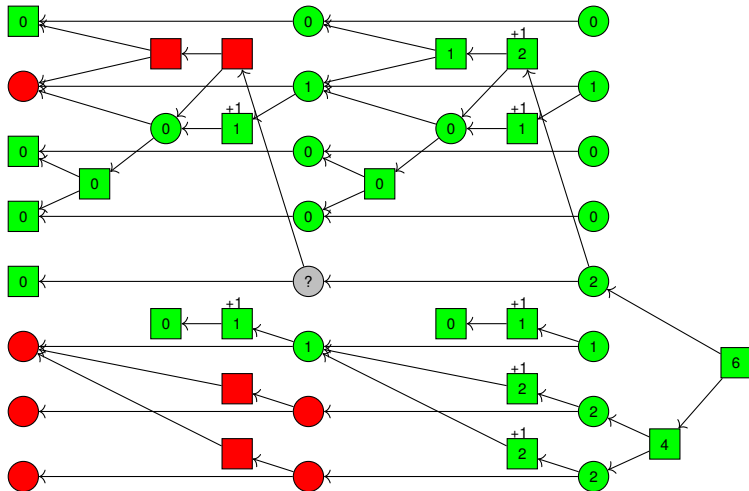
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Summary

# Incremental computation example: $h_{\text{add}}$



Recompute outdated values.



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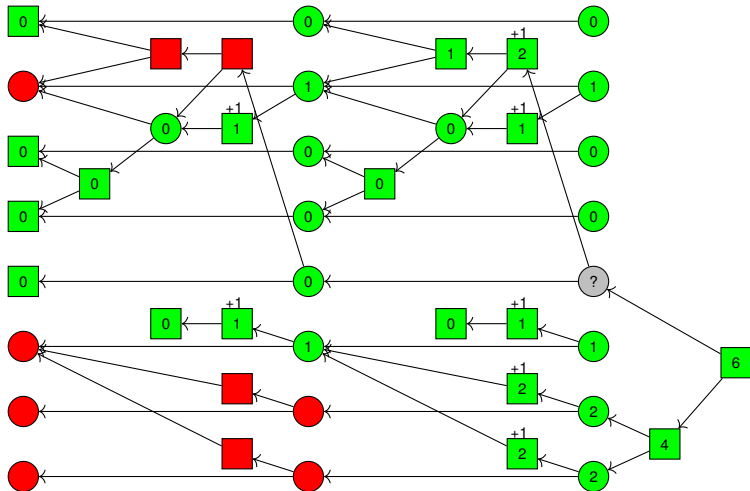
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Summary

# Incremental computation example: $h_{\text{add}}$



Recompute outdated values.



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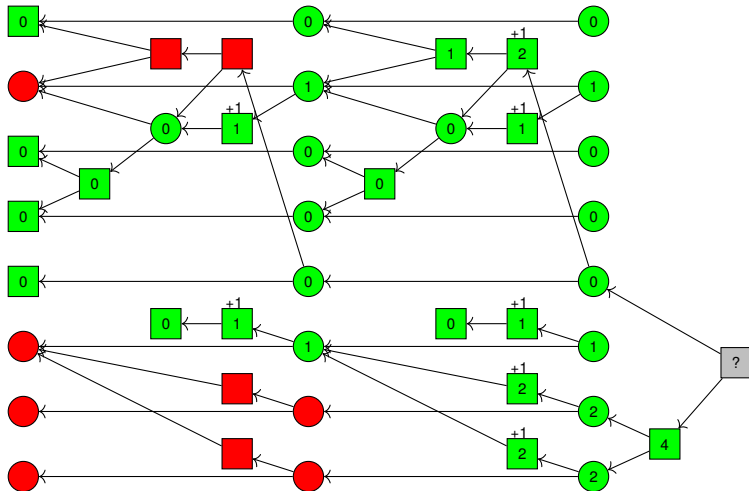
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Summary

# Incremental computation example: $h_{\text{add}}$



Recompute outdated values.



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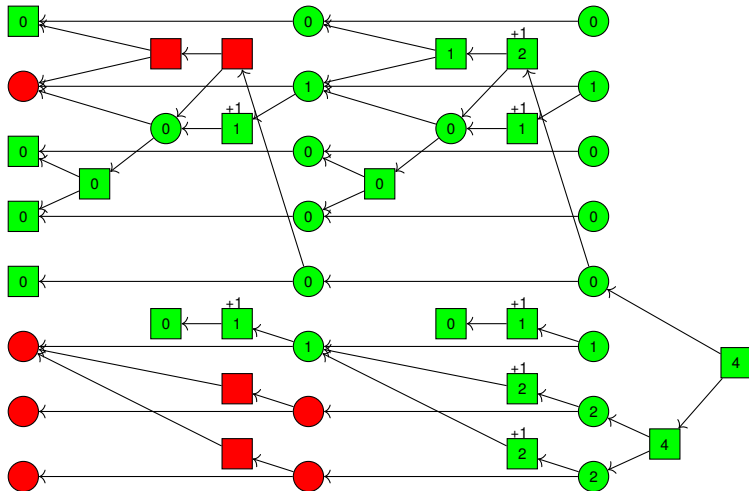
Comparison & practice

Summary

# Incremental computation example: $h_{\text{add}}$



Recompute outdated values.



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Summary





- $h_{sa}$  is more expensive to compute than the other forward propagating heuristics because we must propagate **sets**.
- It is possible to get the same advantage over  $h_{add}$  combined with efficient propagation.
- Key idea of  $h_{FF}$ : perform a **backward propagation** that selects a sufficient subset of nodes to make the goal true (called a **solution graph** in AND/OR dag literature).
- The resulting heuristic is almost as informative as  $h_{sa}$ , yet computable as quickly as  $h_{add}$ .

**Note:** Our presentation inverts the historical order. The set-additive heuristic was defined **after** the FF heuristic (sacrificing speed for even higher informativeness).

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## The FF heuristic $h_{FF}$

### Computing annotations:

- Annotations are **Boolean values**, computed top-down.

A node is **marked** when its annotation is set to 1 and **unmarked** if it is set to 0. Initially, the goal node is marked, and all other nodes are unmarked.

We say that a true AND node is **justified** if all its true successors are marked, and that a true OR node is **justified** if at least one of its true successors is marked.

...

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Summary

## The FF heuristic $h_{FF}$ (ctd.)

### Computing annotations:

■ ...

Apply these rules until **all marked nodes are justified**:

- 1 Mark all true successors of a marked unjustified AND node.
- 2 Mark the true successor of a marked unjustified OR node with only one true successor.
- 3 Mark a true successor of a marked unjustified OR node connected via an idle arc.
- 4 Mark any true successor of a marked unjustified OR node.

The rules are given in priority order: earlier rules are preferred if applicable.

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Incremental computation

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Summary

# FF heuristic: fitting the template (ctd.)



## The FF heuristic $h_{FF}$ (ctd.)

### Termination criterion:

- **Always terminate** at first layer where goal node is true.

### Heuristic value:

- The heuristic value is the **number of operator/effect condition pairs** for which **at least one** effect node is marked.

Parallel plans

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Generic template

$h_{max}$

$h_{add}$

$h_{sa}$

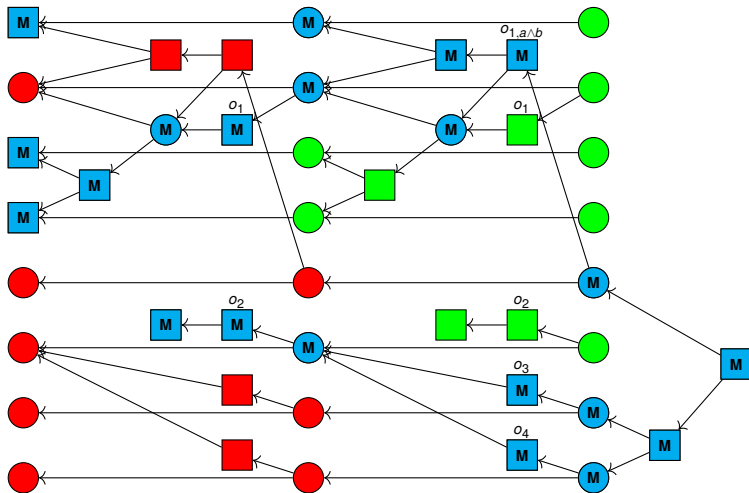
Incremental computation

**$h_{FF}$**

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Summary

# Running example: $h_{FF}$



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Summary



- Like  $h_{add}$  and  $h_{sa}$ ,  $h_{FF}$  is **safe** and **goal-aware**, but neither **admissible** nor **consistent**.
- Its informativeness can be expected to be slightly worse than for  $h_{sa}$ , but is usually not far off.
- Unlike  $h_{sa}$ ,  $h_{FF}$  can be computed in **linear time**.
- Similar to  $h_{sa}$ , the operators corresponding to the marked operator/effect condition pairs define a **relaxed plan**.
- Similar to  $h_{sa}$ , the  $h_{FF}$  value depends on tie-breaking when the marking rules allow several possible choices, so  $h_{FF}$  is **not well-defined** without specifying the tie-breaking rule.
  - The implementation in FF uses additional rules of thumb to try to reduce the size of the generated relaxed plan.

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## Theorem (relationship between relaxation heuristics)

Let  $s$  be a state of planning task  $\langle A, I, O, \gamma \rangle$ . Then:

- $h_{max}(s) \leq h^+(s) \leq h^*(s)$
- $h_{max}(s) \leq h^+(s) \leq h_{sa}(s) \leq h_{add}(s)$
- $h_{max}(s) \leq h^+(s) \leq h_{FF}(s) \leq h_{add}(s)$
- $h^*$ ,  $h_{FF}$  and  $h_{sa}$  are pairwise incomparable
- $h^*$  and  $h_{add}$  are incomparable

Moreover,  $h^+$ ,  $h_{max}$ ,  $h_{add}$ ,  $h_{sa}$  and  $h_{FF}$  assign  $\infty$  to the same set of states.

**Note:** For **inadmissible** heuristics, dominance is in general neither desirable nor undesirable. For relaxation heuristics, the objective is usually to get as close to  $h^+$  as possible.

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## Example (HSP)

**HSP** (Bonet & Geffner) was one of the four top performers at the 1st International Planning Competition (IPC-1998).

Key ideas:

- **hill climbing** search using  $h_{add}$
- on **plateaus**, keep going for a number of iterations, then restart
- use a closed list during exploration of plateaus

**Literature:** Bonet, Loerincs & Geffner (1997), Bonet & Geffner (2001)

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## Example (FF)

FF (Hoffmann & Nebel) won the 2nd International Planning Competition (IPC-2000).

Key ideas:

- **enforced hill-climbing** search using  $h_{FF}$
- **helpful action pruning**: in each search node, only consider successors from operators that add one of the atoms marked in proposition layer 1
- **goal ordering**: in certain cases, FF recognizes and exploits that certain subgoals should be solved one after the other

If main search fails, FF performs greedy best-first search using  $h_{FF}$  without helpful action pruning or goal ordering.

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# Relaxation heuristics in practice: Fast Downward



## Example (Fast Downward)

**Fast Downward** (Helmert & Richter) won the satisficing track of the 4th International Planning Competition (IPC-2004).

Key ideas:

- **greedy best-first search** using  $h_{FF}$  and **causal graph heuristic** (not relaxation-based)
- search enhancements:
  - multi-heuristic best-first search
  - deferred evaluation of heuristic estimates
  - preferred operators (similar to FF's helpful actions)

**Literature:** Helmert (2006)

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Incremental computation

$h_{FF}$

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## Example (SGPlan)

**SGPlan** (Wah, Hsu, Chen & Huang) won the satisficing track of the 5th International Planning Competition (IPC-2006).

Key ideas:

- **FF**
- **problem decomposition** techniques
- **domain-specific techniques**

**Literature:** Chen, Wah & Hsu (2006)

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## Example (LAMA)

**LAMA** (Richter & Westphal) won the satisficing track of the 6th International Planning Competition (IPC-2008).

Key ideas:

- **Fast Downward**
- **landmark pseudo-heuristic** instead of causal graph heuristic (“somewhat” relaxation-based)
- anytime variant of **Weighted A\*** instead of greedy best-first search

**Literature:** Richter, Helmert & Westphal (2008),  
Richter & Westphal (2010)

Parallel  
plans

Relaxed  
planning  
graphs

Relaxation  
heuristics

Generic template

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$h_{add}$

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Summary



- **Relaxed planning graphs** are **AND/OR dags**. They encode which propositions can be made true in  $\Pi^+$  and how.
  - Closely related to **forward sets** and **forward plan steps**, based on the notion of **parallel relaxed plans**.
  - They can be **constructed and evaluated efficiently**, in time  $O((m+1) \|\Pi^+\|)$  for planning task  $\Pi$  and depth  $m$ .
- By annotating RPG nodes with appropriate information, we can compute many useful heuristics.
- Examples: **max** heuristic  $h_{\max}$ , **additive** heuristic  $h_{\text{add}}$ , **set-additive** heuristic  $h_{\text{sa}}$  and **FF** heuristic  $h_{\text{FF}}$ 
  - Of these, only  $h_{\max}$  admissible (but not very accurate).
  - The others are much more informative. The set-additive heuristic is the most sophisticated one.
  - The FF heuristic is often similarly informative. It offers a good trade-off between accuracy and computation time.

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