Principles of AI Planning

17. Strong cyclic planning

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Strong cyclic plans

Planning objectives

Strong plans

- The simplest objective for nondeterministic planning is the one we have considered in the previous lecture: reach a goal state with certainty.
- With this objective the nondeterminism can also be understood as an opponent like in 2-player games.
  The plan guarantees reaching a goal state no matter what the opponent does: plans are winning strategies.

Planning objectives

Limitations of strong plans

- In strong plans, goal states can be reached without visiting any state twice.
- This property guarantees that the length of executions is bounded by some constant (which is smaller than the number of states.)
- Some solvable problems are not solvable this way.
    1. Action may fail to have any effect.
       Hit a coconut to break it.
    2. Action may fail and take us away from the goals.
       Build a house of cards.

  Consequences:
    1. It is impossible to avoid visiting some states several times.
    2. There is no finite upper bound on execution length.

Planning objectives

When strong cyclic plans make sense

Fairness assumption

For any nondeterministic operator $\langle \chi, \{e_1, \dots, e_n\} \rangle$, the "probability" of every effect e_i, $i = 1, \dots, n$, is greater than 0.

Alternatively: For each $s' \in img_o(s)$ the "probability" of reaching s' from s by o is greater than 0.

This assumption guarantees that a strong cyclic plan reaches the goal almost certainly (with probability 1).

This is not compatible with viewing nondeterminism as an opponent in a 2-player game: the opponent's strategy might rule out some of the choices $e_1, \dots, e_n$.

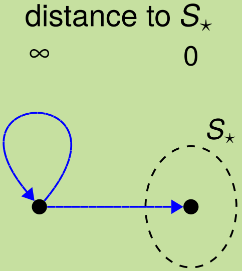
Need for strong cyclic plans

Example

Example (Breaking a coconut)

- Initial state: coconut is intact.
- Goal state: coconut is broken.
- On every hit the coconut may or may not break.
- There is no finite upper bound on the number of hits.

This is equivalent to coin tossing.

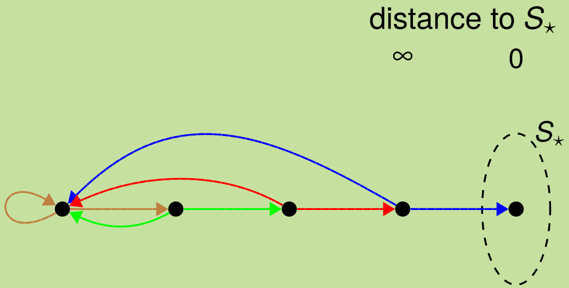


Need for strong cyclic plans

Example

Example (Build a house of cards)

- Initial state: all cards lie on the table.
- Goal state: house of cards is complete.
- At every construction step the house may collapse.



Algorithms for strong cyclic planning

We present two algorithms for strong cyclic planning:

- The nested fixpoint algorithm is conceptually simpler, but typically very costly, especially if not implemented symbolically.
  - Historically older
  - Uninformed
  - Considers entire state space
- The determinization-based incremental planning algorithm is a bit more complicated, but typically more efficient.
  - Historically newer, state of the art
  - Can use informed classical planner as sub-procedure
  - Often only considers small portion of state space

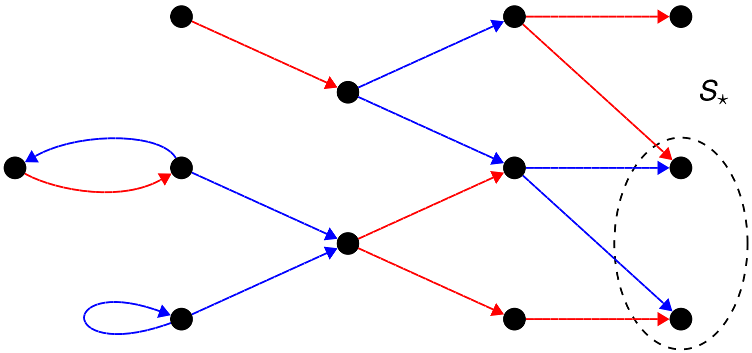
Nested Fixpoint Algorithm

Idea

- Finds plans that may loop (strong cyclic plans).
- The algorithm is rather tricky in comparison to the algorithm for strong plans.
- Every state covered by a plan satisfies two properties:
  1. The state is good: there is at least one execution (= path in the graph defined by the plan) leading to a goal state.
  2. Every successor state is either a goal state or good.
- The algorithm repeatedly eliminates states that are not good.

Nested Fixpoint Algorithm

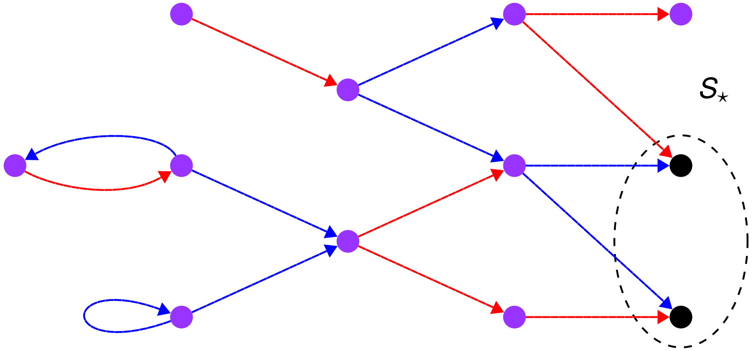
Example



Nested Fixpoint Algorithm

Example

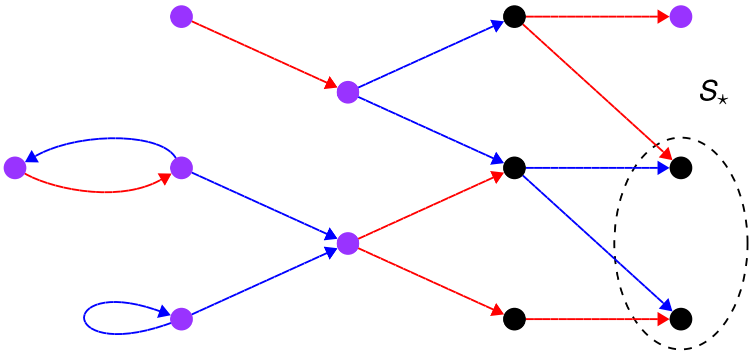
All states are candidates for being good.



Nested Fixpoint Algorithm

Example

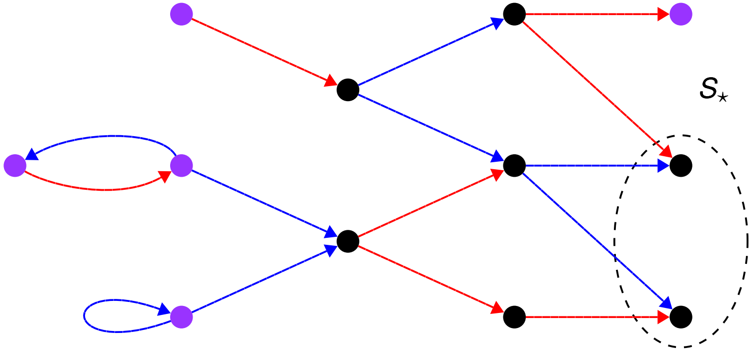
States from which goals are reachable in ≤ 1 steps so that all immediate successors are possibly good.



Nested Fixpoint Algorithm

Example

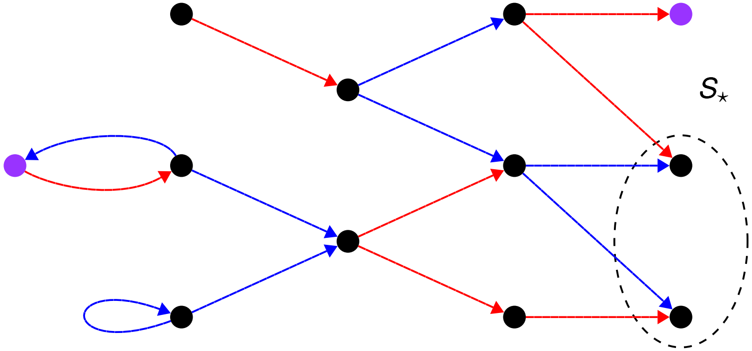
States from which goals are reachable in ≤ 2 steps so that all immediate successors are possibly good.



Nested Fixpoint Algorithm

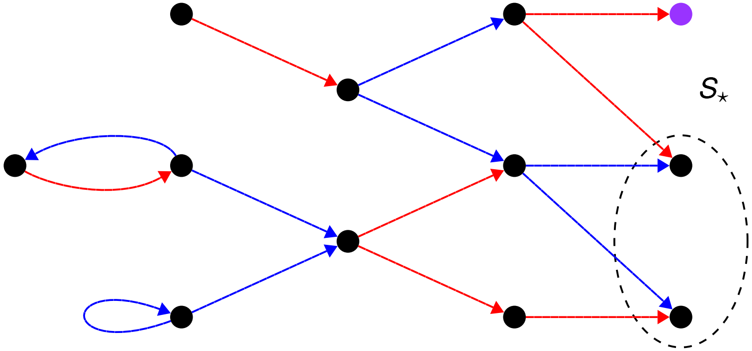
Example

States from which goals are reachable in ≤ 3 steps so that all immediate successors are possibly good.



Nested Fixpoint Algorithm
Example

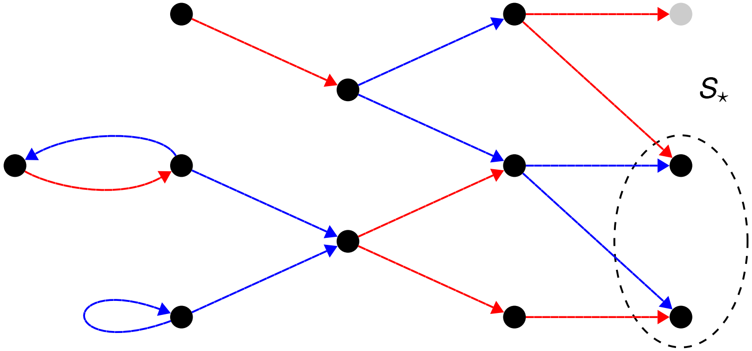
States from which goals are reachable in ≤ 4 steps so that all immediate successors are possibly good.



Nested Fixpoint Algorithm

Example

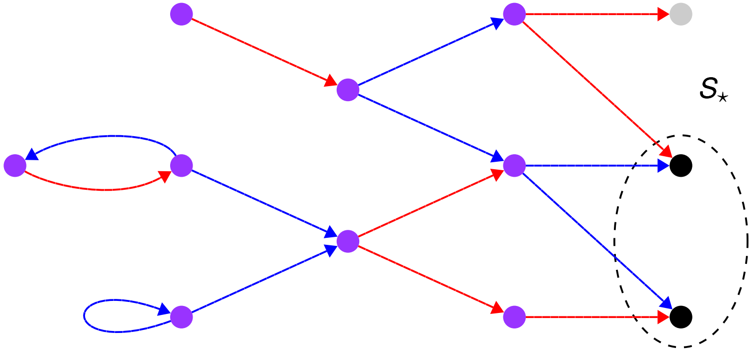
Eliminate states that turned out not to be good.



Nested Fixpoint Algorithm

Example

The set of possibly good states is now smaller.



Nested Fixpoint Algorithm

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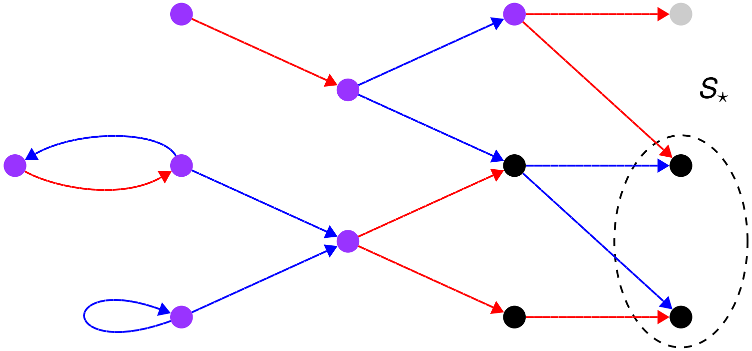

Nested Fixpoint Algorithm
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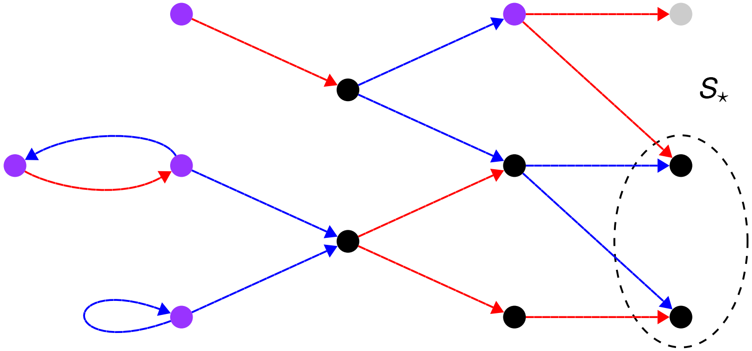

Nested Fixpoint Algorithm

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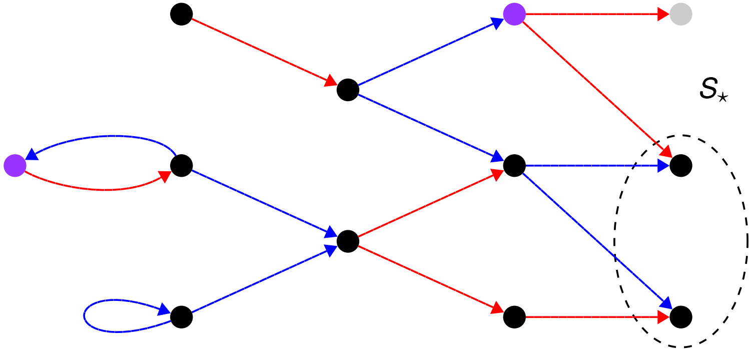

Nested Fixpoint Algorithm

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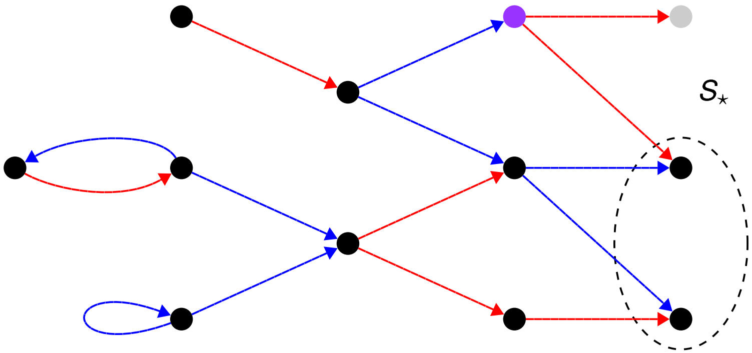

Nested Fixpoint Algorithm

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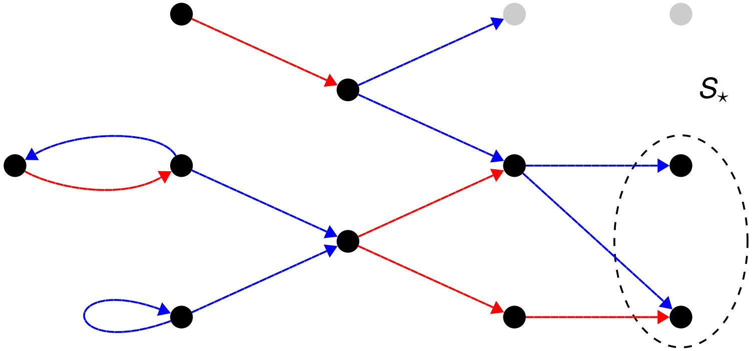

Nested Fixpoint Algorithm

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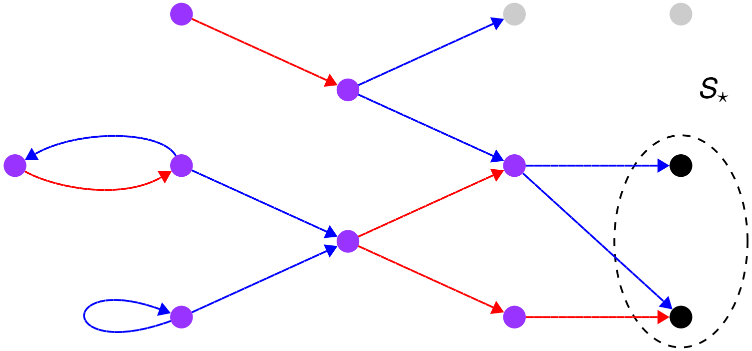

Nested Fixpoint Algorithm

Example

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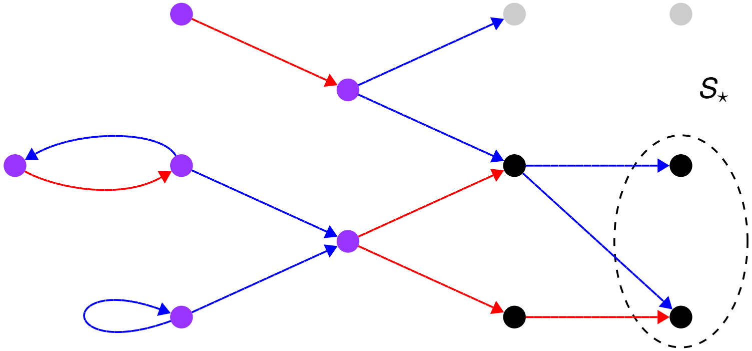
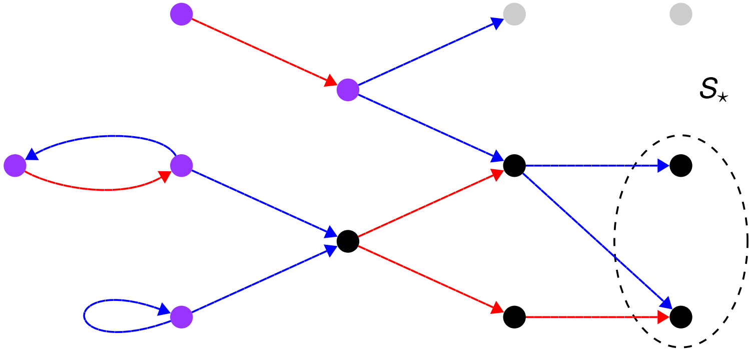


Nested Fixpoint Algorithm

Example

States from which goals are reachable in ≤ 2 steps so that all immediate successors are possibly good.



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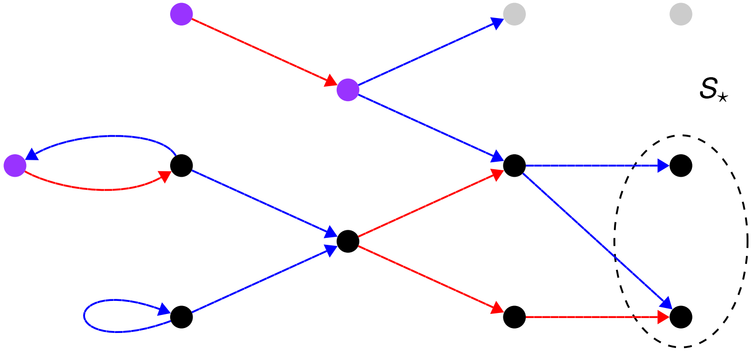

Nested Fixpoint Algorithm

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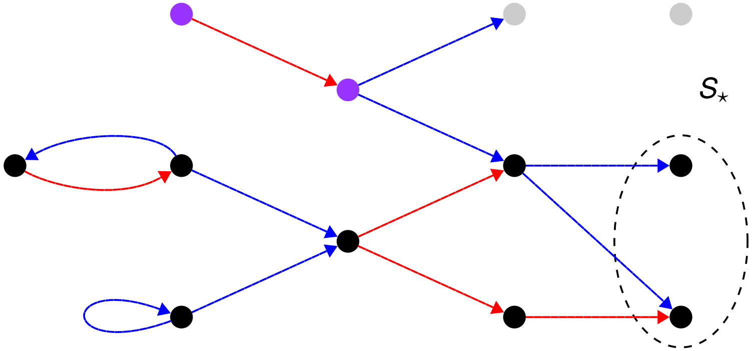
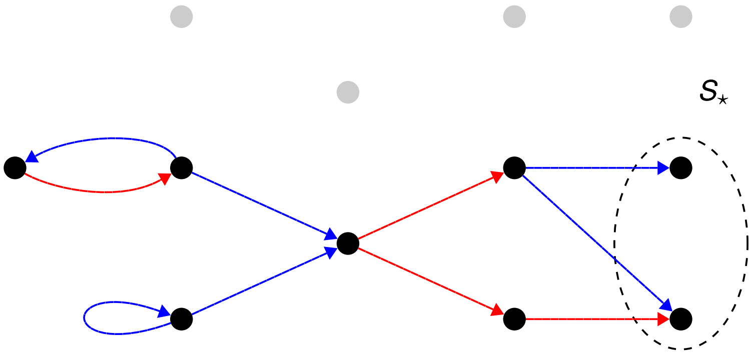


Nested Fixpoint Algorithm

Example

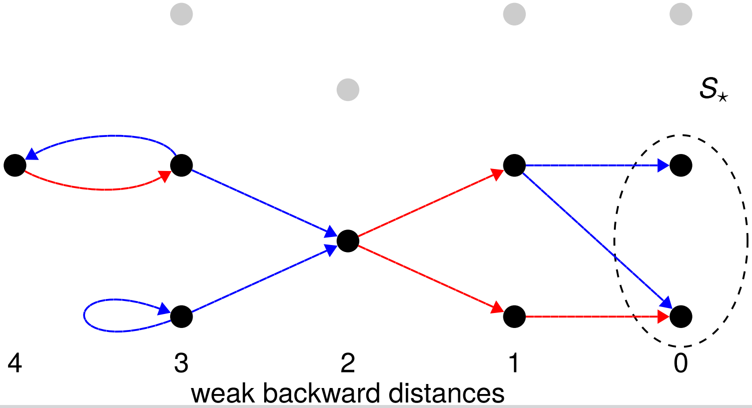
Remaining states are all good.
A further iteration would not eliminate more states.



Nested Fixpoint Algorithm

Example

Assign each state an operator so that the successor states are goal states or good, and some of them are closer to goal states. Use weak distances computed with weak preimages. For this example this is trivial.



Strong cyclic plans

Recall the definition of cyclic strong plans:

Definition (strong cyclic plan)

Let S be the set of states of a planning task Π. Then a strong cyclic plan for Π is a function $\pi : S_\pi \to O$ for some subset $S_\pi \subseteq S$ such that

- $\pi(s)$ is applicable in s for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi \cup S_\star$ (π is closed), and
- $S_\pi(s') \cap S_\star \neq \emptyset$ for all $s' \in S_\pi(s_0)$ (π is proper).

Procedure *prune*

- The procedure prune finds a maximal set of states for which reaching goals with looping is possible.
- It consists of two nested loops:
  1. The outer loop iterates through $i = 0, 1, 2, \ldots$ and produces a shrinking sequence of candidate good state sets $C_0, C_1, \ldots, C_n$ until $C_i = C_{i+1}$.
  2. The inner loop identifies growing sets W_j of states from which a goal state can be reached with j steps without leaving the current set of candidate good states C_i. The union of all $W_0, W_1, \ldots$ will be C_{i+1}.

Procedure *prune*

Definition

Procedure prune

```
def prune($S$, $O$, $S_\star$):
  $C_0 := S$
  for each $i \in \mathbb{N}_1$:
      $W_0 := S_\star$
      for each $j \in \mathbb{N}_1$:
          $W_j := W_{j-1} \cup \bigcup_{o \in O}(wpreimg_o(W_{j-1}) \cap spreimg_o(C_{i-1}))$
          if $W_j = W_{j-1}$:
              break
      $C_i := W_j$
      if $C_i = C_{i-1}$:
          return $\langle C_i, \langle W_0, \ldots, W_{j-1} \rangle \rangle$
```

Procedure *prune*
Correctness

Lemma (Procedure prune)

Let S and $S_\star \subseteq S$ be sets of states and O a set of operators. Then prune$(S, O, S_\star)$ terminates after a finite number of steps and returns $C \subseteq S$ such that there is a strategy $\pi : C \setminus S_\star \to O$ that is a strong cyclic plan (for the states for which it is defined) and maximal in the sense that there is no set $C' \supsetneq C$ and a strong cyclic plan $\pi' : C' \setminus S_\star \to O$.

- The sets W_j also returned by *prune* encode weak distances and can be used to define the strong cyclic plan π.

Nested Fixpoint Algorithm

Main algorithm

The planning algorithm

```
def strong-cyclic-plan($\langle V, I, O, \gamma \rangle$):
    $S$ := set of states over $V$
    $S_\star := \{s \in S \mid s \models \gamma\}$
    $\langle C, (W_j)_{j=0,1,2,\dots} \rangle = prune(S, O, S_\star)$
    if $I \notin C$:
        return no solution
    for each $s \in C$:
        $\delta(s) := \min\{j \in \mathbb{N}_0 \mid s \in W_j\}$
    for each $s \in C \setminus S_\star$:
        $\pi(s)$ := some operator $o \in O$ with $img_o(s) \subseteq C$
                and $\min\{\delta(s') \mid s' \in img_o(s)\} < \delta(s)$
    return $\pi$
```

Nested Fixpoint Algorithm
Complexity

- The procedure *prune* runs in polynomial time in the number of states because the number of iterations of each loop is at most n – hence there are $O(n^2)$ iterations – and computation on each iteration takes polynomial time in the number of states.
- Finding strong cyclic plans for full observability is in the complexity class EXPTIME.
- The problem is also EXPTIME-hard.
- Similar to strong planning, we can speed up the algorithm in many practical cases by using a symbolic implementation (e. g. with BDDs).

Determinization-based Incremental Alg.

Idea [Kuter/Nau/Reisner/Goldman, 2008; Fu/Ng/Bastiani/Yen, 2011]:

1. Pretend the planning task was deterministic: Turn each action $o = \langle \chi, E \rangle$ with $E = \{e_1, \ldots, e_n\}$ into n actions $o_i = \langle \chi, e_i \rangle$ for $i = 1, \ldots, n$. Obtain classical problem Π'.
2. Find classical plan P in Π'. Add state-action mapping corresponding to P to π.
3. For each operator o_i used in P (in state s), identify original nondeterministic operator o and states $S' = img_o(s)$.
4. For each "open" state $s' \in S'$, go to 2.

Remark: May require backtracking, if some state used in a classical plan turns out not to admit a strong cyclic plan.

Determinization-based Incremental Alg.

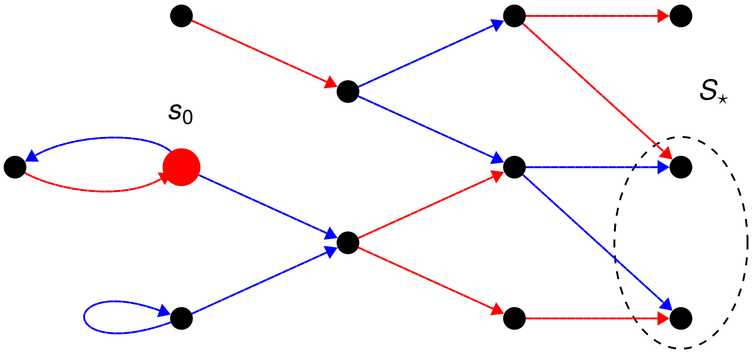
Definition (all-outcomes determinization)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task. The all-outcomes determinization of Π is the deterministic planning task $\Pi_{\mathrm{det}} = \langle V, I, O_{\mathrm{det}}, \gamma \rangle$, where $O_{\mathrm{det}} = \bigcup_{o \in O} o_{\mathrm{det}}$, and $\langle \chi, E \rangle_{\mathrm{det}} = \{ \langle \chi, e \rangle \mid e \in E \}$.

Determinization-based Incremental Alg.

Example

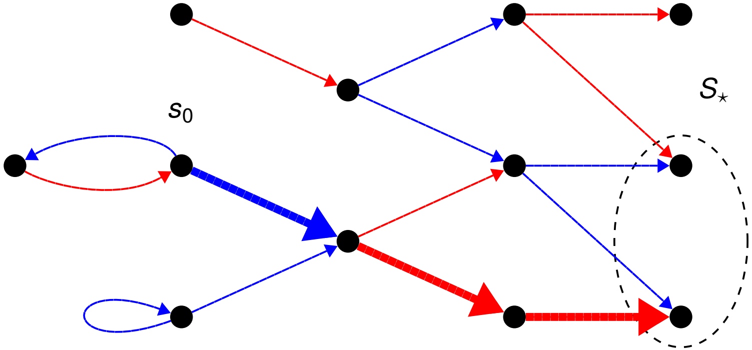
List of states to solve: $\{s_0\}$



Determinization-based Incremental Alg.

Example

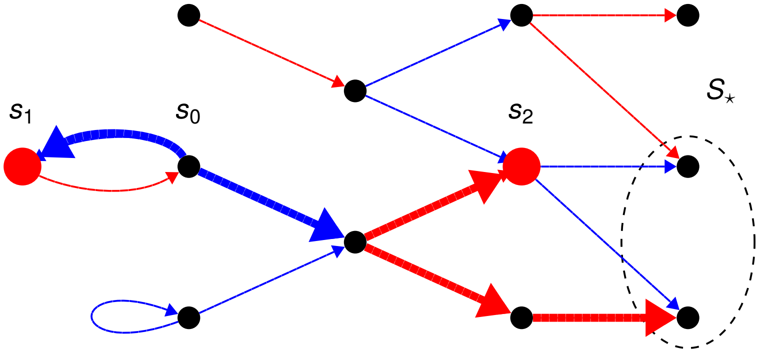
Plan for s_0 in determinization: *blue*$_2$, *red*$_2$, *red*



Determinization-based Incremental Alg.

Example

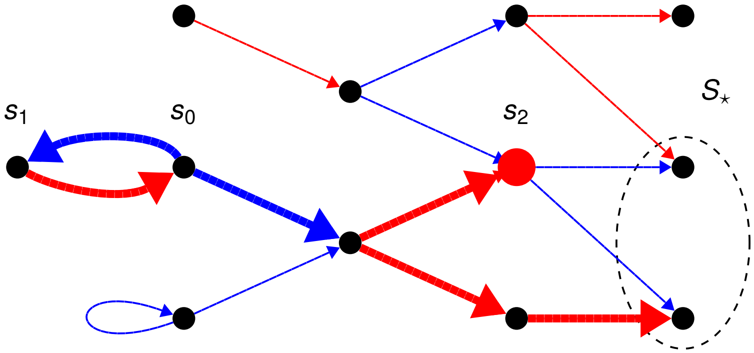
“Undesired” outcomes of *blue*$_1$ and *red*$_1$ lead to new list of states to solve: $\{s_1, s_2\}$



Determinization-based Incremental Alg.

Example

Plan for s_1 in determinization: $red, blue_2, red_2, red$

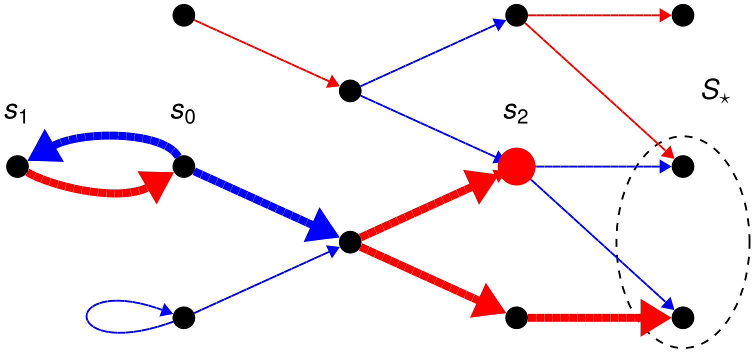


Determinization-based Incremental Alg.

Example

No new “undesired” outcomes.

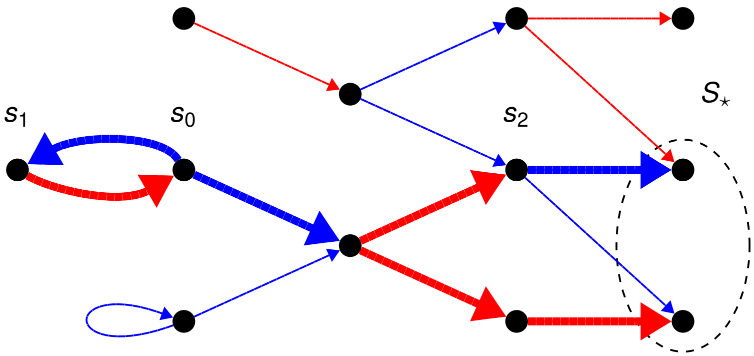
List of states to solve: $\{s_2\}$



Determinization-based Incremental Alg.

Example

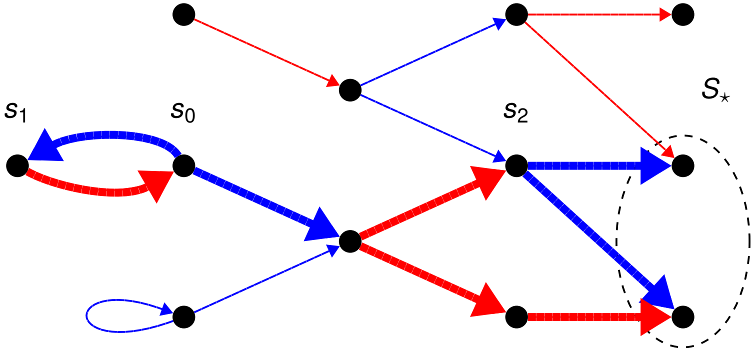
Plan for s_2 in determinization: *$blue_1$*



Determinization-based Incremental Alg.

Example

“Undesired” outcome of *blue*$_2$ in s_2 leads to goal state, too.
List of states to solve: $\emptyset$. Strong cyclic plan found.



Determinization-based Incremental Alg.

Pseudocode

Procedure incremental-strong-cyclic-plan

```
def incremental-strong-cyclic-plan($\langle V, I, O, \gamma \rangle$):
    $\pi \leftarrow \emptyset$; $fail \leftarrow \{I\}$
    while $fail \neq \emptyset$:
        $s \leftarrow$ SelectAndRemoveFrom($fail$)
        $\pi' \leftarrow$ DetSearch($\langle V, s, O_{\mathrm{det}}, \gamma \rangle$)
        if $\pi'$ = Failure:
            if $s = I$: return Failure
            else: Backtrack($s$, $\pi$, $\langle V, I, O, \gamma \rangle$)
        else:
            $\pi \leftarrow \pi \cup \pi'$
        $fail \leftarrow \{s \in S \mid s$ nongoal state reachable from $I$
                          following $\pi$, but $\pi(s)$ undefined$\}$
    return $\pi$
```

Determinization-based Incremental Alg.

If a deterministic search fails, the state *s* from which it started cannot be part of a strong cyclic plan.

- If $s = I$, the whole given planning problem is unsolvable and the algorithm returns FAILURE.
- Otherwise, state *s*, which has already been added to the constructed policy π, has to be removed from π, and the algorithm has to ensure that *s* will never be reconsidered again. This is accomplished by the procedure BACKTRACK.

Determinization-based Incremental Alg.

Procedure backtrack

```
def backtrack($s$, $\pi$, $\langle V, I, O, \gamma \rangle$):
    update $\pi$ by deleting all entries that would immediately
        lead to $s$, i.e. $\pi \leftarrow \pi \setminus \{(s', \pi(s')) \mid s \in img_{\pi(s')}(s')\}$
    add all states $s'$ removed from $\pi$ to the set of fail-states $fail$
    permanently mark all formerly assigned actions $\pi(s')$
        removed from $\pi$ at $s'$ as inapplicable in $s'$ to avoid
        running into the same dead end again.
```

Determinization-based Incremental Alg.

- Iteratively solves all-outcomes determinizations of Π with "fail-states" as initial states.
- Planner can choose desired outcome of each action.
- Deterministic plans are added to policy under construction.
- Corresponding undesired outcomes have to be added to the set of "fail-states" *fail*.
- Deterministic plans for "fail-states" are constructed until no more "fail-states" remain.
- Eventually, the algorithm either returns a strong cyclic plan or FAILURE if no such plan exists.

Determinization-based Incremental Alg.

Correctness

Theorem

Procedure incremental-strong-cyclic-plan, called with task Π*, returns a strong cyclic plan for* Π *iff such a plan exists, and* FAILURE*, otherwise.*

Determinization-based Incremental Alg.

Underlying classical planner

- Can use any classical planner for deterministic searches.
- Can benefit from heuristics etc. used there.
- Classical planner can be configured to prefer short solutions or solutions using deterministic actions induced by nondeterministic actions with few different outcomes (likely fewer new “fail-states”).

Determinization-based Incremental Alg.

Improvements

- When to terminate a deterministic sub-search?
  - At goal states?
  - At states currently part of the partial solution?
  - At parent of currently solved “fail-state”?

  This can make a huge differnce.
- Similarly: Where should the heuristic guide the classical planner? Goals, partial solution, parent node?
- Additional marking of nodes as definitely solved if this can be detected.
- State reuse between subsequent classical planner calls.
- Generalization of solved states by regression search from goal along weak (deterministic) plan (cf. [Muise/McIlraith/Beck, 2012]).

Maintenance goals

Maintenance goals

- In this lecture, we usually limit ourselves to the problem of finding plans that reach a goal state.
- In practice, planning is often about more general goals, where execution cannot be terminated.
  1. An animal: find food, eat, sleep, find food, eat, sleep, . . .
  2. Cleaning robot: keep the building clean.
- These problems cannot be directly formalized in terms of reachability because infinite (unbounded) plan execution is needed.
- We do not discuss this topic in full detail. However, to give at least a little impression of planning for temporally extended goals, we will discuss the simplest objective with infinite plan executions: maintenance.

Plan objectives

Maintenance

Definition

Let $\mathscr{T} = \langle V, I, O, \gamma \rangle$ be a planning task with state set S and set of goal states $S_\star = \{s \in S \mid s \models \gamma\}$.
A strategy π for $\mathscr{T}$ is called a plan for maintenance for $\mathscr{T}$ iff

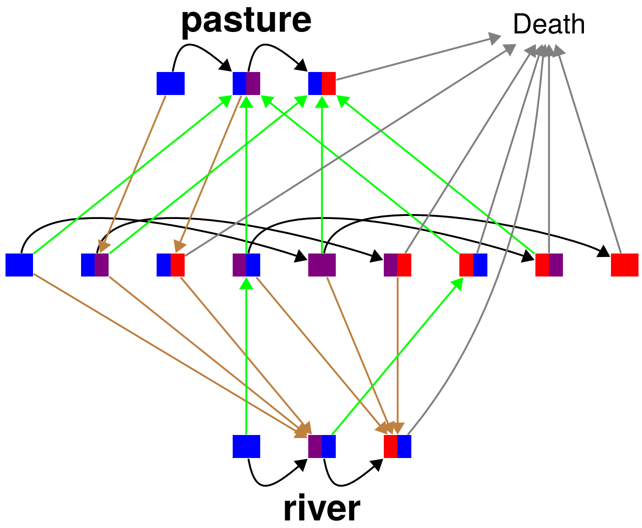
- $\pi(s)$ is applicable in s for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi$, and
- $S_\pi(s_0) \subseteq S_\star$.

Maintenance goals
Example

- The state of an animal is determined by three state values: hunger (0, 1, 2), thirst (0, 1, 2) and location (river, pasture, desert). There is also a special state called death.
- Thirst grows when not at river; at river it is 0.
- Hunger grows when not on pasture; on pasture it is 0.
- If hunger or thirst exceeds 2, the animal dies.
- The goal of the animal is to avoid death.

Maintenance goals

Transition system for the example



Maintenance goals

Plan for the example

We can infer rules backwards starting from the death condition.

1. If in desert and thirst = 2, must go to river.
2. If in desert and hunger = 2, must go to pasture.
3. If on pasture and thirst = 1, must go to desert.
4. If at river and hunger = 1, must go to desert.

If the above rules conflict, the animal will die.

Algorithm for maintenance goals

Idea

Summary of the algorithm idea

Repeatedly eliminate from consideration those states that in one or more steps unavoidably lead to a non-goal state.

- A state is *i*-safe iff there is a plan that guarantees “survival” for the next *i* actions.
- A state is safe (or ∞-safe) iff it is *i*-safe for all $i \in \mathbb{N}_0$.
- The 0-safe states are exactly the goal states: maintenance objective is satisfied for the current state.
- Given all *i*-safe states, compute all $i+1$-safe states by using strong preimages.
- For some $i \in \mathbb{N}_0$, *i*-safe states equal $i+1$-safe states because there are only finitely many states and at each step and $i+1$-safe states are a subset of *i*-safe states. Then *i*-safe states are also ∞-safe.

Algorithm for maintenance goals

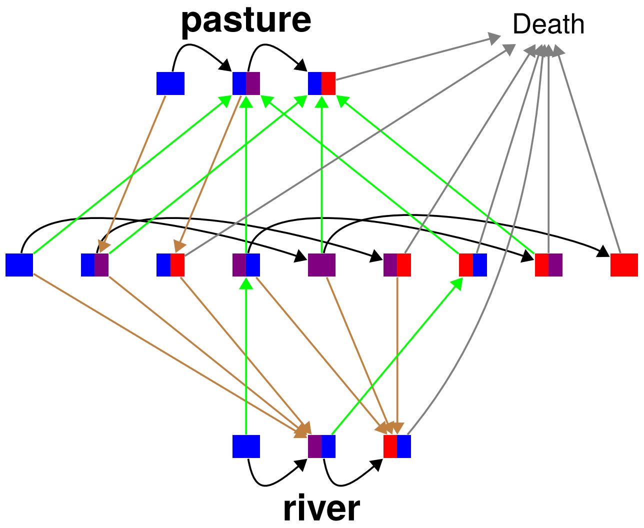
Algorithm

Planning for maintenance goals

```
def maintenance-plan($\langle V, I, O, \gamma \rangle$):
    $S$ := set of states over $V$
    $Safe_0$ := $\{s \in S \mid s \models \gamma\}$
    for each $i \in \mathbb{N}_1$:
        $Safe_i$ := $Safe_{i-1} \cap \bigcup_{o \in O} spreimg_o(Safe_{i-1})$
        if $Safe_i = Safe_{i-1}$:
            break
    if $I \notin Safe_i$:
        return no solution
    for each $s \in Safe_i$:
        $\pi(s)$ := some operator $o \in O$ with $img_o(s) \subseteq Safe_i$
    return $\pi$
```

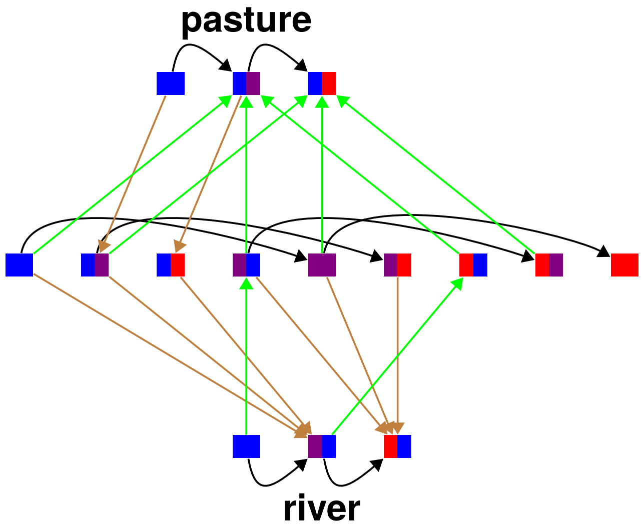
Maintenance goals

Transition system for the example



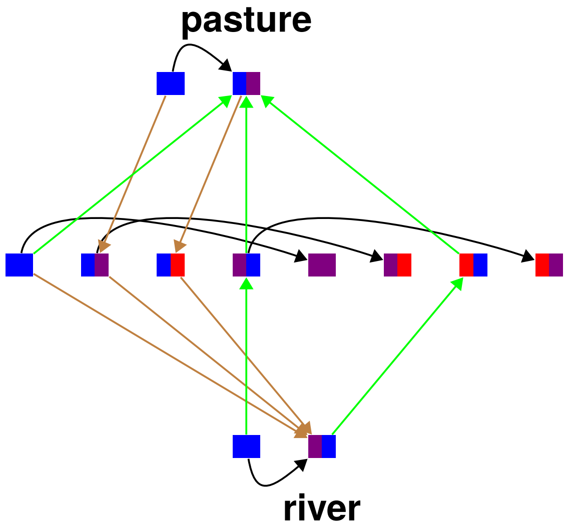
Maintenance goals

0-safe states



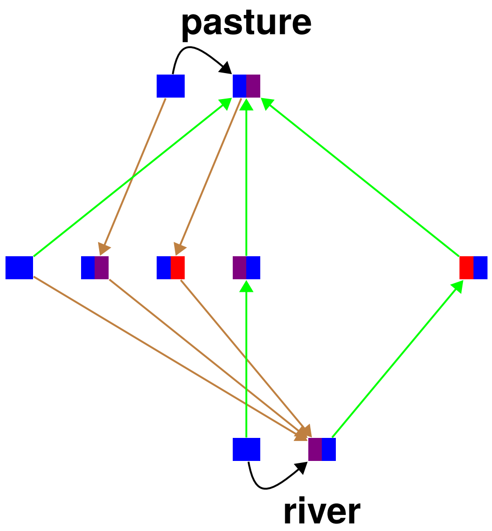
Maintenance goals

1-safe states



Maintenance goals

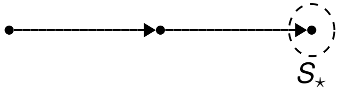
i-safe states for all $i \geq 2$



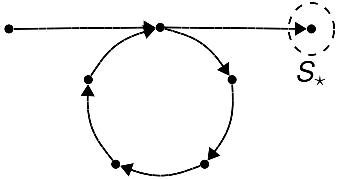
Summary

Different planning objectives

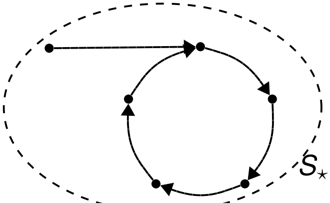
Strong planning



Strong cyclic planning



Maintenance



Outlook: Computational tree logic

- We have considered different classes of solutions for planning tasks by defining different planning problems.
  - strong planning problem: find a strong plan
  - strong cyclic planning problem: find a strong cyclic plan
  - …
- Alternatively, we could allow specifying goals in a modal logic like computational tree logic to directly express the type of plan we are interested in using modalities such as A (all), E (exists), G (globally), and F (finally).
  - Weak planning: EFφ
  - Strong planning: AFφ
  - Strong cyclic planning: AGEFφ
  - Maintenance: AGφ

Summary

- We have extended our earlier planning algorithm from strong plans to strong cyclic plans.
- The story does not end there: When considering infinitely executing plans, many more types of goals are feasible.
- We considered maintenance as a simple example of a temporally extended goal.
- In general, temporally extended goals be expressed in modal logics such as computational tree logic (CTL).
- We presented dynamic programming (backward search) algorithms for strong cyclic and maintenance planning.
- In practice, one might implement both algorithms by using binary decision diagrams (BDDs) as a data structure for state sets.