Principles of AI Planning

2. Transition systems and planning tasks

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October 23rd, 2019
Transition systems
A transition system is a 5-tuple $\mathcal{T} = \langle S, L, T, s_0, S_* \rangle$ where

- $S$ is a finite set of states,
- $L$ is a finite set of (transition) labels,
- $T \subseteq S \times L \times S$ is the transition relation,
- $s_0 \in S$ is the initial state, and
- $S_* \subseteq S$ is the set of goal states.

We say that $\mathcal{T}$ has the transition $\langle s, \ell, s' \rangle$ if $\langle s, \ell, s' \rangle \in T$.

We also write this $s \xrightarrow{\ell} s'$, or $s \rightarrow s'$ when not interested in $\ell$.

Note: Transition systems are also called state spaces.
Transition systems are often depicted as directed arc-labeled graphs with marks to indicate the initial state and goal states.
Transition system terminology

We use common graph theory terms for transition systems:

- $s'$ successor of $s$ if $s \rightarrow s'$
- $s$ predecessor of $s'$ if $s \rightarrow s'$
- $s'$ reachable from $s$ if there exists a sequence of transitions
  
  $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ s.t. $s^0 = s$ and $s^n = s'$

  - **Note:** $n = 0$ possible; then $s = s'$
  - $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called path from $s$ to $s'$
  - $s^0, \ldots, s^n$ is also called path from $s$ to $s'$
  - length of that path is $n$

- additional terms: strongly connected, weakly connected, strong/weak connected components, ...
Transition system terminology (ctd.)

Some additional terminology:

- **s’ reachable** (without reference state) means reachable from initial state $s_0$
- **solution or goal path** from $s$: path from $s$ to some $s’ \in S_*$
  - if $s$ is omitted, $s = s_0$ is implied
- transition system **solvable** if a goal path from $s_0$ exists
A transition system with transitions $T$ is called deterministic if for all states $s$ and labels $\ell$, there is at most one state $s'$ with $s \xrightarrow{\ell} s'$.

**Example:** previously shown transition system
Throughout the course, we will often use the blocks world domain as an example.

In the blocks world, a number of differently coloured blocks are arranged on our table.

Our job is to rearrange them according to a given goal.
Blocks world rules

- Location on the table does not matter.

- Location on a block does not matter.
Blocks world rules (ctd.)

At most one block may be below a block.

At most one block may be on top of a block.
Blocks world transition system for three blocks

(Transition labels omitted for clarity.)
Blocks world computational properties

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<tr>
<th>blocks</th>
<th>states</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>2</td>
<td>3</td>
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<tr>
<td>3</td>
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</table>

- Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration).
- Finding a shortest solution is NP-complete (for a compact description of the problem).
Planning tasks
Compact representations

- Classical (i.e., deterministic) planning is in essence the problem of finding solutions in huge transition systems.
- The transition systems we are usually interested in are too large to explicitly enumerate all states or transitions.
- Hence, the input to a planning algorithm must be given in a more concise form.
- In the rest of chapter, we discuss how to represent planning tasks in a suitable way.
State variables

How to represent huge state sets without enumerating them?

- represent different aspects of the world in terms of different state variables
  - a state is a valuation of state variables
  - $n$ state variables with $m$ possible values each induce $m^n$ different states
  - exponentially more compact than “flat” representations
- Example: $n$ variables suffice for blocks world with $n$ blocks
Blocks world with finite-domain state variables

Describe blocks world state with three state variables:

- \textit{location-of-A}: \{B, C, table\}
- \textit{location-of-B}: \{A, C, table\}
- \textit{location-of-C}: \{A, B, table\}

Example

\[
\begin{align*}
s(\text{location-of-A}) &= \text{table} \\
s(\text{location-of-B}) &= A \\
s(\text{location-of-C}) &= \text{table}
\end{align*}
\]

Not all valuations correspond to intended blocks world states. Example: \(s\) with \(s(\text{location-of-A}) = B, s(\text{location-of-B}) = A\).
Problem:

- How to succinctly represent transitions and goal states?

Idea: Use propositional logic

- state variables: propositional variables (0 or 1)
- goal states: defined by a propositional formula
- transitions: defined by actions given by
  - precondition: when is the action applicable?
  - effect: how does it change the valuation?

Note: general finite-domain state variables can be compactly encoded as Boolean variables
Blocks world with Boolean state variables

Example

\[
\begin{align*}
  s(A\text{-on}\text{-}B) &= 0 \\
  s(A\text{-on}\text{-}C) &= 0 \\
  s(A\text{-on}\text{-}table) &= 1 \\
  s(B\text{-on}\text{-}A) &= 1 \\
  s(B\text{-on}\text{-}C) &= 0 \\
  s(B\text{-on}\text{-}table) &= 0 \\
  s(C\text{-on}\text{-}A) &= 0 \\
  s(C\text{-on}\text{-}B) &= 0 \\
  s(C\text{-on}\text{-}table) &= 1
\end{align*}
\]
Syntax of propositional logic

Definition (propositional formula)
Let $A$ be a set of atomic propositions (here: state variables).
The propositional formulae over $A$ are constructed by finite application of the following rules:
- $\top$ and $\bot$ are propositional formulae (truth and falsity).
- For all $a \in A$, $a$ is a propositional formula (atom).
- If $\varphi$ is a propositional formula, then so is $\neg \varphi$ (negation).
- If $\varphi$ and $\psi$ are propositional formulas, then so are $(\varphi \lor \psi)$ (disjunction) and $(\varphi \land \psi)$ (conjunction).

Note: We often omit the word “propositional”.

Abbreviations:

- $(\varphi \rightarrow \psi)$ is short for $(\neg \varphi \lor \psi)$ (implication)
- $(\varphi \leftrightarrow \psi)$ is short for $((\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi))$ (equivalence)
- parentheses omitted when not necessary
- $(\neg)$ binds more tightly than binary connectives
- $(\land)$ binds more tightly than $(\lor)$ than $(\rightarrow)$ than $(\leftrightarrow)$
Semantics of propositional logic

**Definition (propositional valuation)**

A **valuation** of propositions \( A \) is a function \( \nu : A \rightarrow \{0, 1\} \).

Define the notation \( \nu \models \varphi \) (\( \nu \) satisfies \( \varphi \); \( \nu \) is a **model** of \( \varphi \); \( \varphi \) is **true** under \( \nu \)) for valuations \( \nu \) and formulae \( \varphi \) by

- \( \nu \models \top \)
- \( \nu \not\models \bot \)
- \( \nu \models a \) iff \( \nu(a) = 1 \), for \( a \in A \).
- \( \nu \models \neg \varphi \) iff \( \nu \not\models \varphi \)
- \( \nu \models \varphi \lor \psi \) iff \( \nu \models \varphi \) or \( \nu \models \psi \)
- \( \nu \models \varphi \land \psi \) iff \( \nu \models \varphi \) and \( \nu \models \psi \)
Propositional logic terminology

- A propositional formula $\phi$ is **satisfiable** if there is at least one valuation $v$ so that $v \models \phi$.
- Otherwise it is **unsatisfiable**.
- A propositional formula $\phi$ is **valid** or a **tautology** if $v \models \phi$ for all valuations $v$.
- A propositional formula $\psi$ is a **logical consequence** of a propositional formula $\phi$, written $\phi \models \psi$, if $v \models \psi$ for all valuations $v$ with $v \models \phi$.
- Two propositional formulae $\phi$ and $\psi$ are **logically equivalent**, written $\phi \equiv \psi$, if $\phi \models \psi$ and $\psi \models \phi$.

**Question:** How to phrase these in terms of models?
Propositional logic terminology (ctd.)

- A propositional formula that is a proposition $a$ or a negated proposition $\neg a$ for some $a \in A$ is a literal.
- A formula that is a disjunction of literals is a clause. This includes unit clauses / consisting of a single literal, and the empty clause $\bot$ consisting of zero literals.

Normal forms: NNF, CNF, DNF
Transitions for state sets described by propositions $A$ can be concisely represented as operators or actions $\langle \chi, e \rangle$ where

- the precondition $\chi$ is a propositional formula over $A$ describing the set of states in which the transition can be taken (states in which a transition starts), and

- the effect $e$ describes how the resulting successor states are obtained from the state where the transitions is taken (where the transition goes).
Example: blocks world operators

Blocks world operators

To model blocks world operators conveniently, we use auxiliary state variables $A$-clear, $B$-clear, and $C$-clear to denote that there is nothing on top of a given block.

Then blocks world operators can be modeled as:

- $\langle A\text{-clear} \land A\text{-on}-T \land B\text{-clear}, \neg A\text{-on}-T \land \neg B\text{-clear} \rangle$
- $\langle A\text{-clear} \land A\text{-on}-T \land C\text{-clear}, \neg A\text{-on}-T \land \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \land A\text{-on}-B, \neg A\text{-on}-B \land B\text{-clear} \rangle$
- $\langle A\text{-clear} \land A\text{-on}-C, \neg A\text{-on}-C \land C\text{-clear} \rangle$
- $\langle A\text{-clear} \land A\text{-on}-B \land C\text{-clear}, \neg A\text{-on}-B \land B\text{-clear} \land \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \land A\text{-on}-C \land B\text{-clear}, \neg A\text{-on}-C \land C\text{-clear} \land \neg B\text{-clear} \rangle$
- $\ldots$
(Deterministic) effects are recursively defined as follows:

- If \( a \in A \) is a state variable, then \( a \) and \( \neg a \) are effects (atomic effect).
- If \( e_1, \ldots, e_n \) are effects, then \( e_1 \land \cdots \land e_n \) is an effect (conjunctive effect).
  The special case with \( n = 0 \) is the empty effect \( \top \).
- If \( \chi \) is a propositional formula and \( e \) is an effect, then \( \chi \models e \) is an effect (conditional effect).

Atomic effects \( a \) and \( \neg a \) are best understood as assignments \( a := 1 \) and \( a := 0 \), respectively.
Effect example

\( \chi \triangleright e \) means that change \( e \) takes place if \( \chi \) is true in the current state.

**Example**

Increment 4-bit number \( b_3b_2b_1b_0 \) represented as four state variables \( b_0, \ldots, b_3 \):

\[
(\neg b_0 \triangleright b_0) \land \\
((\neg b_1 \land b_0) \triangleright (b_1 \land \neg b_0)) \land \\
((\neg b_2 \land b_1 \land b_0) \triangleright (b_2 \land \neg b_1 \land \neg b_0)) \land \\
((\neg b_3 \land b_2 \land b_1 \land b_0) \triangleright (b_3 \land \neg b_2 \land \neg b_1 \land \neg b_0))
\]
Operator semantics

**Definition (changes caused by an operator)**

For each effect \( e \) and state \( s \), we define the change set of \( e \) in \( s \), written \([e]_s\) as the following set of literals:

- \([a]_s = \{a\} \) and \([\neg a]_s = \{\neg a\}\) for atomic effects \( a, \neg a \)
- \([e_1 \land \cdots \land e_n]_s = [e_1]_s \cup \cdots \cup [e_n]_s\)
- \([\chi \triangleright e]_s = [e]_s \) if \( s \models \chi \) and \([\chi \triangleright e]_s = \emptyset \) otherwise

**Definition (applicable operators)**

Operator \( \langle \chi, e \rangle \) is applicable in a state \( s \) iff \( s \models \chi \) and \([e]_s\) is consistent (i.e., does not contain two complementary literals).
Definition (successor state)

The successor state \( \text{app}_o(s) \) of \( s \) with respect to operator
\( o = \langle \chi, e \rangle \) is the state \( s' \) with \( s' \models [e]_s \) and \( s'(v) = s(v) \) for all
state variables \( v \) not mentioned in \([e]_s\).
This is defined only if \( o \) is applicable in \( s \).

Example

Consider the operator \( \langle a, \neg a \land (\neg c \triangleright \neg b) \rangle \) and the state
\( s = \{ a \mapsto 1, b \mapsto 1, c \mapsto 1, d \mapsto 1 \} \).

The operator is applicable because \( s \models a \) and
\([\neg a \land (\neg c \triangleright \neg b)]_s = \{\neg a\} \) is consistent.
Applying the operator results in the successor state
\( \text{app}_{\langle a, \neg a \land (\neg c \triangleright \neg b) \rangle}(s) = \{ a \mapsto 0, b \mapsto 1, c \mapsto 1, d \mapsto 1 \} \).
Deterministic planning tasks

Definition (deterministic planning task)

A deterministic planning task is a 4-tuple $\Pi = \langle A, I, O, \gamma \rangle$ where

- $A$ is a finite set of state variables (propositions),
- $I$ is a valuation over $A$ called the initial state,
- $O$ is a finite set of operators over $A$, and
- $\gamma$ is a formula over $A$ called the goal.

Note:

- When we talk about deterministic planning tasks, we usually omit the word “deterministic”.
- When we will talk about nondeterministic planning tasks later, we will explicitly qualify them as “nondeterministic”.
Mapping planning tasks to transition systems

Definition (induced transition system of a planning task)

Every planning task $\Pi = \langle A, I, O, \gamma \rangle$ induces a corresponding deterministic transition system $\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle$:

- $S$ is the set of all valuations of $A$,
- $L$ is the set of operators $O$,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, \text{o applicable in } s, \ s' = \text{app}_o(s) \}$,
- $s_0 = I$, and
- $S_\star = \{ s \in S \mid s \models \gamma \}$
Terminology for transitions systems is also applied to the planning tasks that induce them.

For example, when we speak of the states of $\Pi$, we mean the states of $T(\Pi)$.

A sequence of operators that forms a goal path of $T(\Pi)$ is called a plan of $\Pi$. 
By planning, we mean the following two algorithmic problems:

**Definition (satisficing planning)**
- **Given:** a planning task \( \Pi \)
- **Output:** a plan for \( \Pi \), or **unsolvable** if no plan for \( \Pi \) exists

**Definition (optimal planning)**
- **Given:** a planning task \( \Pi \)
- **Output:** a plan for \( \Pi \) with minimal length among all plans for \( \Pi \), or **unsolvable** if no plan for \( \Pi \) exists
Transition systems are (typically huge) directed graphs that encode how the state of the world can change.

Planning tasks are compact representations for transition systems, suitable as input for planning algorithms.

Planning tasks are based on concepts from propositional logic, enhanced to model state change.

States of planning tasks are propositional valuations.

Operators of planning tasks describe when (precondition) and how (effect) to change the current state of the world.

In satisficing planning, we must find a solution to planning tasks (or show that no solution exists).

In optimal planning, we additionally guarantee that generated solutions are of the shortest possible length.