

Principles of AI Planning

6. Planning as search: search algorithms

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Introduction to search algorithms for planning

- Introduction
- Nodes and states
- Search for planning
- Common procedures
- Uninformed search
- Heuristic search
- Summary

Our plan for the next lectures

Choices to make:

- 1 search direction: progression/regression/both
↪ previous chapter
- 2 search space representation: states/sets of states
↪ previous chapter
- 3 search algorithm: uninformed/heuristic; systematic/local
↪ **this chapter**
- 4 search control: heuristics, pruning techniques
↪ next chapters

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Search

- Search algorithms are used to find solutions (plans) for **transition systems** in general, not just for planning tasks.
- Planning is **one application** of search among many.
- In this chapter, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.
- Most of this is review of material that should be known (details: Russell and Norvig's textbook).

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Search states vs. search nodes



In search, one distinguishes:

- **search states** $s \rightsquigarrow$ states (vertices) of the transition system
- **search nodes** $\sigma \rightsquigarrow$ search states plus information on where/when/how they are encountered during search

What is in a search node?

Different search algorithms store different information in a search node σ , but typical information includes:

- **state**(σ): associated search state
- **parent**(σ): pointer to search node from which σ is reached
- **action**(σ): action leading from $state(parent(\sigma))$ to $state(\sigma)$
- **g**(σ): cost of σ (length of path from the root node)

For the root node, $parent(\sigma)$ and $action(\sigma)$ are undefined.

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Search states vs. planning states



Search states \neq (planning) states:

- **Search states** don't have to correspond to **states** in the planning sense.
 - progression: search states \approx (**planning**) **states**
 - regression: search states \approx **sets of states** (formulae)
- Search algorithms for planning where search states are planning states are called **state-space search** algorithms.
- Strictly speaking, regression is **not** an example of state-space search, although the term is often used loosely.
- However, we will put the emphasis on progression, which is almost always state-space search.

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Required ingredients for search



A general search algorithm can be applied to any transition system for which we can define the following three operations:

- **init**(\cdot): generate the **initial state**
- **is-goal**(s): test if a given state is a **goal state**
- **succ**(s): generate the set of **successor states** of state s , along with the **operators** through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a **search space** (a very similar notion to a transition system).

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Search for planning: progression



Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for progression search

states: all states of Π (assignments to A)

- **init**(\cdot) = I
- **is-goal**(s) = $\begin{cases} \text{true} & \text{if } s \models \gamma \\ \text{false} & \text{otherwise} \end{cases}$
- **succ**(s) = $\{ \langle o, s' \rangle \mid \text{applicable } o \in O, s' = \text{app}_o(s) \}$

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Search for planning: regression



Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for regression search

states: all formulae over A (how many?)

- $\text{init}() = \gamma$
- $\text{is-goal}(\varphi) = \begin{cases} \text{true} & \text{if } I \models \varphi \\ \text{false} & \text{otherwise} \end{cases}$
- $\text{succ}(\varphi) = \{ \langle o, \varphi' \rangle \mid o \in O, \varphi' = \text{regr}_o(\varphi), \varphi' \text{ is satisfiable} \}$
(modified if splitting is used)

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Classification of search algorithms



uninformed search vs. heuristic search:

- **uninformed search algorithms** only use the basic ingredients for general search algorithms
- **heuristic search algorithms** additionally use **heuristic functions** which estimate how close a node is to the goal

systematic search vs. local search:

- **systematic algorithms** consider a large number of search nodes simultaneously
- **local search algorithms** work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are **crossbreeds** (e. g., enforced hill-climbing)

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Classification: what works where in planning?



uninformed vs. heuristic search:

- For **satisficing** planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For **optimal** planning, the difference is less pronounced.

systematic search vs. local search:

- For **satisficing** planning, the most successful algorithms are somewhere between the two extremes.
- For **optimal** planning, systematic algorithms are required.

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Common procedures for search algorithms



Before we describe the different search algorithms, we introduce three procedures used by all of them:

- **make-root-node**: Create a search node without parent.
- **make-node**: Create a search node for a state generated as the successor of another state.
- **extract-solution**: Extract a solution from a search node representing a goal state.

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Procedure make-root-node



make-root-node: Create a search node without parent.

Procedure make-root-node

```
def make-root-node(s):  
     $\sigma :=$  new node  
    state( $\sigma$ ) := s  
    parent( $\sigma$ ) := undefined  
    action( $\sigma$ ) := undefined  
    g( $\sigma$ ) := 0  
    return  $\sigma$ 
```

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Procedure make-node



make-node: Create a search node for a state generated as the successor of another state.

Procedure make-node

```
def make-node( $\sigma$ , o, s):  
     $\sigma' :=$  new node  
    state( $\sigma'$ ) := s  
    parent( $\sigma'$ ) :=  $\sigma$   
    action( $\sigma'$ ) := o  
    g( $\sigma'$ ) := g( $\sigma$ ) + 1  
    return  $\sigma'$ 
```

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Procedure extract-solution



extract-solution: Extract a solution from a search node representing a goal state.

Procedure extract-solution

```
def extract-solution( $\sigma$ ):  
    solution := new list  
    while parent( $\sigma$ ) is defined:  
        solution.push-front(action( $\sigma$ ))  
         $\sigma :=$  parent( $\sigma$ )  
    return solution
```

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Uninformed search algorithms



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Uninformed search algorithms



- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

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Popular uninformed systematic search algorithms:

- **breadth-first search**
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:

- **random walk**

Breadth-first search without duplicate detection



Breadth-first search

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
while not queue.empty():
    sigma = queue.pop-front()
    if is-goal(state(sigma)):
        return extract-solution(sigma)
    for each < o, s > in succ(state(sigma)):
        sigma' := make-node(sigma, o, s)
        queue.push-back(sigma')
return unsolvable
```

- Possible improvement: **duplicate detection** (see next slide).
- Another possible improvement: test if σ' is a goal node; if so, terminate immediately. (We don't do this because it obscures the similarity to some of the later algorithms.)

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Breadth-first search with duplicate detection



Breadth-first search with duplicate detection

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed := {}
while not queue.empty():
    sigma = queue.pop-front()
    if state(sigma) not in closed:
        closed := closed union {state(sigma)}
        if is-goal(state(sigma)):
            return extract-solution(sigma)
        for each < o, s > in succ(state(sigma)):
            sigma' := make-node(sigma, o, s)
            queue.push-back(sigma')
return unsolvable
```

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Breadth-first search with duplicate detection



Breadth-first search with duplicate detection

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queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed := {}
while not queue.empty():
    sigma = queue.pop-front()
    if state(sigma) not in closed:
        closed := closed union {state(sigma)}
        if is-goal(state(sigma)):
            return extract-solution(sigma)
        for each < o, s > in succ(state(sigma)):
            sigma' := make-node(sigma, o, s)
            queue.push-back(sigma')
return unsolvable
```

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Random walk

$\sigma := \text{make-root-node}(\text{init}())$

forever:

if $\text{is-goal}(\text{state}(\sigma))$:

return $\text{extract-solution}(\sigma)$

 Choose a random element $\langle o, s \rangle$ from $\text{succ}(\text{state}(\sigma))$.

$\sigma := \text{make-node}(\sigma, o, s)$

- The algorithm usually does not find any solutions, unless almost every sequence of actions is a plan.
- Often, it runs indefinitely without making progress.
- It can also fail by reaching a **dead end**, a state with no successors. This is a weakness of many local search approaches.

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Heuristic search algorithms

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Heuristic search algorithms: systematic

- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular systematic heuristic search algorithms:

- **greedy best-first search**
- **A***
- **weighted A***
- IDA*
- depth-first branch-and-bound search
- ...

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Heuristic search algorithms: local

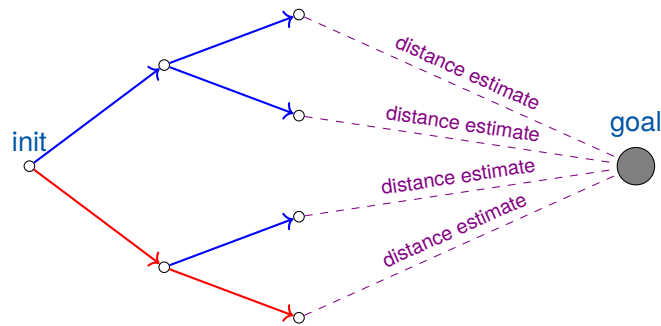
- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic local search algorithms:

- **hill-climbing**
- **enforced hill-climbing**
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- ...

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Heuristic search: idea



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Required ingredients for heuristic search



A **heuristic search algorithm** requires one more operation in addition to the definition of a search space.

Definition (heuristic function)

Let Σ be the set of nodes of a given search space. A **heuristic function** or **heuristic** (for that search space) is a function $h : \Sigma \rightarrow \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the **heuristic estimate** or **heuristic value** of heuristic h for node σ . It is supposed to estimate the distance from σ to the nearest goal node.

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What exactly is a heuristic estimate?



What does it mean that h “estimates the goal distance”?

- For most heuristic search algorithms, h does not need to have any strong properties for the algorithm to work (= be correct and complete).
- However, the **efficiency** of the algorithm closely relates to how accurately h reflects the actual goal distance.
- For some algorithms, like A*, we can prove strong formal relationships between properties of h and properties of the algorithm (optimality, dominance, run-time for bounded error, ...)
- For other search algorithms, “it works well in practice” is often as good an analysis as one gets.

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Heuristics applied to nodes or states?



- Most texts apply heuristic functions to **states**, not **nodes**.
- This is slightly **less general** than our definition:
 - Given a state heuristic h , we can define an equivalent node heuristic as $h'(\sigma) := h(\text{state}(\sigma))$.
 - The opposite is not possible. (Why not?)
- There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on **how** we ended up in a given state s ?
- We call heuristics which don't just depend on $\text{state}(\sigma)$ **pseudo-heuristics**.
- In practice there are sometimes good reasons to have the heuristic value depend on the generating path of σ (e.g., **landmark pseudo-heuristic**, Richter et al. 2008).

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Let Σ be the set of nodes of a given search space.

Definition (optimal/perfect heuristic)

The **optimal** or **perfect heuristic** of a search space is the heuristic h^* which maps each search node σ to the length of a shortest path from $state(\sigma)$ to any goal state.

Note: $h^*(\sigma) = \infty$ iff no goal state is reachable from σ .

A heuristic h is called

- **safe** if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- **goal-aware** if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- **admissible** if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- **consistent** if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that σ' is a successor of σ .¹

Relationships?

¹or: $h(\sigma) \leq h(\sigma') + cost(\sigma, \sigma')$ for non-unit costs, where $cost(\sigma, \sigma')$ is the cost of the transition from σ to σ' .

Greedy best-first search (with duplicate detection)

```

open := new min-heap ordered by ( $\sigma \mapsto h(\sigma)$ )
open.insert(make-root-node(init()))
closed :=  $\emptyset$ 
while not open.empty():
     $\sigma = open.pop-min()$ 
    if  $state(\sigma) \notin closed$ :
        closed := closed  $\cup \{state(\sigma)\}$ 
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in succ(state(\sigma))$ :
             $\sigma' := make-node(\sigma, o, s)$ 
            if  $h(\sigma') < \infty$ :
                open.insert( $\sigma'$ )
return unsolvable
    
```

- one of the three most commonly used algorithms for satisficing planning
- **complete** for safe heuristics (due to duplicate detection)
- **suboptimal** unless h satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of h (e.g., scaling with a positive constant or adding a constant)

A*



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A* (with duplicate detection and reopening)

$open := \text{new}$ min-heap ordered by $(\sigma \mapsto g(\sigma) + h(\sigma))$

$open.insert(\text{make-root-node}(\text{init}()))$

$closed := \emptyset$

$distance := \emptyset$

while not $open.empty()$:

$\sigma = open.pop\text{-min}()$

if $state(\sigma) \notin closed$ **or** $g(\sigma) < distance(state(\sigma))$:

$closed := closed \cup \{state(\sigma)\}$

$distance(state(\sigma)) := g(\sigma)$

if $is\text{-goal}(state(\sigma))$:

return $\text{extract-solution}(\sigma)$

for each $\langle o, s \rangle \in succ(state(\sigma))$:

$\sigma' := \text{make-node}(\sigma, o, s)$

if $h(\sigma') < \infty$: $open.insert(\sigma')$

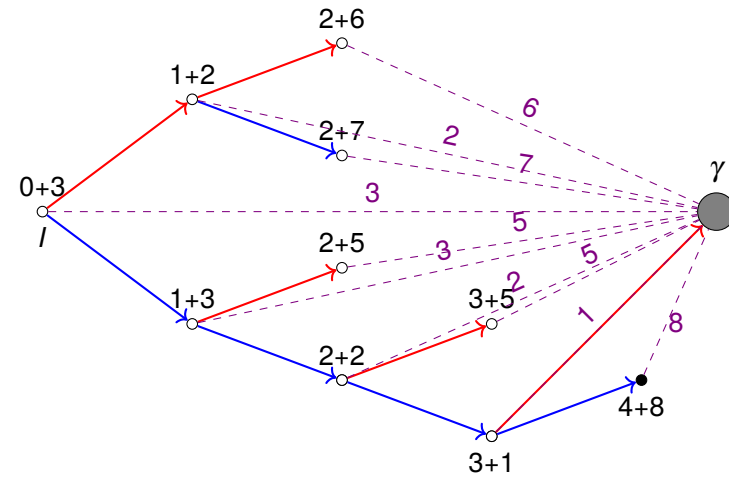
return unsolvable

A* example

Example



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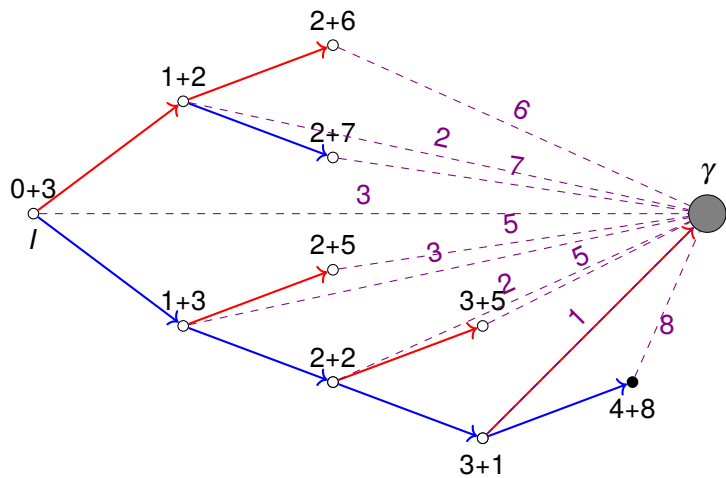


A* example

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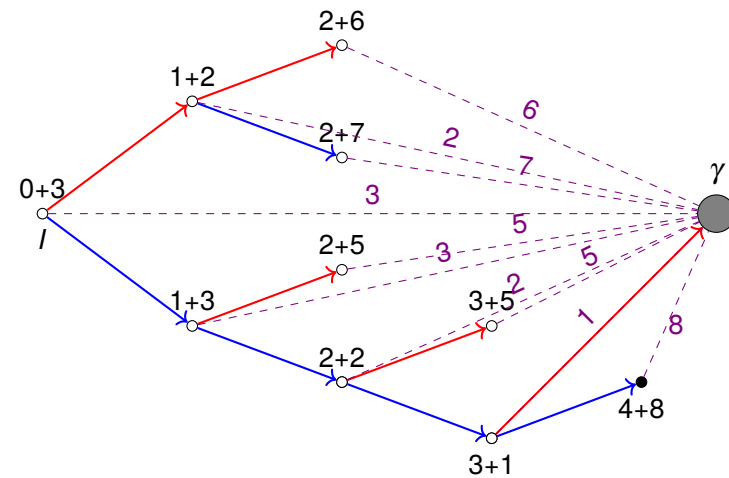


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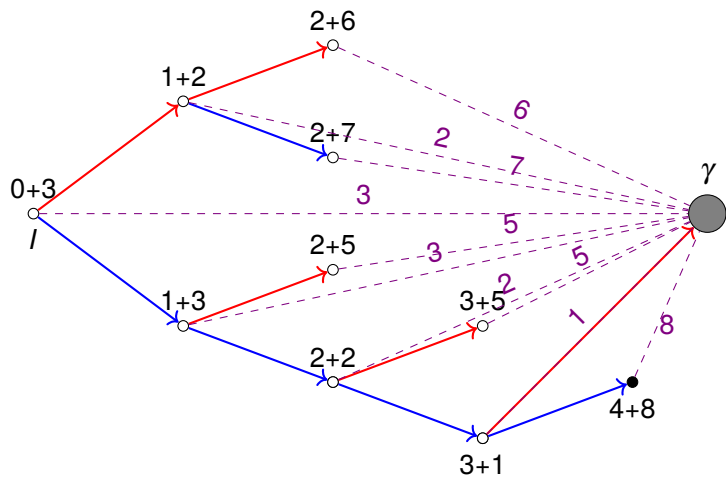


A* example

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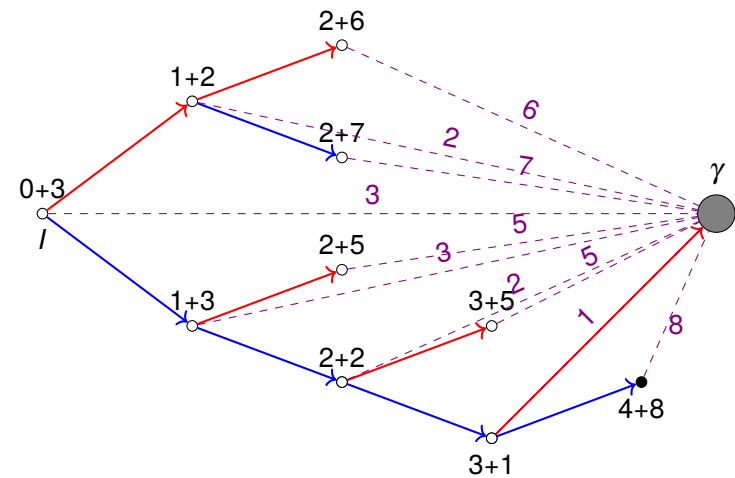


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Terminology for A*



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- **f value** of a node: defined by $f(\sigma) := g(\sigma) + h(\sigma)$
- **generated nodes**: nodes inserted into *open* at some point
- **expanded nodes**: nodes σ popped from *open* for which the test against *closed* and *distance* succeeds
- **reexpanded nodes**: expanded nodes for which $state(\sigma) \in closed$ upon expansion (also called **reopened nodes**)

Properties of A*



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- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- **complete** for safe heuristics (even without duplicate detection)
- **optimal** if h is admissible (even without duplicate detection)
- never reopens nodes if h is consistent

Implementation notes:

- in the heap-ordering procedure, it is considered a good idea to break ties in favour of lower h values
- can simplify algorithm if we know that we only have to deal with consistent heuristics
- common, hard to spot bug: test membership in *closed* at the wrong time

Weighted A*



Weighted A* (with duplicate detection and reopening)

```
open := new min-heap ordered by ( $\sigma \mapsto g(\sigma) + W \cdot h(\sigma)$ )
open.insert(make-root-node(init()))
closed :=  $\emptyset$ 
distance :=  $\emptyset$ 
while not open.empty():
     $\sigma = open.pop\text{-min}()$ 
    if  $state(\sigma) \notin closed$  or  $g(\sigma) < distance(state(\sigma))$ :
        closed := closed  $\cup$  {state( $\sigma$ )}
        distance( $\sigma$ ) :=  $g(\sigma)$ 
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in succ(state(\sigma))$ :
             $\sigma' := make\text{-node}(\sigma, o, s)$ 
            if  $h(\sigma') < \infty$ : open.insert( $\sigma'$ )
return unsolvable
```

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Properties of weighted A*



The weight $W \in \mathbb{R}_0^+$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like A*
- for $W \rightarrow \infty$, behaves like greedy best-first search

Properties:

- one of the most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to A*, replacing optimal with bounded suboptimal: generated solutions are at most a factor W as long as optimal ones

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Hill-climbing



Hill-climbing

```
 $\sigma := make\text{-root}\text{-node}(init())$ 
forever:
    if is-goal(state( $\sigma$ )):
        return extract-solution( $\sigma$ )
     $\Sigma' := \{ make\text{-node}(\sigma, o, s) \mid \langle o, s \rangle \in succ(state(\sigma)) \}$ 
     $\sigma := an\ element\ of\ \Sigma'$  minimizing  $h$  (random tie breaking)
```

- can easily get stuck in local minima where immediate improvements of $h(\sigma)$ are not possible
- many variations: tie-breaking strategies, restarts

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Enforced hill-climbing



Enforced hill-climbing: procedure improve

```
def improve( $\sigma_0$ ):
    queue := new fifo-queue
    queue.push-back( $\sigma_0$ )
    closed :=  $\emptyset$ 
    while not queue.empty():
         $\sigma = queue.pop\text{-front}()$ 
        if  $state(\sigma) \notin closed$ :
            closed := closed  $\cup$  {state( $\sigma$ )}
            if  $h(\sigma) < h(\sigma_0)$ :
                return  $\sigma$ 
        for each  $\langle o, s \rangle \in succ(state(\sigma))$ :
             $\sigma' := make\text{-node}(\sigma, o, s)$ 
            queue.push-back( $\sigma'$ )
    fail
```

↪ breadth-first search for more promising node than σ_0

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Enforced hill-climbing

```
 $\sigma := \text{make-root-node}(\text{init}())$   
while not is-goal(state( $\sigma$ )):  
     $\sigma := \text{improve}(\sigma)$   
return extract-solution( $\sigma$ )
```

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from σ_0)
- complete for **undirected** search spaces (where the successor relation is symmetric) if $h(\sigma) = 0$ for all goal nodes and only for goal nodes

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- distinguish: **planning states**, **search states**, **search nodes**
 - **planning state**: situation in the world modelled by the task
 - **search state**: subproblem remaining to be solved
 - In **state-space search** (usually progression search), planning states and search states are identical.
 - In regression search, search states usually describe sets of states (“subgoals”).
 - **search node**: search state + info on “how we got there”
- search algorithms mainly differ in **order of node expansion**
 - **uninformed** vs. **informed (heuristic)** search
 - **local** vs. **systematic** search

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- **heuristics**: estimators for “distance to goal node”
 - usually: the more accurate, the better performance
 - desiderata: **safe**, **goal-aware**, **admissible**, **consistent**
 - the ideal: **perfect heuristic h^***
- most common algorithms for **satisficing planning**:
 - **greedy best-first search**
 - **weighted A***
 - **enforced hill-climbing**
- most common algorithm for **optimal planning**:
 - **A***

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